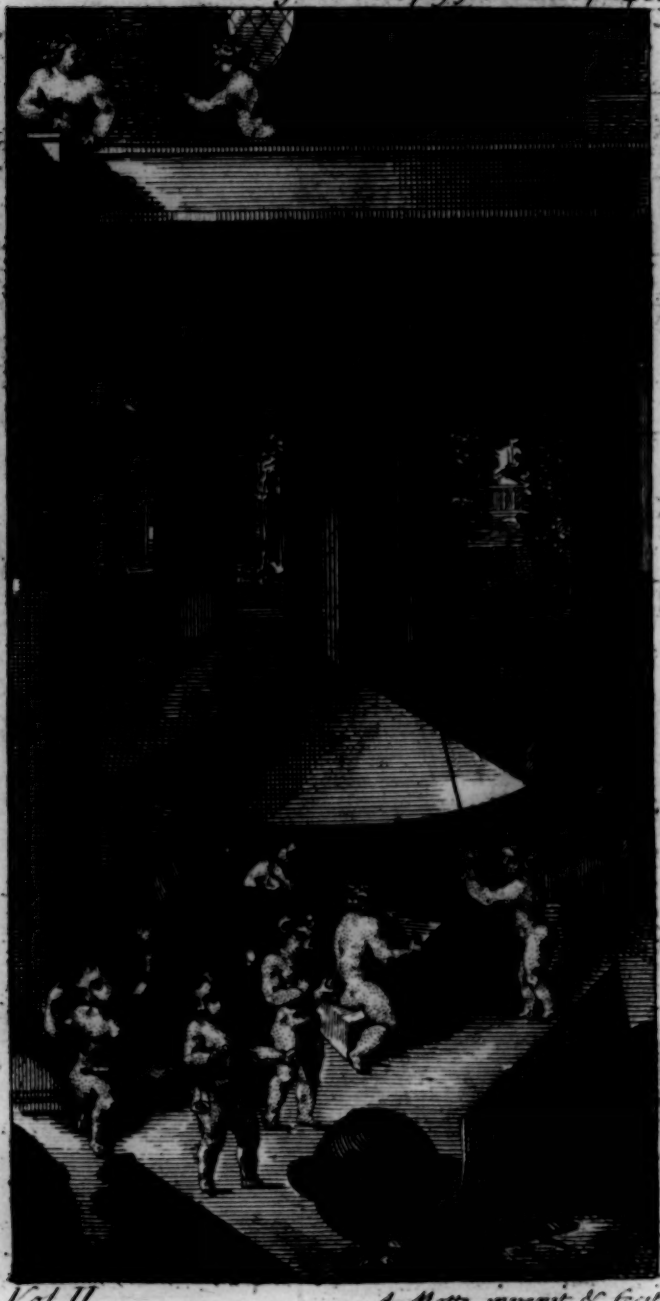
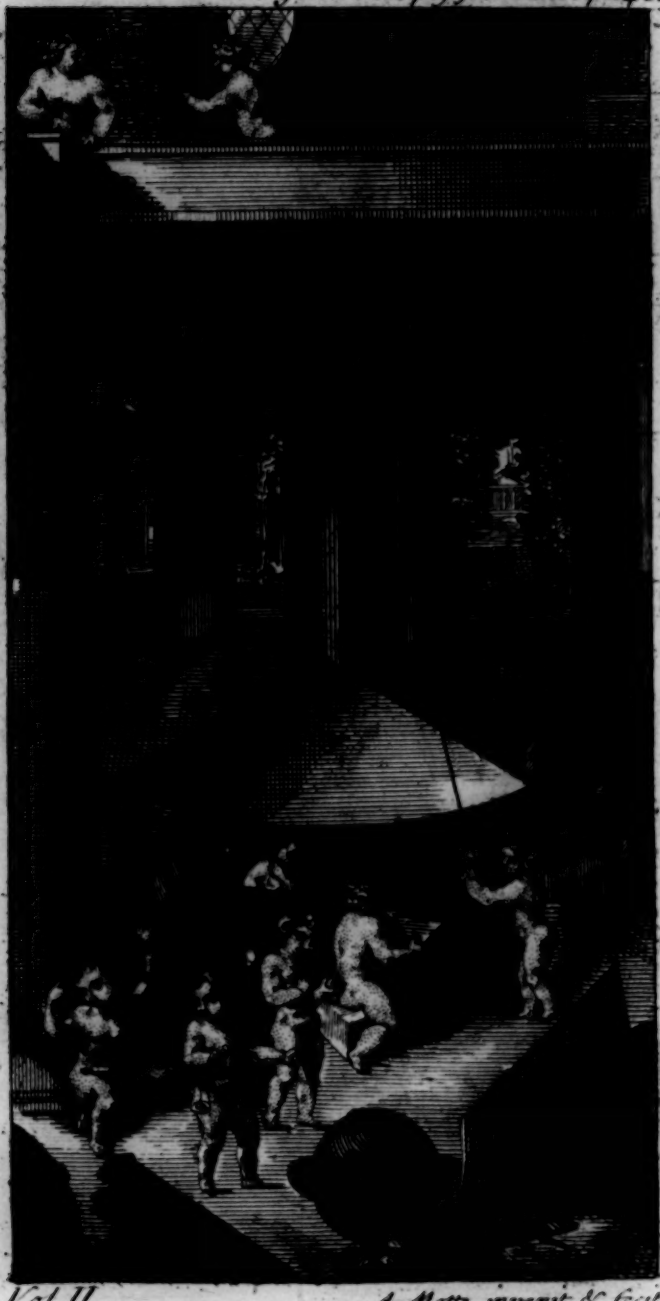
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By Sir *ISAAC NEWTON.*

Tranſlated into *ENGLISH.*

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A. M. f.

OF THE MOTION OF BODIES.

BOOK II.

SECTION I.

Of the Motion of Bodies that are reſiſted in the ratio of the Velocity.

PROPOSITION I. THEOREM I.

If a body is reſiſted in the ratio of its velocity, the motion loſt by reſiſtance is as the ſpace gone over in its motion.

FOR ſince the motion loſt in each equal particle of time is as the velocity, that is, as the particle of ſpace gone over; then, by compoſition, the motion loſt in the whole time will be as the whole ſpace gone over. *Q. E. D.*

Cor. Therefore if the body, deſtitute of all gravity, move by its innate force only in free ſpaces, and there be given both its whole motion at the beginning, and alſo the motion remaining after ſome part of the way is gone over; there will be given alſo the whole ſpace which the body can deſcribe in an infinite time. For that ſpace will be to the ſpace now deſcribed, as the whole motion at the beginning is to the part loſt of that motion.

Lemma I.

Quantities proportional to their differences are continually proportional.

Let A be to $A-B$ as B to $B-C$ and C to $C-D$, &c. and, by converſion, A will be to B as B to C and C to D, &c. *Q. E. D.*

Proposition II. Theorem II.

If a body is reſiſted in the ratio of its velocity, and moves, by its vis inſita *only, through a ſimilar medium, and the times be taken equal; the velocities in the beginning of each of the times are in a geometrical progreſſion, and the ſpaces deſcribed in each of the times are as the velocities.*

Case 1. Let the time be divided into equal particles; and if at the very beginning of each particle we ſuppoſe the reſiſtance to act with one ſingle impulſe which is as the velocity; the decrement of the velocity in each of the particles of time will be as the ſame velocity. Therefore the velocities are proportional to their differences, and therefore (by Lem. 1. Book 2.) con-

continually proportional. Therefore if out of an equal number of particles there be compounded any equal portions of time, the velocities at the beginning of thoſe times will be as terms in a continued progreſſion, which are taken by intervals, omitting every where an equal number of intermediate terms. But the ratio's of theſe terms are compounded of the equal ratio's of the intermediate terms equally repeated; and therefore are equal. Therefore the velocities, being proportional to thoſe terms, are in geometrical progreſſion. Let thoſe equal particles of time be diminiſhed, and their number increaſed *in infinitum*, ſo that the impulſe of reſiſtance may become continual; and the velocities at the beginnings of equal times, always continually proportional, will be alſo in this caſe continually proportional. *Q. E. D.*

Case 2. And, by diviſion, the differences of the velocities, that is, the parts of the velocities loſt in each of the times, are as the wholes: But the ſpaces deſcribed in each of the times are as the loſt parts of the velocities, (by Prop. 1. Book 2.) and therefore are alſo as the wholes. *Q. E. D.*

Corol. Hence if to the rectangular aſymptotes *AC*, *CH*, the Hyperbola *BG* is deſcribed, and *AB*, *DG* be drawn perpendicular to the aſymptote *AC*, and both the velocity of the body, and the reſiſtance of the medium, at the very beginning of the motion, be expreſs'd by any given line *AC*, and after ſome time is elapſed, by the indefinite line *DC*; the time may be expreſs'd by the area *ABGD*, and the ſpace deſcribed in that time by the line *AD*. For if that area, by the motion of the point *D*, be uniformly increaſed in the ſame manner as the time, the right line *DC* will decreaſe in a geometrical ratio in the ſame manner as the velocity, and the parts of the right line *AC*, deſcribed in equal times, will decreaſe in the ſame ratio.

Proposition III. Problem I.

To define the motion of a body which, in a ſimilar medium, aſcends or deſcends in a right line, and is reſiſted in the ratio of its velocity, and acted upon by an uniform force of gravity.

The body aſcending, let the gravity be expounded by any given rectangle *BACH*; and the reſiſtance of the medium, at the beginning of the aſcent, by the rectangle *BADE*, taken on the contrary ſide of the right line *AB*. Through the point *B*, with the rectangular aſymptotes *AC*, *CH*, deſcribe an Hyperbola, cutting the perpendiculars *DE*, *de*, in *G*, *g*; and the body aſcending will in the time *DGgd* deſcribe the ſpace *EGge*; in the time *DGBA*, the ſpace of the whole aſcent *EGB*; in the time *ABKI*, the ſpace of deſcent *BFK*; and in the time *IKki* the ſpace of deſcent *KFfk*; and the velocities of the bodies (proportional to the reſiſtance of the medium) in theſe periods of time, will be *ABED*, *ABed*, o, *ABFI*, *ABfi* reſpectively; and the greateſt velocity which the body can acquire by deſcending, will be *BACH*.

For let the rectangle *BACH* be reſolved into innumerable rectangles *Ak*, *Kl*, *Lm*, *Mn*, *&c.* which ſhall be as the increments of the velocities produced in ſo many equal times; then will o, *Ak*, *Al*, *Am*, *An*, &c. be as the whole velocities, and therefore (by ſuppoſition) as the reſiſtances of the medium in the beginning of each of the equal times. Make *AC* to *AK*, or *ABHC* to *ABkK* as the force of gravity to the reſiſtance in the beginning of the ſecond time; then from the force of gravity ſubduct the reſiſtances, and *ABHC*, *KkHC*, *LlHC*, *MmHC*, &c. will be as the abſolute forces with which the body is acted upon in the beginning of each of the times, and therefore

fore (by Law 2) as the increments of the velocities, that is, as the rectangles Ak, Kl, Lm, Mn, &c. and therefore (by Lem. 1. Book 2.) in a geometrical progreſſion. Therefore if the right lines Kk, Ll, Mm, Nn, &c. are produced ſo as to meet the Hyperbola in q, r, s, t, &c. the areas $ABqK$, $KqrL$, $LrsM$, $MstN$, &c. will be equal, and therefore analogous to the equal times and equal gravitating forces. But the area $ABqK$ (by Corol. 3. Lem. 7 & 8. Book 1.) is to the area Bkq as Kq to $\frac{1}{2}kq$, or AC to $\frac{1}{2}AK$, that is as the force of gravity to the reſiſtance in the middle of the firſt time. And by the like reaſoning the areas $qKLr$, $rLMs$, $sMNt$, &c. are to the areas $qklr$, $rlms$, $smnt$, &c. as the gravitating forces to the reſiſtances in the middle of the ſecond, third, fourth time, and ſo on. Therefore ſince the equal areas $BAKq$, $qKLr$, $rLMs$, $sMNt$, &c. are analogous to the gravitating forces, the areas Bkq, $qklr$, $rlms$, $smnt$, &c. will be analogous to the reſiſtances in the middle of each of the times, that is (by ſuppoſition) to the velocities, and ſo to the ſpaces deſcribed. Take the ſums of the analogous quantities, and the areas Bkq, Blr, Bms, Bnt, &c. will be analogous to the whole ſpaces deſcribed; and alſo the areas $ABqK$, $ABrL$, $ABsM$, $ABtN$, &c. to the times. Therefore the body, in deſcending, will in any time $ABrL$, deſcribe the ſpace Blr, and in the time $LrtN$ the ſpace $rlnt$. *Q. E. D.* And the like demonſtration holds in aſcending motion.

Corol. 1. Therefore the greateſt velocity that the body can acquire by falling, is to the velocity acquired in any given time, as the given force of gravity which perpetually acts upon it, to the reſiſting force which oppoſes it at the end of that time.

Corol. 2. But the time being augmented in an arithmetical progreſſion, the ſum of that greateſt velocity and the velocity in the aſcent, and alſo their difference in the deſcent, decreaſes in a geometrical progreſſion.

Corol. 3. Alſo the differences of the ſpaces, which are deſcribed in equal differences of the times, decreaſe in the ſame geometrical progreſſion.

Corol. 4. The ſpace deſcribed by the body is the difference of two ſpaces, whereof one is as the time taken from the beginning of the deſcent, and the other as the velocity; which [ſpaces] alſo at the beginning of the deſcent are equal among themſelves.

Proposition IV. Problem II.

Suppoſing the force of gravity in any ſimilar medium to be uniform, and to tend perpendicularly to the plane of the horizon; to define the motion of a projectile therein, which ſuffers reſiſtance proportional to its velocity.

Let the projectile go from any place D in the direction of any right line DP, and let its velocity at the beginning of the motion be expounded by the length DP. From the point P let fall the perpendicular PC on the horizontal line DC, and cut DC in A, ſo that DA may be to AC as the reſiſtance of the medium ariſing from its motion upwards at the beginning, to the force of gravity: or (which comes to the ſame) ſo that the rectangle under DA and DP may be to that under AC and CP, as the whole reſiſtance at the beginning of the motion to the force of gravity. With the aſymptotes DC, CP deſcribe any Hyperbola $GTBS$ cutting the perpendiculars DG, AB in G and B; compleat the parallelogram $DGKC$, and let its ſide GK cut AB in Q. Take a line N in the ſame ratio to QB as DC is in to CP; and from any point R of the right line DC, erect RT perpendicular to it, meeting the Hyperbola in T, and the right lines EH, GK, DP in I, t, and V; in that perpendicular

cular take Vr equal to $\frac{tGT}{N}$, or, which is the ſame thing, take Rr equal to $\frac{GTIE}{N}$; and the projectile in the time $DRTG$ will arrive at the point r, deſcribing the curve line $DraF$, the locus of the point r; thence it will come to its greateſt height a in the perpendicular AB; and afterwards ever approach to the aſymptote PC. And its velocity in any point r will be as the tangent rL to the curve. *Q. E. I.*

For N is to QB as DC to CP or DR to RV, and therefore RV is equal to $\frac{DR \times QB}{N}$, and Rr (that is, $RV - Vr$, or $\frac{DR \times QB - tGT}{N}$) is equal to $\frac{DR \times AB - RDGT}{N}$. Now let the time be expounded by the area $RDGT$, and (by Laws Cor. 2.) diſtinguiſh the motion of the body into two others, one of aſcent, the other lateral. And ſince the reſiſtance is as the motion, let that alſo be diſtinguiſhed into two parts proportional and contrary to the parts of the motion: and therefore the length deſcribed by the lateral motion, will be (by Prop. 2. Book 2.) as the line DR, and the height (by Prop. 3. Book 2.) as the area $DR \times AB - RDGT$, that is, as the line Rr. But in the very beginning of the motion the area $RDGT$ is equal to the rectangle $DR \times AQ$, and therefore that line Rr (or $\frac{DR \times AB - DR \times AQ}{N}$) will then be to DR as $AB - AQ$ or QB to N, that is, as CP to DC; and therefore as the motion upwards to the motion lengthwiſe at the beginning. Since therefore Rr is always as the height, and DR always as the length, and Rr is to DR at the beginning, as the height to the length: it

follows, that Rr is always to DR as the height to the length; and therefore that the body will move in the line $DraF$, which is the locus of the point r. *Q. E. D.*

Cor. 1. Therefore Rr is equal to $\frac{DR \times AB}{N} - \frac{RDGT}{N}$; and therefore if RT be produced to X, so that RX may be equal to $\frac{DR \times AB}{N}$, that is, if the parallelogram $ACPY$ be compleated, and DY cutting CP in Z be drawn, and RT be produced till it meets DY in X; Xr will be equal to $\frac{RDGT}{N}$, and therefore proportional to the time.

Cor. 2. Whence if innumerable lines CR, or, which is the same, innumerable lines ZX, be taken in a geometrical progression; there will be as many lines Xr in an arithmetical progression. And hence the curve $DraF$ is easily delineated by the Table of Logarithms.

Cor. 3. If a Parabola be constructed to the vertex D, and the diameter DG, produced downwards, and its latus rectum is to $2DP$ as the whole resistance at the beginning of the motion to the gravitating force: the velocity with which the body ought to go from the place D, in the direction of the right line DP, so as in an uniform resisting medium to describe the curve $DraF$, will be the same as that with which it ought to go from the same place D, in the direction of the same right line DP, so as to describe a Parabola in a non-resisting medium. For the latus rectum of this Parabola, at the very beginning of the motion, is $\frac{DV^2}{Vr}$; and Vr is $\frac{tGT}{N}$ or $\frac{DR \times Tt}{2N}$. But a right line,

line, which, if drawn, would touch the Hyperbola GTS in G, is parallel to DK, and therefore Tt is $\frac{CK \times DR}{DC}$, and N is $\frac{QB \times DC}{CP}$: And therefore Vr is equal to $\frac{DR^2 \times CK \times CP}{2DC^2 \times QB}$, that is, (becauſe DR and DC, DV and DP are proportionals) to $\frac{DV^2 \times CK \times CP}{2DP^2 \times QB}$; and the latus rectum $\frac{DV^2}{Vr}$ comes out $\frac{2DP^2 \times QB}{CK \times CP}$, that is, (becauſe QB and CK, DA and AC are proportional) $\frac{2DP^2 \times DA}{AC \times CP}$, and therefore is to $2DP$, as $DP \times DA$ to $CP \times AC$; that is, as the reſiſtance to the gravity. *Q. E. D.*

Cor. 4. Hence if a body be projected from any place D, with a given velocity, in the direction of a right line DP given by poſition; and the reſiſtance of the medium, at the beginning of the motion, be given: the curve $DraF$, which that body will deſcribe, may be found. For the velocity being given, the latus rectum of the parabola is given, as is well known. And taking $2DP$ to that latus rectum, as the force of gravity to the reſiſting force, DP is alſo given. Then cutting DC in A, ſo that $CP \times AC$ may be to $DP \times DA$ in the ſame ratio of the gravity to the reſiſtance, the point A will be given. And hence the curve $DraF$ is alſo given.

Cor. 5. And on the contrary, if the curve $DraF$ be given, there will be given both the velocity of the body, and the reſiſtance of the medium in each of the places r. For the ratio of $CP \times AC$ to $DP \times DA$ being given, there is given both the reſiſtance of the medium at the beginning of the motion, and the latus rectum of the parabola; and thence the velocity at the begin-

beginning of the motion is given alſo. Then from the length of the tangent rL, there is given both the velocity proportional to it, and the reſiſtance proportional to the velocity in any place r.

Cor. 6. But ſince the length $2DP$ is to the latus rectum of the parabola as the gravity to the reſiſtance in D; and, from the velocity augmented, the reſiſtance is augmented in the ſame ratio, but the latus rectum of the parabola is augmented in the duplicate of that ratio; it is plain that the length $2DP$ is augmented in that ſimple ratio only; and is therefore always proportional to the velocity; nor will it be augmented or diminiſhed by the change of the angle CDP, unleſs the velocity be alſo changed.

Cor. 7. Hence appears the method of determining the curve $DraF$, nearly, from the phænomena, and thence collecting the reſiſtance and velocity with which the body is projected. Let two ſimilar and equal bodies be projected with the ſame velocity, from the place D, in different angles CDP, CDp; and let the places F, f, where they fall upon the horizontal plane DC, be known. Then taking any length for DP or Dp, ſuppoſe the reſiſtance in D to be to the gravity in any ratio whatſoever, and let that ratio be expounded by any length SM. Then by computation, from that aſſumed length DP, find the lengths DF, Df; and from the ratio $\frac{Ff}{DF}$, found by calculation, ſubduct the ſame ratio as found by experiment; and let the difference be expounded by the perpendicular MN. Repeat the ſame a ſecond and a third time, by aſſuming always a new ratio SM of the reſiſtance to the gravity, and collecting a new difference MN. * Draw the affirmative differences on one ſide of the right line SM, and the negative on the other ſide; and through the points N, N, N draw a regular curve NNN, cutting the

the right line *SMMM* in *X*, and *SX* will be the true ratio of the reſiſtance to the gravity, which was to be found. From this ratio the length *DF* is to be collected by calculation; and a length, which is to the aſſumed length *DP*, as the length *DF* known by experiment to the length *DF* juſt now found, will be the true length *DP*. This being known, you will have both the curve line *DraF* which the body deſcribes, and alſo the velocity and reſiſtance of the body in each place.

Scholium.

But yet that the reſiſtance of bodies is in the ratio of the velocity, is more a mathematical hypotheſis than a phyſical one. In mediums void of all tenacity, the reſiſtances made to bodies are in the duplicate ratio of the velocities. For by the action of a ſwifter body, a greater motion, in proportion to a greater velocity, is communicated to the ſame quantity of the medium, in a leſs time; and in an equal time, by reaſon of a greater quantity of the diſturbed medium, a motion is communicated in the duplicate ratio greater; and the reſiſtance (by Law 2 and 3.) is as the motion communicated. Let us therefore ſee what motions ariſe from this law of reſiſtance.

SEC-

SECTION II.

Of the Motion of Bodies that are reſiſted in the duplicate ratio of their Velocities.

PROPOSITION V. THEOREM III.

If a body is reſiſted in the duplicate ratio of its velocity, and moves by its innate force only through a ſimilar medium; and the times be taken in a geometrical progreſſion, proceeding from leſs to greater terms: I ſay that the velocities at the beginning of each of the times are in the ſame geometrical progreſſion inverſely; and that the ſpaces are equal, which are deſcribed in each of the times.

For ſince the reſiſtance of the medium is proportional to the ſquare of the velocity, and the decrement of the velocity is proportional to the reſiſtance; if the time be divided into innumerable equal particles, the ſquares of the velocities at the beginning of each of the times will be proportional to the differences of the ſame velocities. Let thoſe particles of time be *AK*, *KL*, *LM*, &c. taken in the right line *CD*; and erect the perpendiculars *AB*, *Kk*, *Ll*, *Mm*, &c. meeting the Hyperbola *Bklm G*, deſcribed with the centre *C*, and the rectangular aſymptotes *CD*, *CH*, in *B*, *k*, *l*, *m*, &c. then *AB* will be to *Kk*, as

as CK to CA, and, by diviſion, $AB - Kk$ to Kk as AK to CA, and, alternately, $AB - Kk$ to AK as Kk to CA, and therefore as $AB \times Kk$ to $AB \times CA$. Therefore ſince AK and $AB \times CA$ are given, $AB - Kk$ will be as $AB \times Kk$; and laſtly, when AB and Kk coincide, as AB^2. And, by the like reaſoning, $Kk - Ll$, $Ll - Mm$, &c. will be as Kk^2, Ll^2, &c. Therefore the ſquares of the lines AB, Kk, Ll, Mm, &c. are as their differences; and therefore, ſince the ſquares of the velocities were ſhewn above to be as their differences, the progreſſion of both will be alike. This being demonſtrated, it follows alſo that the areas deſcribed by theſe lines are in a like progreſſion with the ſpaces deſcribed by theſe velocities. Therefore if the velocity at the beginning of the firſt time AK be expounded by the line AB, and the velocity at the beginning of the ſecond time KL by the line Kk, and the length deſcribed in the firſt time by the area $AKkB$; all the following velocities will be expounded by the following lines Ll, Mm, &c. and the lengths deſcribed, by the areas Kl, Lm, &c. And, by compoſition, if the whole time be expounded by AM, the ſum of its parts, the whole length deſcribed will be expounded by $AMmB$ the ſum of its parts. Now conceive the time AM to be divided into the parts AK, KL, LM, &c. ſo that CA, CK, CL, CM, &c. may be in a geometrical progreſſion; and thoſe parts will be in the ſame progreſſion, and the velocities AB, Kk, Ll, Mm, &c. will be in the ſame progreſſion inverſly, and the ſpaces deſcribed Ak, Kl, Lm, &c. will be equal. *Q. E. D.*

Cor. 1. Hence it appears, that if the time be expounded by any part AD of the aſymptote, and the velocity in the beginning of the time by the ordinate AB; the velocity at the end of the time will be expounded by the ordinate DG; and the whole ſpace deſcribed, by the adjacent hyperbolic area $ABGD$;

and

and the ſpace which any body can deſcribe in the ſame time AD, with the firſt velocity AB, in a non-reſiſting medium, by the rectangle $AB \times AD$.

Cor. 2. Hence the ſpace deſcribed in a reſiſting medium is given, by taking it to the ſpace deſcribed with the uniform velocity AB in a non-reſiſting medium, as the hyperbolic area $ABGD$ to the rectangle $AB \times AD$.

Cor. 3. The reſiſtance of the medium is alſo given, by making it equal, in the very beginning of the motion, to an uniform centripetal force, which could generate, in a body falling thro' a non-reſiſting medium, the velocity AB, in the time AC. For if BT be drawn touching the hyperbola in B, and meeting the aſymptote in T; the right line AT will be equal to AC, and will expreſs the time, in which the firſt reſiſtance uniformly continued, may take away the whole velocity AB.

Cor. 4. And thence is alſo given the proportion of this reſiſtance to the force of gravity, or any other given centripetal force.

Cor. 5. And *vice verſa*, if there is given the proportion of the reſiſtance to any given centripetal force; the time AC is alſo given, in which a centripetal force equal to the reſiſtance may generate any velocity as AB; and thence is given the point B, through which the hyperbola, having CH, CD for its aſymptotes, is to be deſcribed; as alſo the ſpace $ABGD$, which a body, by beginning its motion with that velocity AB, can deſcribe in any time AD, in a ſimilar reſiſting medium.

PRO-

Proposition VI. Theorem IV.

Homogeneous and equal ſpherical bodies, oppos'd by reſiſtances that are in the duplicate ratio of the velocities, and moving on by their innate force only, will, in times which are reciprocally as the velocities at the beginning, deſcribe equal ſpaces, and loſe parts of their velocities proportional to the wholes.

To the rectangular aſymptotes *CD*, *CH* deſcribe any hyperbola *BbEe*, cutting the perpendiculars *AB*, *ab*, *DE*, *de*, in *B*, *b*, *E*, *e*; let the initial velocities be expounded by the perpendiculars *AB*, *DE*, and the times by the lines *Aa*, *Dd*. Therefore as *Aa* is to *Dd*, ſo (by the hypotheſis) is *DE* to *AB*, and ſo (from the nature of the hyperbola) is *CA* to *CD*; and, by compoſition, ſo is *Ca* to *Cd*. Therefore the areas *ABba*, *DEed*, that is, the ſpaces deſcribed, are equal among themſelves, and the firſt velocities *AB*, *DE* are proportional to the laſt *ab*, *de*; and therefore, by diviſion, proportional to the parts of the velocities loſt, *AB* — *ab*, *DE* — *de*. *Q.E.D.*

Proposition VII. Theorem V.

If ſpherical bodies are reſiſted in the duplicate ratio of their velocities, in times which are as the firſt motions directly and the firſt reſiſtances inverſely, they will loſe parts of their motions proportional to the wholes, and will deſcribe ſpaces proportional to thoſe times and the firſt velocities conjunctly.

For the parts of the motions loſt are as the reſiſtances and times conjunctly. Therefore, that thoſe parts may be

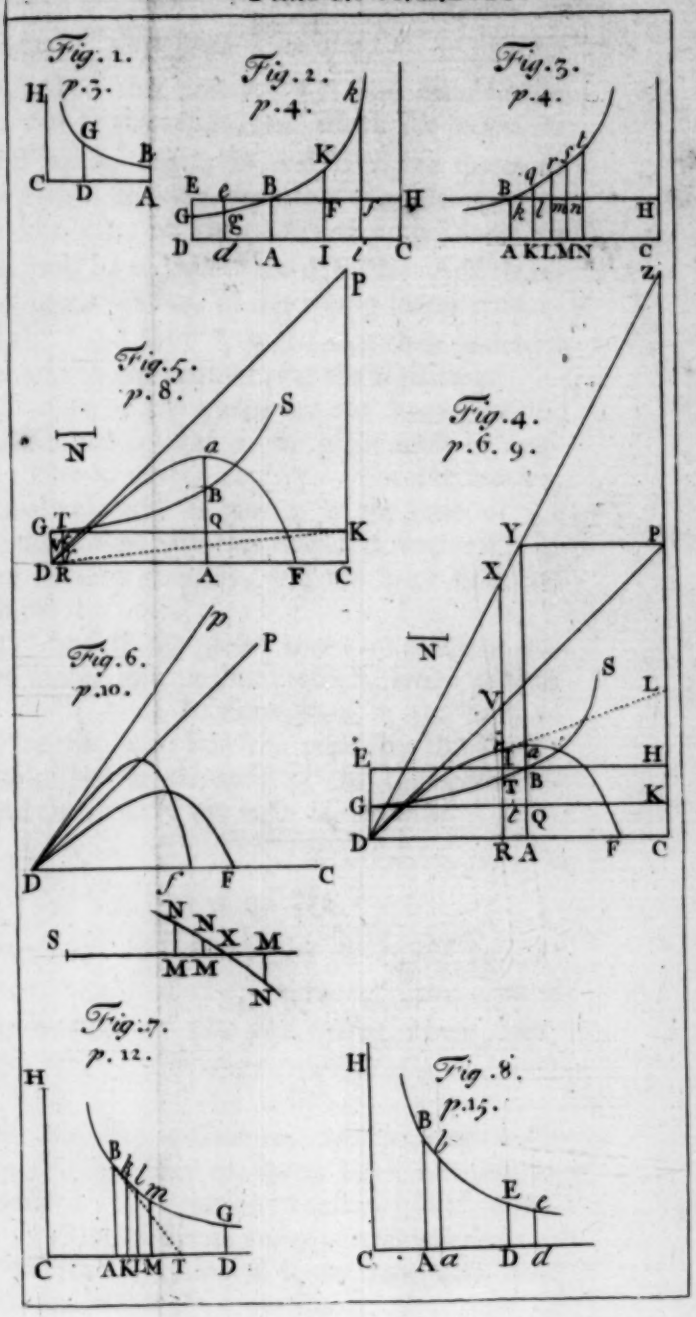
be proportional to the wholes, the reſiſtance and time conjunctly ought to be as the motion. Therefore the time will be as the motion directly and the reſiſtance inverſely. Wherefore the particles of the times being taken in that ratio, the bodies will always loſe parts of their motions proportional to the wholes, and therefore will retain velocities always proportional to their firſt velocities. And becauſe of the given ratio of the velocities, they will always deſcribe ſpaces, which are as the firſt velocities and the times conjunctly. *Q. E. D.*

Cor. 1. Therefore if bodies equally ſwift are reſiſted in a duplicate ratio of their diameters: Homogeneous globes moving with any velocities whatſoever, by deſcribing ſpaces proportional to their diameters, will loſe parts of their motions proportional to the wholes. For the motion of each globe will be as its velocity and maſs conjunctly, that is, as the velocity and the cube of its diameter; the reſiſtance (by ſuppoſition) will be as the ſquare of the diameter and the ſquare of the velocity conjunctly; and the time (by this propoſition) is in the former ratio directly and in the latter inverſely, that is, as the diameter directly and the velocity inverſely; and therefore the ſpace, which is proportional to the time and velocity, is as the diameter.

Cor. 2. If bodies equally ſwift are reſiſted in a ſeſquiplicate ratio of their diameters: Homogeneous globes, moving with any velocities whatſoever, by deſcribing ſpaces that are in a ſeſquiplicate ratio of the diameters, will loſe parts of their motions proportional to the wholes.

Cor. 3. And univerſally, if equally ſwift bodies are reſiſted in the ratio of any power of the diameters: the ſpaces, in which homogeneous globes, moving with any velocity whatſoever, will loſe parts of their motions proportional to the wholes, will be as the cubes of the diameters applied to that power. Let thoſe diameters

Plate I. Vol. II. P. 16.



ameters applied to that power. Let thoſe diameters be D and E; and if the reſiſtances, where the velocities are ſuppoſed equal, are as D^n and E^n: the ſpaces in which the globes, moving with any velocities whatſoever, will loſe parts of their motions proportional to the wholes, will be as D^{3-n} and E^{3-n}. And therefore homogeneous globes, in deſcribing ſpaces proportional to D^{3-n} and E^{3-n}, will retain their velocities in the ſame ratio to one another as at the beginning.

Cor. 4. Now if the globes are not homogeneous, the ſpace deſcribed by the denſer globe muſt be augmented in the ratio of the denſity. For the motion, with an equal velocity, is greater in the ratio of the denſity, and the time (by this Prop.) is augmented in the ratio of motion directly, and the ſpace deſcribed in the ratio of the time.

Cor. 5. And if the globes move in different mediums, the ſpace, in a medium which, *ceteris paribus*, reſiſts the moſt, muſt be diminiſhed in the ratio of the greater reſiſtance. For the time (by this Prop.) will be diminiſhed in the ratio of the augmented reſiſtance, and the ſpace in the ratio of the time.

Lemma II.

The moment of any Genitum *is equal to the moments of each of the generating ſides drawn into the indices of the powers of thoſe ſides, and into their coefficients continually.*

I call any quantity a *Genitum*, which is not made by addition or ſubduction of divers parts, but is generated or produced in arithmetic by the multiplication, diviſion, or extraction of the root of any terms whatſoever; in geometry by the invention of contents and ſides, or of the extreams and means of proportionals. Quantities of

this kind are products, quotients, roots, rectangles, squares, cubes, square and cubic sides, and the like. These quantities I here consider as variable and indetermined, and increasing or decreasing as it were by a perpetual motion or flux; and I understand their momentaneous increments or decrements by the name of Moments; so that the increments may be esteem'd as added, or affirmative moments; and the decrements as subducted, or negative ones. But take care not to look upon finite particles as such. Finite particles are not moments, but the very quantities generated by the moments. We are to conceive them as the just nascent principles of finite magnitudes. Nor do we in this Lemma regard the magnitude of the moments, but their first proportion as nascent. It will be the same thing, if, instead of moments, we use either the Velocities of the increments and decrements (which may also be called the motions, mutations, and fluxions of quantities) or any finite quantities proportional to those velocities. The coefficient of any generating side is the quantity which arises by applying the Genitum to that side.

Wherefore the sense of the Lemma is, that if the moments of any quantities A, B, C, &c. increasing or decreasing by a perpetual flux, or the velocities of the mutations which are proportional to them, be called a, b, c, &c. the moment or mutation of the generated rectangle AB will be $aB + bA$; the moment of the generated content ABC will be $aBC + bAC + cAB$: and the moments of the generated powers, A^2, A^3, A^4, $A^{\frac{4}{2}}$, $A^{\frac{1}{2}}$, $A^{\frac{1}{3}}$, $A^{\frac{2}{3}}$, A^{-1}, A^{-2}, $A^{-\frac{1}{2}}$ will be $2aA$, $3aA^2$, $4aA^3$, $\frac{1}{2}aA^{-\frac{1}{2}}$, $\frac{1}{2}aA^{\frac{1}{2}}$, $\frac{1}{3}aA^{-\frac{2}{3}}$, $\frac{2}{3}aA^{-\frac{1}{3}}$, $-aA^{-2}$, $-2aA^{-3}$, $-\frac{1}{2}aA^{-\frac{3}{2}}$ respectively. And in general, that the moment of any power $A^{\frac{n}{m}}$ will be $\frac{n}{m}aA^{\frac{n-m}{m}}$. Also that the moment

of the generated quantity A^2B will be $2aAB + bA^2$; the moment of the generated quantity $A^3B^4C^2$ will be $3aA^2B^4C^2 + 4bA^3B^3C^2 + 2cA^3B^4C$; and the moment of the generated quantity $\frac{A^3}{B^2}$ or A^3B^{-2} will be $3aA^2B^{-3} - 2bA^3B^{-3}$; and so on. The Lemma is thus demonstrated.

Case 1. Any rectangle as AB augmented by a perpetual flux, when, as yet, there wanted of the sides A and B half their moments $\frac{1}{2}a$ and $\frac{1}{2}b$, was $A - \frac{1}{2}a$ into $B - \frac{1}{2}b$, or $AB - \frac{1}{2}aB - \frac{1}{2}bA + \frac{1}{4}ab$; but as soon as the sides A and B are augmented by the other half moments; the rectangle becomes $A + \frac{1}{2}a$ into $B + \frac{1}{2}b$ or $AB + \frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab$. From this rectangle subduct the former rectangle, and there will remain the excess $aB + bA$. Therefore with the whole increments a and b of the sides, the increment $aB + bA$ of the rectangle is generated. *Q. E. D.*

Case 2. Suppose AB always equal to G, and then the moment of the content ABC or GC (by Case 1.) will be $gC + cG$, that is, (putting AB and $aB + bA$ for G and g) $aBC + bAC + cAB$. And the reasoning is the same for contents under never so many sides. *Q. E. D.*

Case 3. Suppose the sides A, B, and C, to be always equal among themselves; and the moment $aB + bA$, of A^2, that is, of the rectangle AB, will be $2aA$; and the moment $aBC + bAC + cAB$ of A^3, that is, of the content ABC, will be $3aA^2$. And by the same reasoning the moment of any power A^n is naA^{n-1}. *Q. E. D.*

Case 4. Therefore since $\frac{1}{A}$ into A is 1, the moment of $\frac{1}{A}$ drawn into A, together with $\frac{1}{A}$ drawn into a, will be the moment of 1, that is, nothing. Therefore

fore the moment of $\frac{1}{A}$ or of A^{-1} is $\frac{-a}{A^2}$. And generally, since $\frac{1}{A^n}$ into A^n is 1, the moment of $\frac{1}{A^n}$ drawn into A^n together with $\frac{1}{A^n}$ into naA^{n-1} will be nothing. And therefore the moment of $\frac{1}{A^n}$ or A^{-n} will be $-\frac{na}{A^{n+1}}$. *Q. E. D.*

Case 5. And since $A^{\frac{1}{2}}$ into $A^{\frac{1}{2}}$ is A, the moment of $A^{\frac{1}{2}}$ drawn into $2A^{\frac{1}{2}}$ will be a, (by Case 3:) and therefore the moment of $A^{\frac{1}{2}}$ will be $\frac{a}{2A^{\frac{1}{2}}}$ or $\frac{1}{2}aA^{-\frac{1}{2}}$. And generally, putting $A^{\frac{m}{n}}$ equal to B, then A^m will be equal to B^n, and therefore maA^{m-1} equal to nbB^{n-1}, and maA^{-1} equal to nbB^{-1} or $nbA^{-\frac{m}{n}}$; and therefore $\frac{m}{n}aA^{\frac{m-n}{n}}$ is equal to b, that is, equal to the moment of $A^{\frac{m}{n}}$. *Q. E. D.*

Case 6. Therefore the moment of any generated quantity A^mB^n is the moment of A^m drawn into B^n, together with the moment of B^n drawn into A^m, that is, $maA^{m-1}B^n + nbB^{n-1}A^m$; and that whether the indices m and n of the powers be whole numbers or fractions, affirmative or negative. And the reasoning is the same for contents under more powers. *Q. E. D.*

Cor. 1. Hence in quantities continually proportional, if one term is given, the moments of the rest of the terms will be as the same terms multiplied by

the number of intervals between them and the given term. Let A, B, C, D, E, F, be continually proportional; then if the term C is given, the moments of the reſt of the terms will be among themſelves, as —2A, —B, D, 2E, 3F.

Cor. 2. And if in four proportionals the two means are given, the moments of the extremes will be as thoſe extremes. The ſame is to be underſtood of the ſides of any given rectangle.

Cor. 3. And if the ſum or difference of two ſquares is given, the moments of the ſides will be reciprocally as the ſides.

Scholium.

In a letter of mine to Mr. *J. Collins,* dated *December* 10. 1672. having deſcribed a method of Tangents, which I ſuſpected to be the ſame with *Sluſius*'s method, which at that time was not made publick; I ſubjoined theſe words; *This is one particular, or rather a corollary, of a general method, which extends itſelf, without any troubleſome calculation, not only to the drawing of Tangents to any Curve lines, whether Geometrical or Mechanical, or any how reſpecting right lines or other Curves, but alſo to the reſolving other abſtruſer kinds of Problems about the crookedneſs, areas, lengths, centres of gravity of Curves,* &c. *nor is it (as* Hudden's *method* de Maximis & Minimis) *limited to equations which are free from ſurd quantities. This method I have interwoven with that other of working in equations, by reducing them to infinite ſeries.* So far that letter. And theſe laſt words relate to a Treatiſe I compoſed on that ſubject in the year 1671. The foundation of that general method is contained in the preceding Lemma.

Proposition VIII. Theorem VI.

If a body in an uniform medium, being uniformly acted upon by the force of gravity, aſcends or deſcends in a right line; and the whole ſpace deſcribed be diſtinguiſhed into equal parts, and in the beginning of each of the parts, (by adding or ſubducting the reſiſting force of the medium to or from the force of gravity, when the body aſcends or deſcends) you collect the abſolute forces; I ſay that thoſe abſolute forces are in a geometrical progreſſion. Pl. 2. Fig. 1.

For let the force of gravity be expounded by the given line AC; the force of reſiſtance by the indefinite line AK; the abſolute force in the deſcent of the body, by the difference KC; the velocity of the body by a line AP, which ſhall be a mean proportional between AK and AC, and therefore in a ſubduplicate ratio of the reſiſtance; the increment of the reſiſtance made in a given particle of time by the lineola KL, and the contemporaneous increment of the velocity by the lineola PQ; and with the centre C, and rectangular aſymptotes CA, CH, deſcribe any Hyperbola BNS, meeting the erected perpendiculars AB, KN, LO in B, N, and O. Becauſe AK is as AP^2, the moment KL of the one will be as the moment $2APQ$ of the other, that is, as $AP \times KC$; for the increment PQ of the velocity is (by Law 2.) proportional to the generating force KC. Let the ratio of KL be compounded with the ratio of KN, and the rectangle $KL \times KN$ will become as $AP \times KC \times KN$; that is, (becauſe the rectangle $KC \times KN$ is given) as AP. But the ultimate ratio of the hyperbolic area $KNOL$ to the rectangle $KL \times KN$ becomes, when the points K and L coincide, the ratio of

of equality. Therefore that hyperbolic evaneſcent area is as *AP*. Therefore the whole hyperbolic area *ABOL* is compoſed of particles *KNOL* which are always proportional to the velocity *AP*; and therefore is itſelf proportional to the ſpace deſcribed with that velocity. Let that area be now divided into equal parts, as *ABMI*, *IMNK*, *KNOL*, &c. and the abſolute forces *AC*, *IC*, *KC*, *LC*, &c. will be in a geometrical progreſſion. *Q.E.D.* And by a like reaſoning, in the aſcent of the body, taking, on the contrary ſide of the point *A*, the equal areas *ABmi*, *imnk*, *knol*, &c. it will appear that the abſolute forces *AC*, *iC*, *kC*, *lC*, &c. are continually proportional. Therefore if all the ſpaces in the aſcent and deſcent are taken equal; all the abſolute forces *lC*, *kC*, *iC*, *AC*, *IC*, *KC*, *LC*, &c. will be continually proportional. *Q.E.D.*

Cor. 1. Hence if the ſpace deſcribed be expounded by the hyperbolic area *ABNK*; the force of gravity, the velocity of the body, and the reſiſtance of the medium, may be expounded by the lines *AC*, *AP*, and *AK* reſpectively; and *vice verſa*.

Cor. 2. And the greateſt velocity, which the body can ever acquire in an infinite deſcent, will be expounded by the line *AC*.

Cor. 3. Therefore if the reſiſtance of the medium anſwering to any given velocity be known, the greateſt velocity will be found, by taking it to that given velocity in a ratio ſubduplicate of the ratio which the force of gravity bears to that known reſiſtance of the medium.

Proposition IX. Theorem VII.

Suppoſing what is above demonſtrated, I ſay that if the tangents of the angles of the ſector of a circle, and of an hyperbola, be taken proportional to the velocities, the radius being of a fit magnitude; all the time of the aſcent to the higheſt place will be as the ſector of the circle, and all the time of deſcending from the higheſt place as the ſector of the hyperbola. Pl. 2. Fig. 2.

To the right line AC, which expreſſes the force of gravity, let AD be drawn perpendicular and equal. From the centre D with the ſemidiameter AD deſcribe as well the quadrant AtE of a Circle; as the rectangular Hyperbola AVZ, whoſe axe is AX, principal vertex A, and aſymptote DC. Let Dp, DP be drawn; and the circular ſector AtD will be as all the time of the aſcent to the higheſt place; and the hyperbolic ſector ATD as all the time of deſcent from the higheſt place: If ſo be that the tangents Ap, AP of thoſe ſectors be as the velocities. *Fig.* 2.

Case 1. Draw Dvq cutting off the moments or leaſt particles tDv and qDp, deſcribed in the ſame time, of the ſector ADt and of the triangle ADp. Since thoſe particles (becauſe of the common angle D) are in a duplicate ratio of the ſides, the particle tDv will be as $\frac{qDp \times tD^2}{pD^2}$, that is, (becauſe tD is given) as $\frac{qDp}{pD^2}$. But pD^2 is $AD^2 + Ap^2$, that is, $AD^2 + AD \times Ak$, or $AD \times Ck$; and qDp is $\frac{1}{2} AD \times pq$. Therefore tDv, the particle of the ſector, is as $\frac{pq}{Ck}$; that is,

is, as the leaſt decrement pq of the velocity directly, and the force Ck, which diminiſhes the velocity, inverſely; and therefore as the particle of time anſwering to the decrement of the velocity. And, by compoſition, the ſum of all the particles tDv in the ſector ADt, will be as the ſum of the particles of time anſwering to each of the loſt particles pq, of the decreaſing velocity Ap, till that velocity, being diminiſhed into nothing, vaniſhes; that is, the whole ſector ADt is as the whole time of aſcent to the higheſt place. *Q. E. D.*

Case 2. Draw DQV cutting off the leaſt particles TDV and PDQ of the ſector DAV, and of the triangle DAQ; and theſe particles will be to each other as DT^2 to DP^2, that is, (if TX and AP are parallel) as DX^2 to DA^2 or TX^2 to AP^2; and, by diviſion, as $DX^2 - TX^2$ to $DA^2 - AP^2$. But, from the nature of the hyperbola, $DX^2 - TX^2$ is AD^2; and, by the ſuppoſition, AP^2 is $AD \times AK$. Therefore the particles are to each other as AD^2 to $AD^2 - AD \times AK$; that is, as AD to $AD - AK$ or AC to CK: and therefore the particle TDV of the ſector is $\frac{PDQ \times AC}{CK}$; and therefore (becauſe AC and AD are given) as $\frac{PQ}{CK}$; that is, as the increment of the velocity directly, and as the force generating the increment inverſely; and therefore as the particle of the time anſwering to the increment. And, by compoſition, the ſum of the particles of time, in which all the particles PQ of the velocity AP are generated, will be as the ſum of the particles of the ſector ATD; that is, the whole time will be as the whole ſector. *Q. E. D.*

Cor. 1. Hence if AB be equal to a fourth part of AC, the ſpace which a body will deſcribe by falling in any time will be to the ſpace which the body could deſcribe, by moving uniformly on in the ſame time with

with its greateſt velocity AC, as the area $ABNK$, which expreſſes the ſpace deſcribed in falling, to the area ATD, which expreſſes the time. For ſince AC is to AP as AP to AK, then (by Cor. 1. Lem. 2. of this Book) LK is to PQ as $2AK$ to AP, that is, as $2AP$ to AC, and thence LK is to $\frac{1}{2}PQ$ as AP to $\frac{1}{2}AC$ or AB; and KN is to AC or AD as AB to CK; and therefore, *ex æquo*, $LKNO$ to DPQ as AP to CK. But DPQ was to DTV as CK to AC. Therefore, *ex æquo*, $LKNO$ is to DTV as AP to AC; that is, as the velocity of the falling body to the greateſt velocity which the body by falling can acquire. Since therefore the moments $LKNO$ and DTV of the areas $ABNK$ and ATD are as the velocities, all the parts of thoſe areas generated in the ſame time, will be as the ſpaces deſcribed in the ſame time; and therefore the whole areas $ABNK$ and ADT generated from the beginning, will be as the whole ſpaces deſcribed from the beginning of the deſcent. *Q. E. D.*

Cor. 2. The ſame is true alſo of the ſpace deſcribed in the aſcent. That is to ſay, that all that ſpace is to the ſpace deſcribed in the ſame time with the uniform velocity AC, as the area $ABnk$ is to the ſector ADt.

Cor. 3. The velocity of the body, falling in the time ATD, is to the velocity which it would acquire in the ſame time in a non-reſiſting ſpace, as the triangle APD to the hyperbolic ſector ATD. For the velocity in a non-reſiſting medium would be as the time ATD, and in a reſiſting medium is as AP, that is, as the triangle APD. And thoſe velocities at the beginning of the deſcent, are equal among themſelves, as well as thoſe areas ATD, APD.

Cor. 4. By the ſame argument, the velocity in the aſcent is to the velocity with which the body in the ſame time, in a non-reſiſting ſpace, would loſe all

all its motion of aſcent, as the triangle *ApD* to the circular ſector *AtD*; or as the right line *Ap* to the arc *At*.

Cor. 5. Therefore the time in which a body by falling in a reſiſting medium, would acquire the velocity *AP*, is to the time in which it would acquire its greateſt velocity *AC* by falling in a non-reſiſting ſpace, as the ſector *ADT* to the triangle *ADC*: and the time in which it would loſe its velocity *Ap* by aſcending in a reſiſting medium, is to the time in which it would loſe the ſame velocity by aſcending in a non-reſiſting ſpace, as the arc *At* to its tangent *Ap*.

Cor. 6. Hence from the given time there is given the ſpace deſcribed in the aſcent or deſcent. For the greateſt velocity of a body deſcending *in infinitum* is given (by Corol. 2 and 3. Theor. 6. of this Book) and thence the time is given in which a body would acquire that velocity by falling in a non-reſiſting ſpace. And taking the ſector *ADT* or *ADt* to the triangle *ADC* in the ratio of the given time to the time juſt now found; there will be given both the velocity *AP* or *Ap*, and the area *ABNK* or *ABnk*, which is to the ſector *ADT*, or *ADt*, as the ſpace ſought to the ſpace which would, in the given time, be uniformly deſcribed with that greateſt velocity found juſt before.

Cor. 7. And by going backward, from the given ſpace of aſcent or deſcent *ABnk* or *ABNK*, there will be given the time *ADt* or *ADT*.

Pro-

Proposition X. Problem III.

Suppose the uniform force of gravity to tend directly to the plane of the horizon, and the resistance to be as the density of the medium and the square of the velocity conjunctly: it is proposed to find the density of the medium in each place, which shall make the body move in any given curve line; the velocity of the body, and the resistance of the medium in each place. Pl. 2. Fig. 3.

Let PQ be a plane perpendicular to the plane of the scheme itself; $PFHQ$ a curve line meeting that plane in the points P and Q; G, H, I, K four places of the body going on in this curve from F to Q; and GB, HC, ID, KE four parallel ordinates let fall from these points to the horizon, and standing on the horizontal line PQ at the points B, C, D, E; and let the distances BC, CD, DE, of the ordinates be equal among themselves. From the points G and H let the right lines GL, HN, be drawn touching the curve in G and H, and meeting the ordinates CH, DI, produced upwards, in L and N; and compleat the parallelogram $HCDM$. And the times, in which the body describes the arcs GH, HI, will be in a subduplicate ratio of the altitudes LH, NI, which the bodies would describe in those times, by falling from the tangents; and the velocities will be as the lengths described GH, HI directly and the times inversely. Let the times be expounded by T and t, and the velocities by $\frac{GH}{T}$ and $\frac{HI}{t}$; and the decrement of the velocity pro-

produced in the time t will be expounded by $\frac{GH}{T} - \frac{HI}{t}$. This decrement ariſes from the reſiſtance which retards the body, and from the gravity which accelerates it. Gravity, in a falling body, which in its fall deſcribes the ſpace NI, produces a velocity, with which it would be able to deſcribe twice that ſpace in the ſame time, as *Galileo* has demonſtrated; that is, the velocity $\frac{2NI}{t}$: but if the body deſcribes the arc HI, it augments that arc only by the length $HI - HN$ or $\frac{MI \times NI}{HI}$; and therefore generates only the velocity $\frac{2MI \times NI}{t \times HI}$. Let this velocity be added to the beforementioned decrement, and we ſhall have the decrement of the velocity ariſing from the reſiſtance alone, that is, $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times HI}$. Therefore ſince in the ſame time, the action of gravity generates, in a falling body, the velocity $\frac{2NI}{t}$; the reſiſtance will be to the gravity as $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times HI}$ to $\frac{2NI}{t}$, or as $\frac{t \times GH}{T} - HI + \frac{2MI \times NI}{HI}$ to $2NI$.

Now for the abſciſſa's CB, CD, CE put $-o$, o, $2o$. For the ordinate CH put P; and for MI put any ſeries $Qo + Ro^2 + So^3$ &c. And all the terms of the ſeries after the firſt, that is, $Ro^2 + So^3 +$ &c. will be NI; and the ordinates DI, EK and BG will be $P - Qo - Ro^2 - So^3 -$ &c. $P - 2Qo - 4Ro^2 - 8So^3 -$ &c. and $P + Qo - Ro^2 + So^3 -$ &c. reſpectively. And by ſquaring the dif-

differences of the ordinates $BG - CH$ and $CH - DI$, and to the ſquares thence produced adding the ſquares of BC and CD themſelves, you will have $oo + QQoo - 2QRo^3 +$ &c. and $oo + QQoo + 2QRo^3 +$ &c. the ſquares of the arcs GH, HI; whoſe roots $o\sqrt{1+QQ} - \frac{QRoo}{\sqrt{1+QQ}}$, and $o\sqrt{1+QQ} + \frac{QRoo}{\sqrt{1+QQ}}$ are the arcs GH and HI. Moreover, if from the ordinate CH there be ſubducted half the ſum of the ordinates BG and DI, and from the ordinate DI there be ſubducted half the ſum of the ordinates CH and EK, there will remain Roo and $Roo + 3So^3$ the verſed ſines of the arcs GI and HK. And theſe are proportional to the lineolæ LH and NI, and therefore in the duplicate ratio of the infinitely ſmall times T and t: and thence the ratio $\frac{t}{T}$ is $\sqrt{\frac{R + 3So}{R}}$ or $\frac{R + \frac{3}{2}So}{R}$; and $\frac{t \times GH}{T} - HI + \frac{2MI \times NI}{HI}$, by ſubſtituting the values of $\frac{t}{T}$, GH, HI, MI and NI juſt found, becomes $\frac{3Soo}{2R}\sqrt{1+QQ}$. And ſince $2NI$ is $2Roo$, the reſiſtance will be now to the gravity as $\frac{3Soo}{2R}\sqrt{1+QQ}$ to $2Roo$, that is, as $3S\sqrt{1+QQ}$ to $4RR$.

And the velocity will be ſuch, that a body going off therewith from any place H, in the direction of the tangent HN, would deſcribe, in vacuo, a Parabola, whoſe diameter is HC, and its latus rectum $\frac{HN^2}{NI}$ or $\frac{1+QQ}{R}$.

And

And the reſiſtance is as the denſity of the medium and the ſquare of the velocity conjunctly; and therefore the denſity of the medium is as the reſiſtance directly, and the ſquare of the velocity inverſely; that is, as $\frac{3S\sqrt{1+QQ}}{4RR}$ directly and $\frac{1+QQ}{R}$ inverſely; that is, as $\frac{S}{R\sqrt{1+QQ}}$ *Q. E. I.*

Cor. 1. If the tangent HN be produced both ways, ſo as to meet any ordinate AF in T: $\frac{HT}{AC}$ will be equal to $\sqrt{1+QQ}$, and therefore in what has gone before may be put for $\sqrt{1+QQ}$. By this means the reſiſtance will be to the gravity as $3S \times HT$ to $4RR \times AC$; the velocity will be as $\frac{HT}{AC\sqrt{R}}$, and the denſity of the medium will be as $\frac{S \times AC}{R \times HT}$.

Cor. 2. And hence, if the curve line $PFHQ$ be defined by the relation between the baſe or abſciſſa AC and the ordinate CH, as is uſual; and the value of the ordinate be reſolved into a converging ſeries: The problem will be expeditiouſly ſolved by the firſt terms of the ſeries; as in the following examples.

Example 1. Let the line $PFHQ$ be a ſemi-circle deſcribed upon the diameter PQ; to find the denſity of the medium that ſhall make a projectile move in that line.

Biſect the diameter PQ in A; and call AQ, n; AC, a; CH, e; and CD, o: then DI^2 or $AQ^2 - AD^2 = nn - aa - 2ao - oo$, or $ee - 2ao - oo$; and the root being extracted by our method, will give

$$DI = e - \frac{ao}{e} - \frac{oo}{2e} - \frac{aaoo}{2e^3} - \frac{ao^3}{2e^3} - \frac{a^3o^3}{2e^5} -$$

&c. Here put nn for $ee + aa$, and DI will become $= e - \frac{ao}{e} - \frac{nnoo}{2e^3} - \frac{annn^3}{2e^5}$ &c.

Such series I distinguish into successive terms after this manner: I call that the first term, in which the infinitely small quantity o is not found; the second, in which that quantity is of one dimension only; the third, in which it arises to two dimensions; the fourth, in which it is of three; and so *ad infinitum*. And the first term, which here is e, will always denote the length of the ordinate CH, standing at the beginning of the indefinite quantity o. The second term, which here is $\frac{ao}{e}$, will denote the difference between CH and DN; that is, the lineola MN which is cut off by compleating the parallelogram $HCDM$; and therefore always determines the position of the tangent HN; as, in this case, by taking MN to HM as $\frac{ao}{e}$ to o, or a to e. The third term, which here is $\frac{nnoo}{2e^3}$, will represent the lineola IN, which lies between the tangent and the curve; and therefore determines the angle of contact IHN, or the curvature which the curve line has in H. If that lineola IN is of a finite magnitude, it will be expres'd by the third term together with those that follow *in infinitum*. But if that lineola be diminished *in infinitum*, the terms following become infinitely less than the third term, and therefore may be neglected. The fourth term determines the variation of the curvature; the fifth, the variation of the variation; and so on. Whence, by the way, appears no contemptible use of these series in the solution of problems that depend upon tangents, and the curvature of curves.

Now

Now compare the ſeries $e - \frac{ao}{e} - \frac{nnoo}{2e^3} - \frac{annno^3}{2e^5}$ — &c. with the ſeries $P - Qo - Roo - So^3$ — &c. and for P, Q, R and S put e, $\frac{a}{e}$, $\frac{nn}{2e^3}$ and $\frac{ann}{2e^5}$, and for $\sqrt{1+QQ}$ put $\sqrt{1+\frac{aa}{ee}}$ or $\frac{n}{e}$; and the denſity of the medium will come out as $\frac{a}{ne}$, that is, (becauſe n is given) as $\frac{a}{e}$, or $\frac{AC}{CH}$, that is, as that length of the tangent HT, which is terminated at the ſemidiameter AF ſtanding perpendicularly on PQ: and the reſiſtance will be to the gravity as $3a$ to $2n$, that is, as $3AC$ to the diameter PQ of the circle; and the velocity will be as $\sqrt{CH}$. Therefore if the body goes from the place F, with a due velocity, in the direction of a line parallel to PQ, and the denſity of the medium in each of the places H is as the length of the tangent HT, and the reſiſtance alſo in any place H is to the force of gravity as $3AC$ to PQ, that body will deſcribe the quadrant FHQ of a circle. *Q. E. I.*

But if the ſame body ſhould go from the place P, in the direction of a line perpendicular to PQ, and ſhould begin to move in an arc of the ſemi-circle PFQ, we muſt take AC or a on the contrary ſide of the centre A; and therefore its ſign muſt be changed, and we muſt put $-a$ for $+a$. Then the denſity of the medium would come out as $-\frac{a}{e}$. But nature does not admit of a negative denſity, that is, a denſity which accelerates the motion of bodies; and therefore it cannot naturally come to paſs, that a body by aſcending from P ſhould deſcribe the quadrant PF of a circle. To produce ſuch an effect, a body ought to be acce-

lerated by an impelling medium, and not impeded by a reſiſting one.

EXAMPLE 2. Let the line PFQ be a Parabola, having its axis AF perpendicular to the horizon PQ; to find the denſity of the medium, which will make a projectile move in that line. *Fig.* 4.

From the nature of the Parabola, the rectangle PDQ is equal to the rectangle under the ordinate DI and ſome given right line: that is, if that right line be called b; PC, a; PQ, c; CH, e; and CD, o; the rectangle $a+o$ into $c-a-o$ or $ac-aa-2ao+co-oo$ is equal to the rectangle b into DI, and therefore DI is equal to $\frac{ac-aa}{b}+\frac{c-2a}{b}o-\frac{oo}{b}$. Now the ſecond term $\frac{c-2a}{b}o$ of this ſeries is to be put for Qo, and the third term $\frac{oo}{b}$ for Roo. But ſince there are no more terms, the coefficient S of the fourth term will vaniſh; and therefore the quantity $\frac{S}{R\sqrt{1+QQ}}$, to which the denſity of the medium is proportional, will be nothing. Therefore, where the medium is of no denſity, the projectile will move in a Parabola; as *Galileo* hath heretofore demonſtrated. *Q. E. I.*

EXAMPLE 3. Let the line AGK be an Hyperbola, having its aſymptote NX perpendicular to the horizontal plane AK; to find the denſity of the medium, that will make a projectile move in that line. *Fig.* 5.

Let MX be the other aſymptote, meeting the ordinate DG produced in V; and from the nature of the Hyperbola, the rectangle of XV into VG will be given. There is alſo given the ratio of DN to VX, and therefore the rectangle of DN into VG is given. Let that be bb: and, compleating the parallelogram

$DNXZ$; let BN be called a; BD, o; NX, c; and let the given ratio of VZ to ZX or DN be $\frac{m}{n}$. Then DN will be equal to $a-o$, VG equal to $\frac{bb}{a-o}$, VZ equal to $\frac{m}{n}\times\overline{a-o}$, and GD or $NX-VZ-VG$ equal to $c-\frac{m}{n}a+\frac{m}{n}o-\frac{bb}{a-o}$. Let the term $\frac{bb}{a-o}$ be reſolved into the converging ſeries $\frac{bb}{a}+\frac{bb}{aa}o+\frac{bb}{a^3}oo+\frac{bb}{a^4}o^3$ &c. and GD will become equal to $c-\frac{m}{n}a-\frac{bb}{a}+\frac{m}{n}o-\frac{bb}{aa}o-\frac{bb}{a^3}o^2-\frac{bb}{a^4}o^3$ &c. The ſecond term $\frac{m}{n}o-\frac{bb}{aa}o$ of this ſeries is to be uſed for Qo, the third $\frac{bb}{a^3}o^2$ with its ſign changed for Ro^2, and the fourth $\frac{bb}{a^4}o^3$ with its ſign changed alſo for So^3, and their coefficients $\frac{m}{n}-\frac{bb}{aa}$, $\frac{bb}{a^3}$ and $\frac{bb}{a^4}$ are to be put for Q, R and S in the former Rule. Which being done, the denſity of the medium will come out as $\frac{\frac{bb}{a^4}}{\frac{bb}{a^3}\sqrt{1+\frac{mm}{nn}-\frac{2mbb}{naa}+\frac{b^4}{a^4}}}$ or $\frac{1}{\sqrt{aa+\frac{mm}{nn}aa-\frac{2mbb}{n}+\frac{b^4}{aa}}}$, that is, if in VZ

you take VY equal to VG, as $\frac{1}{XY}$. For aa and $\frac{m^2}{n^2}a^2 - \frac{2mbb}{n} + \frac{b^4}{aa}$ are the squares of XZ and ZY. But the ratio of the resistance to gravity is found to be that of $3XY$ to $2YG$; and the velocity is that with which the body would describe a Parabola, whose vertex is G, diameter DG, latus rectum $\frac{XY^2}{VG}$. Suppose therefore that the densities of the medium in each of the places G are reciprocally as the distances XY, and that the resistance in any place G is to the gravity as $3XY$ to $2YG$; and a body let go from the place A, with a due velocity, will describe that Hyperbola AGK. *Q.E.I.*

Example 4. Suppose indefinitely, the line AGK to be an Hyperbola, described with the centre X, and the asymptotes MX, NX, so that, having constructed the rectangle $XZDN$, whose side ZD cuts the Hyperbola in G and its asymptote in V, VG may be reciprocally as any power DN^n of the line ZX or DN, whose index is the number n: To find the density of the medium in which a projected body will describe this curve. *Fig.* 5.

For BN, BD, NX put A, O, C respectively, and let VZ be to XZ or DN as d to e, and VG be equal to $\frac{bb}{DN^n}$; then DN will be equal to $A - O$, $VG = \frac{bb}{\overline{A-O}|^n}$, $VZ = \frac{d}{e}\overline{A-O}$, and GD or $NX - VZ - VG$ equal to $C - \frac{d}{e}A + \frac{d}{e}O - \frac{bb}{\overline{A-O}|^n}$. Let the term $\frac{bb}{\overline{A-O}|^n}$ be resolved into an infinite series $\frac{bb}{A^n} +$

$+\frac{nbb}{A^{n+1}}\times O+\frac{nn+n}{2A^{n+2}}\times bbO^2+\frac{n^3+3nn+2n}{6A^{n+3}}$ $\times bbO^3$ &c. and GD will be equal to $C-\frac{d}{e}A-\frac{bb}{A^n}+\frac{d}{e}O-\frac{nbb}{A^{n+1}}O-\frac{+nn+n}{2A^{n+2}}bbO^2-\frac{+n^3+3nn+2n}{6A^{n+3}}bbO^3$ &c. The second term $\frac{d}{e}O-\frac{nbb}{A^{n+1}}O$ of this series is to be used for Qo, the third $\frac{nn+n}{2A^{n+2}}bbO^2$ for Roo, the fourth $\frac{n^3+3nn+2n}{6A^{n+3}}bbO^3$ for So^3. And thence the density of the medium $\frac{S}{R\sqrt{1+QQ}}$, in any place G, will be $\frac{n+2}{3\sqrt{A^2+\frac{dd}{ee}A^2-\frac{2dnbb}{eA^n}A+\frac{nnb^4}{A^{2n}}}}$, and therefore if in VZ you take VY equal to $n\times VG$, that density is reciprocally as XY. For A^2 and $\frac{dd}{ee}A^2-\frac{2dnbb}{eA^n}A+\frac{nnb^4}{A^{2n}}$ are the squares of XZ and ZY. But the resistance in the same place G is to the force of gravity as $3S\times\frac{XY}{A}$ to $4RR$, that is, as XY to $\frac{2nn+2n}{n+2}VG$. And the velocity there, is the same wherewith the projected body would move in a Parabola, whose vertex is G, diameter GD, and latus rectum $\frac{1+QQ}{R}$ or $\frac{2XY^2}{nn+n\times VG}$. *Q.E.I.*

SCHOLIUM.

In the ſame manner that the denſity of the medium comes out to be as $\frac{S \times AC}{R \times HT}$, in Corol. 1. if the reſiſtance is put as any power V^n of the velocity V, the denſity of the medium will come out to be as $\frac{S}{R^{\frac{4-n}{2}}} \times \left|\frac{AC}{HT}\right|^{n-1}$. *Fig.* 3.

And therefore if a curve can be found, ſuch that the ratio of $\frac{S}{R^{\frac{4-n}{2}}}$ to $\left|\frac{HT}{AC}\right|^{n-1}$, or of $\frac{S^2}{R^{4-n}}$ to $\left|1 + QQ\right|^{n-1}$ may be given: the body, in an uniform medium, whoſe reſiſtance is as the power V^n of the velocity V, will move in this curve. But let us return to more ſimple curves.

Becauſe there can be no motion in a Parabola except in a non-reſiſting medium, but in the Hyperbola's here deſcribed 'tis produced by a perpetual reſiſtance; it is evident that the line which a projectile deſcribes in an uniformly reſiſting medium, approaches nearer to theſe Hyperbola's than to a Parabola. That line is certainly of the hyperbolic kind, but about the vertex it is more diſtant from the aſymptotes, and in the parts remote from the vertex draws nearer to them, than theſe Hyperbola's here deſcribed. The difference however is not ſo great between the one and the other, but that theſe latter may be commodiouſly enough uſed in practice inſtead of the former. And perhaps theſe may prove more uſeful, than an Hyperbola that is more accurate, and at the ſame time more compounded. They may be made uſe of then in this manner. *Fig.* 5.

Com-

Compleat the parallelogram *XYGT*, and the right line *GT* will touch the hyperbola in *G*, and therefore the denſity of the medium in *G* is reciprocally as the tangent *GT*, and the velocity there, as $\sqrt{\frac{GT^2}{GV}}$, and the reſiſtance is to the force of gravity as *GT* to $\frac{2nn+2n}{n+2} \times GV$.

Therefore if a body projected from the place *A* in the direction of the right line *AH*, (*Fig.* 6.) deſcribes the Hyperbola *AGK*, and *AH* produced meets the aſymptote *NX* in *H*, and *AI* drawn parallel to it meets the other aſymptote *MX* in *I*; the denſity of the medium in *A* will be reciprocally as *AH*, and the velocity of the body as $\sqrt{\frac{AH^2}{AI}}$, and the reſiſtance there to the force of gravity as *AH* to $\frac{2nn+2n}{n+2} \times AI$. Hence the following rules are deduced.

Rule 1. If the denſity of the medium at *A*, and the velocity with which the body is projected remain the ſame, and the angle *NAH* be changed; the lengths *AH*, *AI*, *HX* will remain. Therefore if thoſe lengths, in any one caſe, are found, the Hyperbola may afterwards be eaſily determined from any given angle *NAH*.

Rule 2. If the angle *NAH*, and the denſity of the medium at *A* remain the ſame, and the velocity with which the body is projected be changed, the length *AH* will continue the ſame; and *AI* will be changed in a duplicate ratio of the velocity reciprocally.

Rule 3. If the angle *NAH*, the velocity of the body at *A*, and the accelerative gravity remain the ſame, and the proportion of the reſiſtance at *A* to the motive gravity be augmented in any ratio; the proportion of *AH* to *AI* will be augmented in the ſame ratio,

the latus rectum of the abovementioned Parabola remaining the ſame, and alſo the length $\frac{AH^2}{AI}$ proportional to it; and therefore AH will be diminiſhed in the ſame ratio, and AI will be diminiſhed in the duplicate of that ratio. But the proportion of the reſiſtance to the weight is augmented, when either the ſpecific gravity is made leſs, the magnitude remaining equal, or when the denſity of the medium is made greater, or when, by diminiſhing the magnitude, the reſiſtance becomes diminiſhed in a leſs ratio than the weight.

Rule 4. Becauſe the denſity of the medium is greater near the vertex of the Hyperbola, than it is in the place A; that a mean denſity may be preſerv'd, the ratio of the leaſt of the tangents GT to the tangent AH ought to be found, and the denſity in A augmented in a ratio a little greater than that of half the ſum of thoſe tangents to the leaſt of the tangents GT.

Rule 5. If the lengths AH, AI are given, and the figure AGK is to be deſcribed: produce HN to X, ſo that HX may be to AI as $n+1$ to 1; and with the centre X, and the aſymptotes MX, NX deſcribe an Hyperbola thro' the point A, ſuch that AI may be to any of the lines VG as XV^n to XI^n.

Rule 6. By how much the greater the number n is, ſo much the more accurate are theſe Hyperbola's in the aſcent of the body from A, and leſs accurate in its deſcent to K; and the contrary. The Conic Hyperbola keeps a mean ratio between theſe, and is more ſimple than the reſt. Therefore if the Hyperbola be of this kind, and you are to find the point K, where the projected body falls upon any right line AN paſſing thro' the point A: let AN produced meet the aſymptotes MX, NX in M and N, and take NK equal to AM.

Rule 7. And hence appears an expeditious method of determining this Hyperbola from the phænomena.

Let

Let two ſimilar and equal bodies be projected with the ſame velocity, in different angles HAK, hAk, (*Fig.* 6.) and let them fall upon the plane of the horizon in K and k; and note the proportion of AK to Ak. Let it be as d to e. Then erecting a perpendicular AI of any length, aſſume any how the length AH or Ah, and thence graphically, or by ſcale and compaſs, collect the lengths AK, Ak (by Rule 6.) If the ratio of AK to Ak be the ſame with that of d to e, the length of AH was rightly aſſumed. If not, take on the indefinite right line SM, (*Fig.* 7.) the length SM equal to the aſſumed AH; and erect a perpendicular MN, equal to the difference $\frac{AK}{Ak} - \frac{d}{e}$ of the ratio's drawn into any given right line. By the like method, from ſeveral aſſumed lengths AH, you may find ſeveral points N; and draw thro' them all a regular curve $NNXN$, cutting the right line $SMMM$ in X. Laſtly, aſſume AH equal to the abſciſſa SX, and thence find again the length AK; and the lengths, which are to the aſſumed length AI and this laſt AH, as the length AK known by experiment, to the length AK laſt found, will be the true lengths AI and AH, which were to be found. But theſe being given, there will be given alſo the reſiſting force of the medium in the place A, it being to the force of gravity as AH to $2AI$. Let the denſity of the medium be increaſed by Rule 4. and if the reſiſting force juſt found be increaſed in the ſame ratio, it will become ſtill more accurate.

Rule 8. The lengths AH, HX being found; let there be now required the poſition of the line AH, according to which a projectile thrown with that given velocity, ſhall fall upon any point K. At the points A and K, (*Fig.* 6.) erect the lines AC, KF perpendicular to the horizon; whereof let AC be drawn downwards, and be equal to AI or $\frac{1}{2}HX$. With the aſymptotes AK, KF, deſcribe an Hyperbola, whoſe con-

conjugate ſhall paſs thro' the point C; and from the centre A, with the interval AH, deſcribe a circle cutting that Hyperbola in the point H; then the projectile thrown in the direction of the right line AH will fall upon the point K. *Q. E. I.* For the point H, becauſe of the given length AH, muſt be ſomewhere in the circumference of the deſcribed circle. Draw CH meeting AK and KF in E and F; and becauſe CH, MX are parallel, and AC, AI equal, AE will be equal to AM, and therefore alſo equal to KN. But CE is to AE as FH to KN, and therefore CE and FH are equal. Therefore the point H falls upon the hyperbolic curve deſcribed with the aſymptotes AK, KF, whoſe conjugate paſſes thro' the point C; and is therefore found in the common interſection of this hyperbolic curve and the circumference of the deſcribed circle. *Q. E. D.* It is to be obſerved that this operation is the ſame, whether the right line AKN be parallel to the horizon, or inclined thereto in any angle; and that from two interſections H, H, there ariſe two angles NAH, NAH; and that in mechanical practice it is ſufficient once to deſcribe a circle, then to apply a ruler CH, of an indeterminate length, ſo to the point C, that its part FH, intercepted between the circle and the right line FK, may be equal to its part CE placed between the point C and the right line AK.

What has been ſaid of Hyperbola's may be eaſily applied to Parabola's. For if (*Fig.* 8.) a Parabola be repreſented by $XAGK$, touched by a right line XV in the vertex X; and the ordinates IA, VG be as any powers XI^n, XV^n of the abſciſſa's XI, XV; draw XT, GT, AH, whereof let XT be parallel to VG, and let GT, AH touch the Parabola in G and A; and a body projected from any place A, in the direction of the right line AH, with a due velocity, will deſcribe this Parabola, if the denſity of the medium in each

each of the places G, be reciprocally as the tangent GT. In that caſe the velocity in G will be the ſame as would cauſe a body, moving in a non-reſiſting ſpace, to deſcribe a Conic Parabola, having G for its vertex, VG produced downwards for its diameter, and $\frac{2GT^2}{nn-n\times VG}$ for its latus rectum. And the reſiſting force in G will be to the force of gravity, as GT to $\frac{2nn-2n}{n-2}VG$. Therefore if NAK repreſent an horizontal line, and, both the denſity of the medium at A and the velocity with which the body is projected, remaining the ſame, the angle NAH be any how alter'd; the lengths AH, AI, HX will remain; and thence will be given the vertex X of the Parabola, and the poſition of the right line XI, and by taking VG to IA as XV^n to XI^n, there will be given all the points G of the Parabola, thro' which the projectile will paſs.

SEC-

SECTION III.

Of the Motions of Bodies which are reſiſted partly in the ratio of the Velocities, and partly in the duplicate of the ſame ratio.

Proposition XI. Theorem VIII.

If a body be reſiſted partly in the ratio, and partly in the duplicate ratio of its velocity, and moves in a ſimilar medium by its innate force only; and the times be taken in arithmetical progreſſion: then quantities reciprocally proportional to the velocities, increaſed by a certain given quantity, will be in geometrical progreſſion. Pl. 3. Fig. 1.

With the centre *C*, and the rectangular aſymptotes *CADd* and *CH* deſcribe an Hyperbola *BEe*, and let *AB*, *DE*, *de*, be parallel to the aſymptote *CH*. In the aſymptote *CD* let *A*, *G* be given points: And if the time be expounded by the hyperbolic area *ABED* uniformly increaſing; I ſay that the velocity may be expreſs'd by the length *DF*, whoſe reciprocal *GD* together with the given line *CG*, compoſe the length *CD* increaſing in a geometrical progreſſion.

Fig. 1.
p. 22.



Fig. 2.
p. 24.

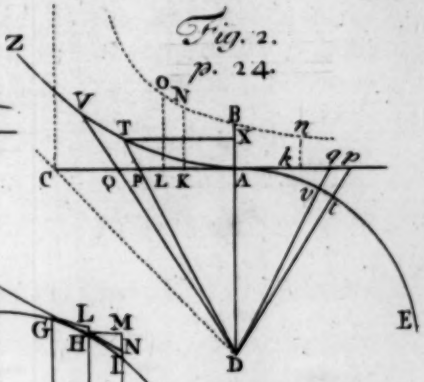


Fig. 3.
p. 28. 38.

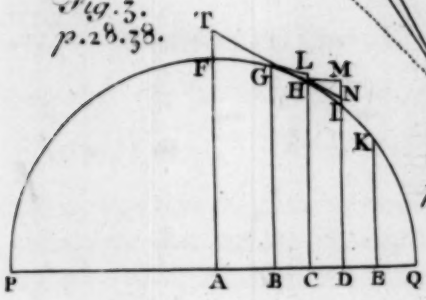


Fig. 4.
p. 34.

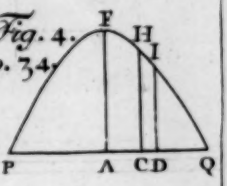


Fig. 5.
p. 34. 38.

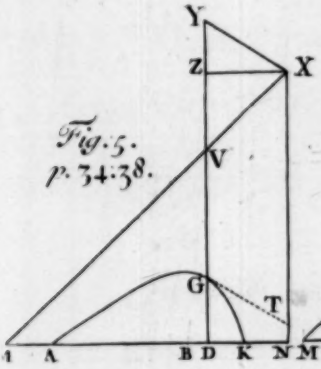


Fig. 6.
p. 39.

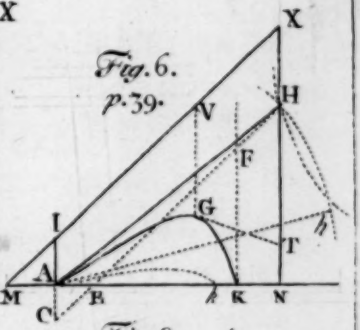


Fig. 7.
p. 41.

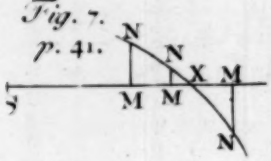
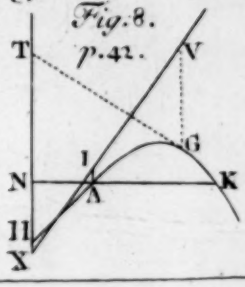


Fig. 8.
p. 42.



For let the areola $DEed$ be the leaſt given increment of the time, and Dd will be reciprocally as DE, and therefore directly as CD. Therefore the decrement of $\frac{1}{GD}$, which (by Lem. 2. Book 2.) is $\frac{Dd}{GD^2}$ will be alſo as $\frac{CD}{GD^2}$ or $\frac{CG+GD}{GD^2}$, that is, as $\frac{1}{GD}+\frac{CG}{GD^2}$. Therefore the time $ABED$ uniformly increaſing by the addition of the given particles $EDde$, it follows that $\frac{1}{GD}$ decreaſes in the ſame ratio with the velocity. For the decrement of the velocity is as the reſiſtance, that is, (by the ſuppoſition) as the ſum of two quantities, whereof one is as the velocity, and the other as the ſquare of the velocity; and the decrement of $\frac{1}{GD}$ is as the ſum of the quantities $\frac{1}{GD}$ and $\frac{CG}{GD^2}$, whereof the firſt is $\frac{1}{GD}$ it ſelf, and the laſt $\frac{CG}{GD^2}$ is as $\frac{1}{GD^2}$: therefore $\frac{1}{GD}$ is as the velocity, the decrements of both being analogous. And if the quantity GD, reciprocally proportional to $\frac{1}{GD}$, be augmented by the given quantity CG; the ſum CD, the time $ABED$ uniformly increaſing, will increaſe in a geometrical progreſſion. *Q. E. D.*

Cor. 1. Therefore, if, having the points A and G given, the time be expounded by the hyperbolic area $ABED$, the velocity may be expounded by $\frac{1}{GD}$ the reciprocal of GD.

Cor. 2. And by taking GA to GD as the reciprocal of the velocity at the beginning, to the reciprocal of

of the velocity at the end of any time *ABED*, the point *G* will be found. And that point being found, the velocity may be found from any other time given.

Proposition XII. Theorem IX.

The ſame things being ſuppoſed, I ſay, that if the ſpaces deſcribed are taken in arithmetical progreſſion, the velocities augmented by a certain given quantity will be in geometrical progreſſion. Pl. 3. Fig. 2.

In the aſymptote *CD* let there be given the point *R*, and erecting the perpendicular *RS* meeting the Hyperbola in *S*, let the ſpace deſcribed be expounded by the hyperbolic area *RSED*; and the velocity will be as the length *GD*, which, together with the given line *CG*, compoſes a length *CD* decreaſing in a geometrical progreſſion, while the ſpace *RSED* increaſes in an arithmetical progreſſion.

For, becauſe the increment *EDde* of the ſpace is given, the lineola *Dd*, which is the decrement of *GD*, will be reciprocally as *ED*, and therefore directly as *CD*; that is, as the ſum of the ſame *GD* and the given length *CG*. But the decrement of the velocity, in a time reciprocally proportional thereto, in which the given particle of ſpace *DdeE* is deſcribed, is as the reſiſtance and the time conjunctly, that is, directly as the ſum of two quantities, whereof one is as the velocity, the other as the ſquare of the velocity, and inverſely as the velocity; and therefore directly as the ſum of two quantities, one of which is given, the other is as the velocity. Therefore the decrement both of the velocity and the line *GD*, is as a given quantity and a decreaſing quantity conjunctly; and, becauſe the decrements are analogous, the decreaſing quantities will always

always be analogous; *viz.* the velocity, and the line *GD*. *Q.E.D.*

Cor. 1. If the velocity be expounded by the length *GD*, the ſpace deſcribed will be as the hyperbolic area *DESR*.

Cor. 2. And if the point *R* be aſſumed any how, the point *G* will be found, by taking *GR* to *GD*, as the velocity at the beginning to the velocity after any ſpace *RSED* is deſcribed. The point *G* being given, the ſpace is given from the given velocity: and the contrary.

Cor. 3. Whence ſince (by Prop. 11.) the velocity is given from the given time, and (by this Prop.) the ſpace is given from the given velocity; the ſpace will be given from the given time: and the contrary.

Proposition XIII. Theorem X.

Suppoſing that a body attracted downwards by an uniform gravity aſcends or deſcends in a right line; and that the ſame is reſiſted, partly in the ratio of its velocity, and partly in the duplicate ratio thereof: I ſay that, if right lines parallel to the diameters of a Circle and an Hyperbola be drawn thro' the ends of the conjugate diameters, and the velocities be as ſome ſegments of thoſe parallels drawn from a given point; the times will be as the ſectors of the areas, cut off by right lines drawn from the centre to the ends of the ſegments; and the contrary. Pl. 3. Fig. 3.

Case 1. Suppoſe firſt that the body is aſcending, and from the centre *D*, with any ſemidiameter *DB*, deſcribe a quadrant *BETF* of a circle, and thro' the end

end B of the ſemidiameter DB draw the indefinite line BAP, parallel to the ſemidiameter DF. In that line let there be given the point A, and take the ſegment AP proportional to the velocity. And ſince one part of the reſiſtance is as the velocity, and another part as the ſquare of the velocity; let the whole reſiſtance be as $AP^2 + 2BAP$. Join DA, DP cutting the circle in E and T, and let the gravity be expounded by DA^2, ſo that the gravity ſhall be to the reſiſtance in P, as DA^2 to $AP^2 + 2BAP$; and the time of the whole aſcent will be as the ſector EDT of the circle.

For draw DVQ, cutting off the moment PQ of the velocity AP, and the moment DTV of the ſector DET anſwering to a given moment of time; and that decrement PQ of the velocity will be as the ſum of the forces of gravity DA^2 and of reſiſtance $AP^2 + 2BAP$, that is, (by 12 Prop. 2 Book Elem.) as DP^2. Then the area DPQ, which is proportional to PQ, is as DP^2, and the area DTV, which is to the area DPQ as DT^2 to DP^2, is as the given quantity DT^2. Therefore the area EDT decreaſes uniformly according to the rate of the future time, by ſubduction of given particles DTV, and is therefore proportional to the time of the whole aſcent. *Q. E. D.*

Case 2. If the velocity in the aſcent of the body be expounded by the length AP as before, and the reſiſtance be made as $AP^2 + 2BAP$, and if the force of gravity be leſs than can be expreſſed by DA^2; take BD (*Fig.* 4.) of ſuch a length, that $AB^2 - BD^2$ may be proportional to the gravity, and let DF be perpendicular and equal to DB, and thro' the vertex F deſcribe the Hyperbola $FTVE$, whoſe conjugate ſemidiameters are DB and DF, and which cuts DA in E, and DP, DQ in T and V; and the time of the whole aſcent will be as the hyperbolic ſector TDE.

For the decrement PQ of the velocity produced in a given particle of time, is as the ſum of the reſiſtance AP^2

$AP^2 + 2BAP$ and of the gravity $AB^2 - BD^2$, that is, as $BP^2 - BD^2$. But the area DTV is to the area DPQ as DT^2 to DP^2; and therefore, if GT be drawn perpendicular to DF, as GT^2 or $GD^2 - DF^2$ to BD^2, and as GD^2 to BP^2, and, by diviſion, as DF^2 to $BP^2 - BD^2$. Therefore ſince the area DPQ is as PQ, that is, as $BP^2 - BD^2$; the area DTV will be as the given quantity DF^2. Therefore the area EDT decreaſes uniformly in each of the equal particles of time, by the ſubduction of ſo many given particles DTV, and therefore is proportional to the time. *Q. E. D.*

Case 3. Let AP be the velocity in the deſcent of the body, and $AP^2 + 2BAP$ the force of reſiſtance, and $BD^2 - AB^2$ the force of gravity, the angle DBA being a right one. And if with the centre D, and the principal vertex B, there be deſcribed a rectangular Hyperbola $BETV$ (*Fig.* 5.) cutting DA, DP, and DQ produced in E, T, and V; the ſector DET of this Hyperbola will be as the whole time of deſcent.

For the increment PQ of the velocity, and the area DPQ proportional to it, is as the exceſs of the gravity above the reſiſtance, that is, as $BD^2 - AB^2 - 2BAP - AP^2$ or $BD^2 - BP^2$. And the area DTV is to the area DPQ, as DT^2 to DP^2; and therefore as GT^2 or $GD^2 - BD^2$ to BP^2, and as GD^2 to BD^2, and, by diviſion, as BD^2 to $BD^2 - BP^2$. Therefore ſince the area DPQ is as $BD^2 - BP^2$, the area DTV will be as the given quantity BD^2. Therefore the area EDT increaſes uniformly in the ſeveral equal particles of time by the addition of as many given particles DTV, and therefore is proportional to the time of the deſcent. *Q. E. D.*

Cor. If with the centre D and the ſemidiameter DA there be drawn thro' the vertex A an arc At ſimilar to the arc ET, and ſimilarly ſubtending the angle

ADT: the velocity *AP* will be to the velocity, which the body in the time *EDT*, in a non-reſiſting ſpace, can loſe in its aſcent, or acquire in its deſcent, as the area of the triangle *DAP* to the area of the ſector *DAt*; and therefore is given from the time given. For the velocity in a non-reſiſting medium, is proportional to the time, and therefore to this ſector; in a reſiſting medium it is as the triangle; and in both mediums, where it is leaſt, it approaches to the ratio of equality, as the ſector and triangle do.

Scholium.

One may demonſtrate alſo that caſe in the aſcent of the body, where the force of gravity is leſs than can be expreſs'd by DA^2 or $AB^2 + BD^2$, and greater than can be expreſs'd by $AB^2 - DB^2$, and muſt be expreſs'd by AB^2. But I haſten to other things.

Proposition XIV. Theorem XI.

The ſame things being ſuppoſed, I ſay, that the ſpace deſcribed in the aſcent or deſcent, is as the difference of the area by which the time is expreſs'd, and of ſome other area which is augmented or diminiſhed in an arithmetical progreſſion; if the forces compounded of the reſiſtance and the gravity be taken in a geometrical progreſſion. Pl. 3. Fig. 5, 6, 7.

Take *AC* (in the three laſt figures) proportional to the gravity, and *AK* to the reſiſtance. But take them on the ſame ſide of the point *A*, if the body is deſcending, otherwiſe on the contrary. Erect *Ab*, which make to *DB* as DB^2 to $4BAC$: and to the rectangular aſymptotes *CK*, *CH*, deſcribe the Hyperbola

bola bN, and erecting KN-perpendicular to CK, the area $AbNK$ will be augmented or diminiſhed in an arithmetical progreſſion, while the forces CK are taken in a geometrical progreſſion. I ſay therefore that the diſtance of the body from its greateſt altitude is as the exceſs of the area $AbNK$ above the area DET.

For ſince AK is as the reſiſtance, that is, as $AP^2 + 2BAP$; aſſume any given quantity Z, and put AK equal to $\frac{AP^2 + 2BAP}{Z}$; then (by Lem. 2. of this Book) the moment KL of AK will be equal to $\frac{2APQ + 2BA \times PQ}{Z}$ or $\frac{2BPQ}{Z}$, and the moment $KLON$ of the area $AbNK$, will be equal to $\frac{2BPQ \times LO}{Z}$ or $\frac{BPQ \times BD^3}{2Z \times CK \times AB}$.

Case 1. Now if the body aſcends, and the gravity be as $AB^2 + BD^2$, BET, (in *Fig.* 5.) being a circle, the line AC, which is proportional to the gravity, will be $\frac{AB^2 + BD^2}{Z}$, and DP^2 or $AP^2 + 2BAP + AB^2 + BD^2$ will be $AK \times Z + AC \times Z$ or $CK \times Z$; and therefore the area DTV will be to the area DPQ as DT^2 or DB^2 to $CK \times Z$.

Case 2. If the body aſcends, and the gravity be as $AB^2 - BD^2$, the line AC (in *Fig.* 6.) will be $\frac{AB^2 - BD^2}{Z}$ and DT^2 will be to DP^2 as DF^2 or DB^2 to $BP^2 - BD^2$ or $AP^2 + 2BAP + AB^2 - BD^2$, that is, to $AK \times Z + AC \times Z$ or $CK \times Z$. And therefore the area DTV will be to the area DPQ as DB^2 to $CK \times Z$.

Case 3. And by the ſame reaſoning, if the body deſcends, and therefore the gravity is as $BD^2 - AB^2$, and the line AC (in *Fig.* 7.) becomes equal to

$\frac{BD^2 - AB^2}{Z}$; the area DTV will be to the area DPQ as DB^2 to $CK \times Z$: as above.

Since therefore theſe areas are always in this ratio; if for the area DTV, by which the moment of the time, always equal to itſelf, is expreſſed, there be put any determinate rectangle, as $BD \times m$, the area DPQ, that is, $\frac{1}{2} BD \times PQ$, will be to $BD \times m$ as $CK \times Z$ to BD^2. And thence $PQ \times BD^3$ becomes equal to $2BD \times m \times CK \times Z$, and the moment $KLON$ of the area $AbNK$, found before, becomes $\frac{BP \times BD \times m}{AB}$. From the area DET ſubduct its moment DTV or $BD \times m$, and there will remain $\frac{AP \times BD \times m}{AB}$. Therefore the difference of the moments, that is, the moment of the difference of the areas is equal to $\frac{AP \times BD \times m}{AB}$; and therefore (becauſe of the given quantity $\frac{BD \times m}{AB}$) as the velocity AP; that is, as the moment of the ſpace which the body deſcribes in its aſcent or deſcent. And therefore the difference of the areas, and that ſpace, increaſing or decreaſing by proportional moments, and beginning together or vaniſhing together, are proportional. *Q. E. D.*

Cor. If the length, which ariſes by applying the area DET to the line BD, be called M; and another length V be taken in that ratio to the length M, which the line DA has to the line DE: the ſpace which a body, in a reſiſting medium, deſcribes in its whole aſcent or deſcent, will be to the ſpace, which a body, in a non-reſiſting medium, falling from reſt can deſcribe in the ſame time, as the difference of the aforeſaid areas to $\frac{BD \times V^2}{AB}$: and therefore is given from the

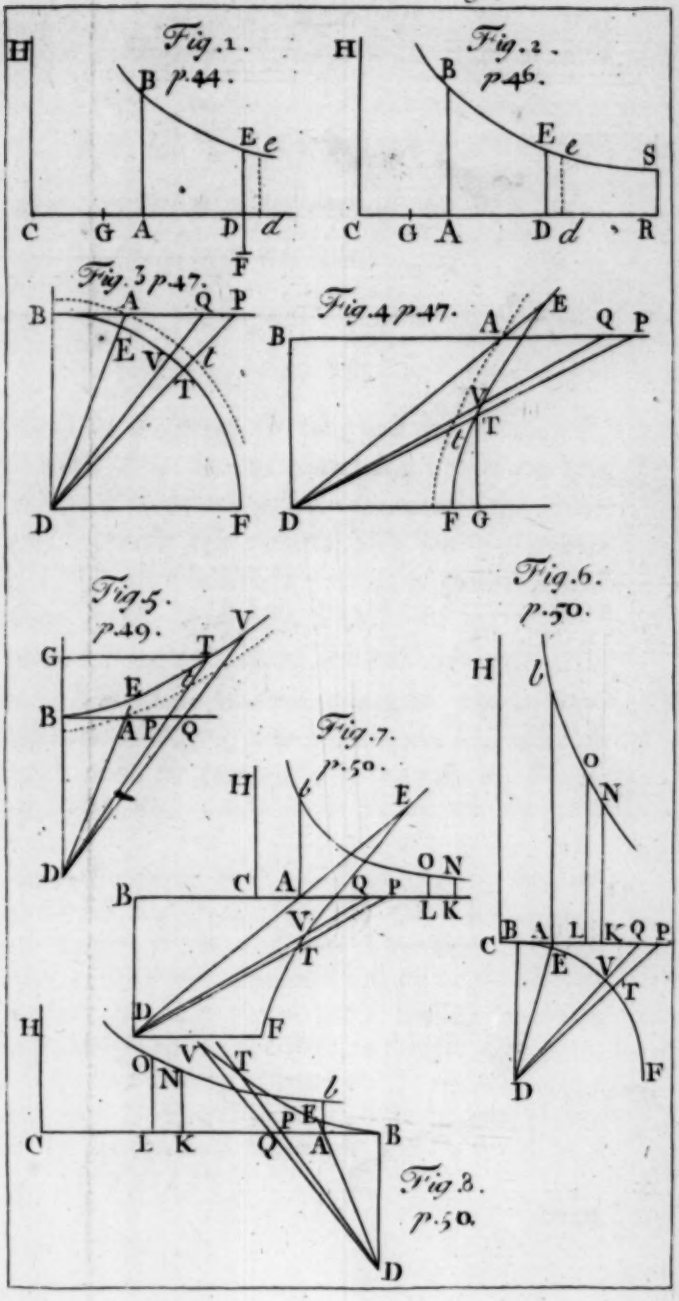
the time given. For the ſpace in a non-reſiſting medium is in a duplicate ratio of the time, or as V^2; and, becauſe BD and AB are given, as $\frac{BD \times V^2}{AB}$. This area is equal to the area $\frac{DA^2 \times BD \times M^2}{DE^2 \times AB}$, and the moment of M is m; and therefore the moment of this area is $\frac{DA^2 \times BD \times 2M \times m}{DE^2 \times AB}$. But this moment is to the moment of the difference of the aforeſaid areas DET and $AbNK$, *viz.* to $\frac{AP \times BD \times m}{AB}$, as $\frac{DA^2 \times BD \times M}{DE^2}$ to $\frac{1}{2}BD \times AP$, or as $\frac{DA^2}{DE^2}$ into DET to DAP; and therefore, when the areas DET and DAP are leaſt, in the ratio of equality. Therefore the area $\frac{BD \times V^2}{AB}$ and the difference of the areas DET and $AbNK$, when all theſe areas are leaſt, have equal moments; and are therefore equal. Therefore ſince the velocities, and therefore alſo the ſpaces in both mediums deſcribed together, in the beginning of the deſcent, or the end of the aſcent, approach to equality, and therefore are then one to another as the area $\frac{BD \times V^2}{AB}$, and the difference of the areas DET and $AbNK$; and moreover ſince the ſpace, in a non-reſiſting medium, is perpetually as $\frac{BD \times V^2}{AB}$, and the ſpace, in a reſiſting medium, is perpetually as the difference of the areas DET and $AbNK$; it neceſſarily follows, that the ſpaces, in both mediums, deſcribed in any equal times, are one to another as that area $\frac{BD \times V^2}{AB}$, and the difference of the areas DET and $AbNK$. *Q.E.D.*

Scholium.

The reſiſtance of ſphærical bodies in fluids ariſes partly from the tenacity, partly from the attrition, and partly from the denſity of the medium. And that part of the reſiſtance, which ariſes from the denſity of the fluid, is, as I ſaid, in a duplicate ratio of the velocity, the other part, which ariſes from the tenacity of the fluid, is uniform, or as the moment of the time: and therefore we might now proceed to the motion of bodies, which are reſiſted partly by an uniform force, or in the ratio of the moments of the time, and partly in the duplicate ratio of the velocity. But it is ſufficient to have cleared the way to this ſpeculation in the 8th and 9th Prop. foregoing, and their Corollaries. For in thoſe Propoſitions, inſtead of the uniform reſiſtance made to an aſcending body ariſing from its gravity, one may ſubſtitute the uniform reſiſtance which ariſes from the tenacity of the medium, when the body moves by its *vis inſita* alone; and when the body aſcends in a right line, add this uniform reſiſtance to the force of gravity, and ſubduct it when the body deſcends in a right line. One might alſo go on to the motion of bodies which are reſiſted in part uniformly, in part in the ratio of the velocity, and in part in the duplicate ratio of the ſame velocity. And I have opened a way to this in the 13th and 14th Prop. foregoing, in which the uniform reſiſtance ariſing from the tenacity of the medium, may be ſubſtituted for the force of gravity, or be compounded with it as before. But I haſten to other things.

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Plate III. Vol. II. P. 54.



SECTION IV.

Of the circular motion of bodies in reſiſting mediums.

LEMMA III.

Let P Q R *be a ſpiral cutting all the radii* S P, S Q, S R, *&c. in equal angles. Draw the right line* P T *touching the ſpiral in any point* P, *and cutting the radius* S Q *in* T; *draw* P O, Q O *perpendicular to the ſpiral, and meeting in* O, *and join* S O. *I ſay, that if the points* P *and* Q *approach and coincide, the angle* P S O *will become a right angle, and the ultimate ratio of the rectangle* T Q × 2 P S *to* P Q^2 *will be the ratio of equality.* Pl. 4. Fig. 1.

For from the right angles OPQ, OQR, ſubduct the equal angles SPQ, SQR, and there will remain the equal angles OPS, OQS. Therefore a circle which paſſes thro' the points O, S, P, will paſs alſo thro' the point Q. Let the points P and Q coincide, and this circle will touch the ſpiral in the place of coincidence PQ, and will therefore cut the right line QP perpendicularly. Therefore OP will become a diameter of this circle, and the angle OSP, being in a ſemicircle, becomes a right one. $Q.E.D.$

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For from the right angles *O P Q*, *O Q R*, ſubduct the equal angles *S P Q*, *S Q R*, and there will remain the equal angles *O P S*, *O Q S*. Therefore a circle which paſſes thro' the points *O*, *S*, *P*, will paſs alſo thro' the point *Q*. Let the points *P* and *Q* coincide, and this circle will touch the ſpiral in the place of coincidence *P Q*, and will therefore cut the right line *O P* perpendicularly. Therefore *O P* will become a diameter of this circle, and the angle *O S P*, being in a ſemicircle, becomes a right one. *Q. E. D.*

Draw QD, SE perpendicular to OP, and the ultimate ratio's of the lines will be as follows; TQ to PD as TS or PS to PE, or $2PO$ to $2PS$; and PD to PQ as PQ to $2PO$; and, *ex æquo perturbatè*, TQ to PQ as PQ to $2PS$. Whence PQ^2 becomes equal to $TQ \times 2PS$. *Q. E. D.*

Proposition XV. Theorem XII.

If the denſity of a medium in each place thereof be reciprocally as the diſtance of the places from an immoveable centre, and the centripetal force be in the duplicate ratio of the denſity: I ſay, that a body may revolve in a ſpiral which cuts all the radii drawn from that centre in a given angle. Pl. 4. Fig. 2.

Suppoſe every thing to be as in the foregoing Lemma, and produce SQ to V, ſo that SV may be equal to SP. In any time let a body, in a reſiſting medium, deſcribe the leaſt arc PQ, and in double the time, the leaſt arc PR; and the decrements of thoſe arcs ariſing from the reſiſtance, or their differences from the arcs which would be deſcribed in a non-reſiſting medium in the ſame times, will be to each other, as the ſquares of the times in which they are generated: Therefore the decrement of the arc PQ is the fourth part of the decrement of the arc PR. Whence alſo if the area QSr be taken equal to the area PSQ, the decrement of the arc PQ will be equal to half the lineola Rr; and therefore the force of reſiſtance and the centripetal force are to each other as the lineola's $\frac{1}{2}Rr$ and TQ which they generate in the ſame time. Becauſe the centripetal force with which the body is urged in P, is reciprocally as SP^2, and (by Lem. 10. Book 1.) the lineola TQ, which is generated by that force, is in a ratio

tio compounded of the ratio of this force and the duplicate ratio of the time in which the arc PQ is deſcribed, (for in this caſe I neglect the reſiſtance, as being infinitely leſs than the centripetal force,) it follows, that $TQ \times SP^2$, that is, (by the laſt Lemma) $\frac{1}{2} PQ^2 \times SP$, will be in a duplicate ratio of the time, and therefore the time is as $PQ \times \sqrt{SP}$; and the velocity of the body, with which the arc PQ is deſcribed in that time, as $\frac{PQ}{PQ \times \sqrt{SP}}$ or $\frac{1}{\sqrt{SP}}$, that is, in the ſubduplicate ratio of SP reciprocally. And by a like reaſoning, the velocity with which the arc QR is deſcribed, is in the ſubduplicate ratio of SQ reciprocally. Now thoſe arcs PQ and QR are as the deſcribing velocities to each other; that is, in the ſubduplicate ratio of SQ to SP, or as SQ to $\sqrt{SP \times SQ}$; and, becauſe of the equal angles SPQ, SQr, and the equal areas PSQ, QSr, the arc PQ is to the arc Qr as SQ to SP. Take the differences of the proportional conſequents, and the arc PQ will be to the arc Rr as SQ to $SP - \sqrt{SP \times SQ}$, or $\frac{1}{2} VQ$. For the points P and Q coinciding, the ultimate ratio of $SP - \sqrt{SP \times SQ}$ to $\frac{1}{2} VQ$ is the ratio of equality. Becauſe the decrement of the arc PQ ariſing from the reſiſtance, or its double Rr, is as the reſiſtance and the ſquare of the time conjunctly; the reſiſtance will be as $\frac{Rr}{PQ^2 \times SP}$. But PQ was to Rr, as SQ to $\frac{1}{2} VQ$, and thence $\frac{Rr}{PQ^2 \times SP}$ becomes as $\frac{\frac{1}{2} VQ}{PQ \times SP \times SQ}$ or as $\frac{\frac{1}{2} OS}{OP \times SP^2}$. For the points P and Q coinciding, SP and SQ coincide alſo, and the angle PVQ becomes a right one; and, becauſe of the ſimilar triangles PVQ, PSO, PQ becomes to $\frac{1}{2} VQ$ as OP to $\frac{1}{2} OS$. Therefore $\frac{OS}{OP \times SP^2}$ is as the reſiſtance, that is, in the ratio of

the

the denſity of the medium in P and the duplicate ratio of the velocity conjunctly. Subduct the duplicate ratio of the velocity, namely the ratio $\frac{1}{SP}$, and there will remain the denſity of the medium in P as $\frac{OS}{OP \times SP^2}$. Let the ſpiral be given, and, becauſe of the given ratio of OS to OP, the denſity of the medium in P will be as $\frac{1}{SP^2}$. Therefore in a medium whoſe denſity is reciprocally as SP the diſtance from the centre, a body will revolve in this ſpiral. *Q. E. D.*

Cor. 1. The velocity in any place P, is always the ſame wherewith a body in a non-reſiſting medium with the ſame centripetal force would revolve in a circle, at the ſame diſtance SP from the centre.

Cor. 2. The denſity of the medium, if the diſtance SP be given, is as $\frac{OS}{OP^2}$, but if that diſtance is not given, as $\frac{OS}{OP \times SP^2}$. And thence a ſpiral may be fitted to any denſity of the medium.

Cor. 3. The force of the reſiſtance in any place P, is to the centripetal force in the ſame place as $\frac{1}{2}OS$ to OP. For thoſe forces are to each other as $\frac{1}{2}Rr$ and TQ or as $\frac{\frac{1}{4}VQ \times PQ}{SQ}$ and $\frac{\frac{1}{2}PQ^2}{SP}$, that is, as $\frac{1}{2}VQ$ and PQ, or $\frac{1}{2}OS$ and OP. The ſpiral therefore being given, there is given the proportion of the reſiſtance to the centripetal force; and *vice verſa*, from that proportion given the ſpiral is given.

Cor. 4. Therefore the body can't revolve in this ſpiral, except where the force of reſiſtance is leſs than half the centripetal force. Let the reſiſtance be made equal to half the centripetal force, and the ſpiral will coincide with the right line PS, and in that right line

the

the body will deſcend to the centre with a velocity, that is to the velocity, with which it was proved before in the caſe of the Parabola, (Theor. 10. Book 1.) the deſcent would be made in a non-reſiſting medium, in the ſubduplicate ratio of unity to the number two. And the times of the deſcent will be here reciprocally as the velocities, and therefore given.

Cor. 5. And becauſe at equal diſtances from the centre, the velocity is the ſame in the ſpiral PQR as it is in the right line SP, and the length of the ſpiral is to the length of the right line PS, in a given ratio, namely in the ratio of OP to OS; the time of the deſcent in the ſpiral will be to the time of the deſcent in the right line SP in the ſame given ratio, and therefore given.

Cor. 6. If from the centre S with any two given intervals, two circles are deſcribed; and theſe circles remaining, the angle which the ſpiral makes with the radius PS be any how changed; the number of revolutions which the body can compleat in the ſpace between the circumferences of thoſe circles, going round in the ſpiral from one circumference to another, will be as $\frac{PS}{OS}$, or as the tangent of the angle which the ſpiral makes with the radius PS; and the time of the ſame revolutions will be as $\frac{OP}{OS}$, that is, as the ſecant of the ſame angle, or reciprocally as the denſity of the medium.

Cor. 7. If a body, in a medium whoſe denſity is reciprocally as the diſtances of places from the centre, revolves in any curve AEB (*Fig.* 3.) about that centre, and cuts the firſt radius AS in the ſame angle in B as it did before in A, and that with a velocity, that ſhall be to its firſt velocity in A reciprocally in a ſubduplicate ratio of the diſtances from the centre (that is, as AS to a mean proportional between AS and BS) that

that body will continue to deſcribe innumerable ſimilar revolutions BFC, CGD, &c. and by its interſections will diſtinguiſh the radius AS into parts AS, BS, CS, DS, &c. that are continually proportional. But the times of the revolutions will be as the perimeters of the orbits AEB, BFC, CGD, &c. directly, and the velocities at the beginnings A, B, C of thoſe orbits, inverſely; that is, as $AS^{\frac{1}{2}}$, $BS^{\frac{1}{2}}$, $CS^{\frac{1}{2}}$. And the whole time in which the body will arrive at the centre, will be to the time of the firſt revolution, as the ſum of all the continued proportionals $AS^{\frac{1}{2}}$, $BS^{\frac{1}{2}}$, $CS^{\frac{1}{2}}$, going on *ad infinitum*, to the firſt term $AS^{\frac{1}{2}}$; that is, as the firſt term $AS^{\frac{1}{2}}$ to the difference of the two firſt $AS^{\frac{1}{2}} - BS^{\frac{1}{2}}$, or as $\frac{2}{3}AS$ to AB very nearly. Whence the whole time may be eaſily found.

Cor. 8. From hence alſo may be deduced, near enough, the motions of bodies in mediums whoſe denſity is either uniform or obſerves any other aſſigned law. From the centre S, with intervals SA, SB, SC, &c. continually proportional, deſcribe as many circles; and ſuppoſe the time of the revolutions between the perimeters of any two of thoſe circles, in the medium whereof we treated, to be to the time of the revolutions between the ſame in the medium propoſed, as the mean denſity of the propoſed medium between thoſe circles, to the mean denſity of the medium whereof we treated, between the ſame circles, nearly: And that the ſecant of the angle in which the ſpiral above determined, in the medium whereof we treated, cuts the radius AS, is in the ſame ratio to the ſecant of the angle in which the new ſpiral, in the propoſed medium, cuts the ſame radius: And alſo that the number of all the revolutions between the ſame two circles is nearly as the tangents of thoſe angles. If this be done every where between every two circles, the motion will

will be continued thro' all the circles. And by this means one may without difficulty conceive at what rate and in what time bodies ought to revolve in any regular medium.

Cor. 9. And altho theſe motions becoming excentrical ſhould be performed in ſpirals approaching to an oval figure; yet conceiving the ſeveral revolutions of thoſe ſpirals to be at the ſame diſtances from each other, and to approach to the centre by the ſame degrees as the ſpiral above deſcribed, we may alſo underſtand how the motions of bodies may be performed in ſpirals of that kind.

Proposition XVI. Theorem XIII.

If the denſity of the medium in each of the places be reciprocally as the diſtance of the places from the immoveable centre, and the centripetal force be reciprocally as any power of the ſame diſtance, I ſay, that the body may revolve in a ſpiral interſecting all the radii drawn from that centre in a given angle. Pl. 4. Fig. 2.

This is demonſtrated in the ſame manner as the foregoing propoſition. For if the centripetal force in P be reciprocally as any power SP^{n+1} of the diſtance SP whoſe index is $n+1$: it will be collected as above, that the time in which the body deſcribes any arc PQ will be as $PQ \times PS^{\frac{1}{2}n}$; and the reſiſtance in P as $\frac{Rr}{PQ^2 \times SP^n}$, or as $\frac{\overline{1-\frac{1}{2}n} \times VQ}{PQ \times SP^n \times SQ}$, and therefore as $\frac{\overline{1-\frac{1}{2}n} \times OS}{OP \times SP^{n+1}}$, that is, (becauſe $\frac{\overline{1-\frac{1}{2}n} \times OS}{OP}$ is a given

given quantity) reciprocally as SP^{n+1}. And therefore, ſince the velocity is reciprocally as $SP^{\frac{1}{2}n}$, the denſity in P will be reciprocally as SP.

Cor. 1. The reſiſtance is to the centripetal force as $1-\frac{1}{2}n \times OS$ to OP.

Cor. 2. If the centripetal force be reciprocally as SP^3, $1-\frac{1}{2}n$ will be $=0$; and therefore the reſiſtance and denſity of the medium will be nothing, as in Prop. 9. Book 1.

Cor. 3. If the centripetal force be reciprocally as any power of the radius SP, whoſe index is greater than the number 3, the affirmative reſiſtance will be changed into a negative.

Scholium.

This Propoſition and the former which relate to mediums of unequal denſity, are to be underſtood of the motion of bodies that are ſo ſmall, that the greater denſity of the medium on one ſide of the body, above that on the other, is not to be conſider'd. I ſuppoſe alſo the reſiſtance, *ceteris paribus*, to be proportional to its denſity. Whence in mediums whoſe force of reſiſtance is not as the denſity, the denſity muſt be ſo much augmented or diminiſhed, that either the exceſs of the reſiſtance may be taken away, or the defect ſupplied.

Proposition XVII. Problem IV.

To find the centripetal force and the reſiſting force of the medium, by which a body, the law of the velocity being given, ſhall revolve in a given ſpiral. Pl. 4. Fig. 4.

Let that ſpiral be PQR. From the velocity, with which the body goes over the very ſmall arc PQ, the time

time will be given; and from the altitude TQ, which is as the centripetal force, and the ſquare of the time, that force will be given. Then from the difference RSr, of the areas PSQ and QSR deſcribed in equal particles of time, the retardation of the body will be given; and from the retardation will be found the reſiſting force and denſity of the medium.

Proposition XVIII. Problem V.

The law of centripetal force being given, to find the denſity of the medium in each of the places thereof, by which a body may deſcribe a given ſpiral.

From the centripetal force the velocity in each place muſt be found; then from the retardation of the velocity, the denſity of the medium is found, as in the foregoing Propoſition.

But I have explain'd the method of managing theſe Problems in the tenth Propoſition and ſecond Lemma of this Book; and will no longer detain the reader in theſe perplex'd diſquiſitions. I ſhall now add ſome things relating to the forces of progreſſive bodies, and to the denſity and reſiſtance of thoſe mediums in which the motions hitherto treated of, and thoſe akin to them, are performed.

SEC-

SECTION V.

Of the denſity and compreſſion of fluids; and of Hydroſtatics.

The Definition of a Fluid.

A fluid is any body whoſe parts yield to any force impreſſed on it, and, by yielding, are eaſily moved among themſelves.

PROPOSITION XIX. THEOREM XIV.

All the parts of a homogeneous and unmoved fluid included in any unmoved veſſel, and compreſſed on every ſide, (ſetting aſide the conſideration of condenſation, gravity, and all centripetal forces) will be equally preſſed on every ſide, and remain in their places without any motion ariſing from that preſſure. Pl. 4. Fig. 5.

CASE 1. Let a fluid be included in the ſphærical veſſel *ABC* and uniformly compreſſed on every ſide: I ſay, that no part of it will be moved by that preſſure. For if any part, as *D*, be moved, all ſuch parts at the ſame diſtance from the centre on every ſide, muſt neceſſarily be moved at the ſame time by a like motion; becauſe the preſſure of them all is ſimilar and equal; and all other mōtion is excluded that does not ariſe from that

that preſſure. But if theſe parts come all of them nearer to the centre, the fluid muſt be condenſed towards the centre, contrary to the ſuppoſition. If they recede from it, the fluid muſt be condenſed towards the circumference; which is alſo contrary to the ſuppoſition. Neither can they move in any one direction retaining their diſtance from the centre, becauſe for the ſame reaſon they may move in a contrary direction; but the ſame part cannot be moved contrary ways at the ſame time. Therefore no part of the fluid will be moved from its place. *Q. E. D.*

Case 2. I ſay now, that all the ſphærical parts of this fluid are equally preſſed on every ſide. For let *E F* be a ſphærical part of the fluid; if this be not preſſed equally on every ſide, augment the leſſer preſſure till it be preſſed equally on every ſide; and its parts (by Caſe 1.) will remain in their places. But before the increaſe of the preſſure, they would remain in their places, (by Caſe 1.) and by the addition of a new preſſure, they will be moved, by the definition of a fluid, from thoſe places. Now theſe two concluſions contradict each other. Therefore it was falſe to ſay, that the ſphere *E F* was not preſſed equally on every ſide. *Q E. D.*

Case 3. I ſay beſides, that different ſphærical parts have equal preſſures. For the contiguous ſphærical parts preſs each other mutually and equally in the point of contact, (by Law 3.) But (by Caſe 2.) they are preſſed on every ſide with the ſame force. Therefore any two ſphærical parts not contiguous, ſince an intermediate ſphærical part can touch both, will be preſſed with the ſame force. *Q. E. D.*

Case 4. I ſay now, that all the parts of the fluid are every where preſſed equally. For any two parts may be touched by ſphærical parts in any points whatever; and there they will equally preſs thoſe ſphærical

parts, (by Caſe 3.) and are, reciprocally, equally preſſed by them, (by Law 3.) *Q. E. D.*

Case 5. Since therefore any part *GHI* of the fluid is incloſed by the reſt of the fluid as in a veſſel, and is equally preſſed on every ſide; and alſo its parts equally preſs one another, and are at reſt among themſelves; it is manifeſt that all the parts of any fluid as *GHI*, which is preſſed equally on every ſide, do preſs each other mutually and equally, and are at reſt among themſelves. *Q. E. D.*

Case 6. Therefore if that fluid be included in a veſſel of a yielding ſubſtance, or that is not rigid, and be not equally preſſed on every ſide; the ſame will give way to a ſtronger preſſure, by the definition of fluidity.

Case 7. And therefore in an inflexible or rigid veſſel, a fluid will not ſuſtain a ſtronger preſſure on one ſide than on the other, but will give way to it, and that in a moment of time; becauſe the rigid ſide of the veſſel does not follow the yielding liquor. But the fluid, by thus yielding, will preſs againſt the oppoſite ſide, and ſo the preſſure will tend on every ſide to equality. And becauſe the fluid, as ſoon as it endeavours to recede from the part that is moſt preſſed, is withſtood by the reſiſtance of the veſſel on the oppoſite ſide; the preſſure will on every ſide be reduced to equality, in a moment of time, without any local motion: and from thence the parts of the fluid, (by Caſe 5.) will preſs each other mutually and equally, and be at reſt among themſelves. *Q. E. D.*

Cor. Whence neither will a motion of the parts of the fluid among themſelves, be changed by a preſſure communicated to the external ſuperficies, except ſo far as either the figure of the ſuperficies may be ſomewhere alter'd, or that all the parts of the fluid, by preſſing one another more intenſely or remiſsly, may ſlide with more or leſs difficulty among themſelves.

Pro-

Proposition XX. Theorem XV.

If all the parts of a ſphærical fluid, homogeneous at equal diſtances from the centre, lying on a ſphærical concentric bottom, gravitate towards the centre of the whole; the bottom will ſuſtain the weight of a cylinder, whoſe baſe is equal to the ſuperficies of the bottom, and whoſe altitude is the ſame with that of the incumbent fluid. Pl. 4. Fig. 6.

Let *DHM* be the ſuperficies of the bottom, and *AEI* the upper ſuperficies of the fluid. Let the fluid be diſtinguiſhed into concentric orbs of equal thickneſs, by the innumerable ſphærical ſuperficies *BFK*, *CGL*; and conceive the force of gravity to act only in the upper ſuperficies of every orb, and the actions to be equal on the equal parts of all the ſuperficies. Therefore the upper ſuperficies *AE* is preſſed by the ſingle force of its own gravity, by which all the parts of the upper orb, and the ſecond ſuperficies *BFK* will, (by Prop. 19.) according to its meaſure, be equally preſſed. The ſecond ſuperficies *BFK* is preſſed likewiſe by the force of its own gravity, which added to the former force, makes the preſſure double. The third ſuperficies *CGL* is, according to its meaſure, acted on by this preſſure and the force of its own gravity beſides, which makes its preſſure triple. And in like manner the fourth ſuperficies receives a quadruple preſſure, the fifth ſuperficies a quintuple, and ſo on. Therefore the preſſure acting on every ſuperficies, is not as the ſolid quantity of the incumbent fluid, but as the number of the orbs reaching to the upper ſurface of the fluid; and is equal to the gravity of the loweſt orb multiplied by the number of orbs: that is, to the gravity of a ſolid

whoſe ultimate ratio to the cylinder abovementioned (when the number of the orbs is increaſed and their thickneſs diminiſhed *ad infinitum*, ſo that the action of gravity from the loweſt ſuperficies to the uppermoſt may become continued) is the ratio of equality. Therefore the loweſt ſuperficies ſuſtains the weight of the cylinder above-determined. *Q.E.D.* And by a like reaſoning the Propoſition will be evident, where the gravity of the fluid decreaſes in any aſſigned ratio of the diſtance from the centre, and alſo where the fluid is more rare above and denſer below. *Q.E.D.*

Cor. 1. Therefore the bottom is not preſſed by the whole weight of the incumbent fluid, but only ſuſtains that part of it which is deſcribed in the Propoſition; the reſt of the weight being ſuſtained archwiſe by the ſphærical figure of the fluid.

Cor. 2. The quantity of the preſſure is the ſame always at equal diſtances from the centre, whether the ſuperficies preſſed be parallel to the horizon, or perpendicular, or oblique; or whether the fluid, continued upwards from the compreſſed ſuperficies, riſes perpendicularly in a rectilinear direction, or creeps obliquely thro' crooked cavities and canals, whether thoſe paſſages be regular or irregular, wide or narrow. That the preſſure is not alter'd by any of theſe circumſtances, may be collected by applying the demonſtration of this Theorem to the ſeveral caſes of fluids.

Cor. 3. From the ſame demonſtration it may alſo be collected, (by Prop. 19.) that the parts of an heavy fluid acquire no motion among themſelves, by the preſſure of the incumbent weight; except that motion which ariſes from condenſation.

Cor. 4. And therefore if another body of the ſame ſpecific gravity, incapable of condenſation, be immerſed in this fluid, it will acquire no motion by the preſſure of the incumbent weight: it will neither deſcend, nor aſcend, nor change its figure. If it be ſphærical,

Plate IV. Vol. II. P. 68.

Fig. 1.
p

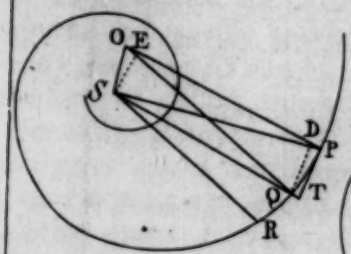


Fig. 2.
p. 56. 61.

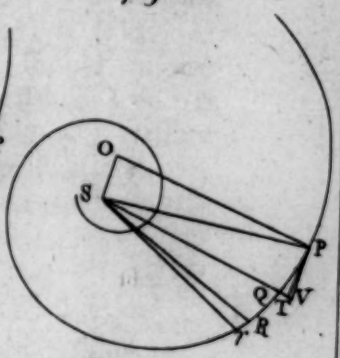


Fig. 3.
p. 59.

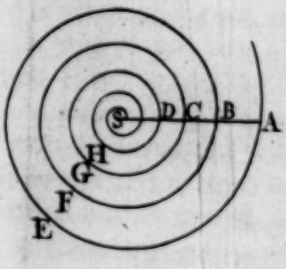


Fig. 4.
p. 62.

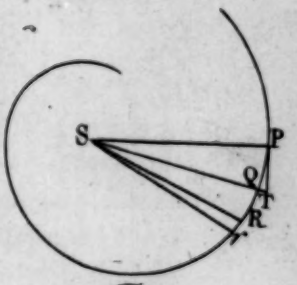


Fig. 5.
p. 64.

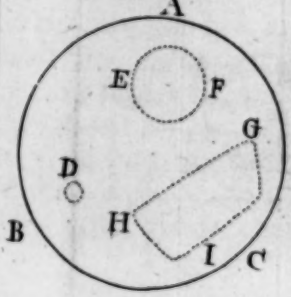
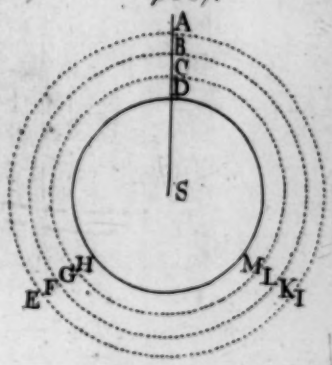


Fig. 6.
p. 67.



ſphærical, it will remain ſo notwithſtanding the preſſure; if it be ſquare, it will remain ſquare: and that whether it be ſoft, or fluid; whether it ſwims freely in the fluid, or lies at the bottom. For any internal part of a fluid is in the ſame ſtate with the ſubmerſed body; and the caſe of all ſubmerſed bodies that have the ſame magnitude, figure, and ſpecific gravity, is alike. If a ſubmerſed body retaining its weight, ſhould diſſolve and put on the form of a fluid, this body, if before it would have aſcended, deſcended, or from any preſſure aſſume a new figure, would now likewiſe aſcend, deſcend, or put on a new figure; and that becauſe its gravity and the other cauſes of its motion remain. But (by Caſe 5. Prop. 19.) it would now be at reſt and retain its figure. Therefore alſo in the former caſe.

Cor. 5. Therefore a body that is ſpecifically heavier than a fluid contiguous to it, will ſink, and that which is ſpecifically lighter will aſcend, and attain ſo much motion and change of figure, as that exceſs or defect of gravity is able to produce. For that exceſs or defect is the ſame thing as an impulſe, by which a body, otherwiſe *in equilibrio* with the parts of the fluid, is acted on; and may be compared with the exceſs or defect of a weight in one of the ſcales of a balance.

Cor. 6. Therefore bodies placed in fluids have a twofold gravity; the one true and abſolute, the other apparent, vulgar and comparative. Abſolute gravity is the whole force with which the body tends downwards: relative and vulgar gravity is the exceſs of gravity with which the body tends downwards more than the ambient fluid. By the firſt kind of gravity, the parts of all fluids and bodies gravitate in their proper places; and therefore their weights taken together, compoſe the weight of the whole. For the whole taken together is heavy, as may be experienced in veſſels full of liquor; and the weight of the whole is equal to the weights of all the parts, and is therefore compoſed of

them. By the other kind of gravity bodies do not gravitate in their places, that is, compared with one another, they do not preponderate, but hindering one another's endeavours to deſcend, remain in their proper places, as if they were not heavy. Thoſe things which are in the air and do not preponderate, are commonly looked on as not heavy. Thoſe which do preponderate are commonly reckoned heavy, in as much as they are not ſuſtained by the weight of the air. The common weights are nothing elſe but the exceſs of the true weights above the weight of the air. Hence alſo vulgarly thoſe things are called light, which are leſs heavy; and by yielding to the preponderating air, mount upwards. But theſe are only comparatively light, and not truly ſo, becauſe they deſcend *in vacuo*. Thus in water, bodies which, by their greater or leſs gravity, deſcend or aſcend, are comparatively and apparently heavy or light, and their comparative and apparent gravity or levity is the exceſs or defect by which their true gravity either exceeds the gravity of the water or is exceeded by it. But thoſe things which neither by preponderating deſcend, nor, by yielding to the preponderating fluid, aſcend, altho' by their true weight they do increaſe the weight of the whole, yet comparatively, and in the ſenſe of the vulgar, they do not gravitate in the water. For theſe caſes are alike demonſtrated.

Cor. 7. Theſe things which have been demonſtrated concerning gravity, take place in any other centripetal forces.

Cor. 8. Therefore if the medium in which any body moves be acted on either by its own gravity, or by any other centripetal force, and the body be urged more powerfully by the ſame force; the difference of the forces is that very motive force, which in the foregoing Propoſitions I have conſider'd as a centripetal force. But if the body be more lightly urg'd by

that

that force, the difference of the forces becomes a centrifugal force, and is to be conſider'd as ſuch.

Cor. 9. But ſince fluids by preſſing the included bodies do not change their external figures, it appears alſo, (by Cor. Prop. 19.) that they will not change the ſituation of their internal parts in relation to one another; and therefore if animals were immerſed therein, and that all ſenſation did ariſe from the motion of their parts; the fluid will neither hurt the immerſed bodies, nor excite any ſenſation, unleſs ſo far as thoſe bodies may be condenſed by the compreſſion. And the caſe is the ſame of any ſyſtem of bodies encompaſſed with a compreſſing fluid. All the parts of the ſyſtem will be agitated with the ſame motions, as if they were placed in a vacuum, and would only retain their comparative gravity; unleſs ſo far as the fluid may ſomewhat reſiſt their motions, or be requiſite to conglutinate them by compreſſion.

Proposition XXI. Theorem XVI.

Let the denſity of any fluid be proportional to the compreſſion, and its parts be attracted downwards by a centripetal force reciprocally proportional to the diſtances from the centre: I ſay, that, if thoſe diſtances be taken continually proportional, the denſities of the fluid at the ſame diſtances will be alſo continually proportional. Pl. 5. Fig. 1.

Let *ATV* denote the ſphærical bottom of the fluid, *S* the centre, *SA*, *SB*, *SC*, *SD*, *SE*, *SF*, &c. diſtances continually proportional. Erect the perpendiculars *AH*, *BI*, *CK*, *DL*, *EM*, *FN*, &c. which ſhall be as the denſities of the medium in the places *A*, *B*, *C*, *D*, *E*, *F*; and the ſpecific gravities in thoſe places will be

as $\frac{AH}{AS}, \frac{BI}{BS}, \frac{CK}{CS}$, &c. or, which is all one, as $\frac{AH}{AB}, \frac{BI}{BC}, \frac{CK}{CD}$, &c. Suppose first these gravities to be uniformly continued from *A* to *B*, from *B* to *C*, from *C* to *D*, &c. the decrements in the points *B*, *C*, *D*, &c. being taken by steps. And these gravities drawn into the altitudes *AB*, *BC*, *CD*, &c. will give the pressures *AH*, *BI*, *CK*, &c. by which the bottom *ATV* is acted on, (by Theor. 15.) Therefore the particle *A* sustains all the pressures *AH*, *BI*, *CK*, *DL*, &c. proceeding *in infinitum*; and the particle *B* sustains the pressures of all but the first *AH*; and the particle *C* all but the two first *AH*, *BI*; and so on: and therefore the density *AH* of the first particle *A* is to the density *BI* of the second particle *B* as the sum of all *AH* + *BI* + *CK* + *DL*, *in infinitum*, to the sum of all *BI* + *CK* + *DL*, &c. And *BI* the density of the second particle *B* is to *CK* the density of the third *C*, as the sum of all *BI* + *CK* + *DL*, &c. to the sum of all *CK* + *DL*, &c. Therefore these sums are proportional to their differences *AH*, *BI*, *CK*, &c. and therefore continually proportional, (by Lem. 1. of this Book) and therefore the differences *AH*, *BI*, *CK*, &c. proportional to the sums, are also continually proportional. Wherefore since the densities in the places *A*, *B*, *C*, &c. are as *AH*, *BI*, *CK*, &c. they will also be continually proportional. Proceed intermissively, and, *ex æquo*, at the distances *SA*, *SC*, *SE* continually proportional, the densities *AH*, *CK*, *EM* will be continually proportional. And by the same reasoning, at any distances *SA*, *SD*, *SG* continually proportional, the densities *AH*, *DL*, *GO* will be continually proportional. Let now the points *A*, *B*, *C*, *D*, *E*, &c. coincide, so that the progression of the specific gravities from the bottom *A* to the top of the fluid may be made continual; and at any distances *SA*, *SD*,

SG

SG continually proportional, the denſities *AH*, *DL*, *GO* being all along continually proportional, will ſtill remain continually proportional. *Q. E. D.*

Cor. Hence if the denſity of the fluid in two places as *A* and *E* be given, its denſity in any other place *Q* may be collected. With the centre *S*, and the rectangular aſymptotes *SQ*, *SX* deſcribe (*Fig.* 2.) an Hyperbola cutting the perpendiculars *AH*, *EM*, *QT* in *a*, *e*, and *q*, as alſo the perpendiculars *HX*, *MT*, *TZ* let fall upon the aſymptote *SX* in *h*, *m*, and *t*. Make the area *YmtZ* to the given area *YmhX* as the given area *EeqQ* to the given area *EeaA*; and the line *Zt* produced will cut off the line *QT* proportional to the denſity. For if the lines *SA*, *SE*, *SQ* are continually proportional, the areas *EeqQ*, *EeaA* will be equal, and thence the areas *YmtZ*, *XhmY* proportional to them will be alſo equal, and the lines *SX*, *SY*, *SZ*, that is, *AH*, *EM*, *QT* continually proportional as they ought to be. And if the lines *SA*, *SE*, *SQ* obtain any other order in the ſeries of continued proportionals, the lines *AH*, *EM*, *QT*, becauſe of the proportional hyperbolic areas, will obtain the ſame order in another ſeries of quantities continually proportional.

Proposition XXII. Theorem XVII.

Let the denſity of any fluid be proportional to the compreſſion, and its parts be attracted downwards by a gravitation reciprocally proportional to the ſquares of the diſtances from the centre: I ſay, that, if the diſtances be taken in harmonic progreſſion, the denſities of the fluid at thoſe diſtances will be in a geometrical progreſſion. Pl. 5. Fig. 3.

Let *S* denote the centre, and *SA*, *SB*, *SC*, *SD*, *SE*, the diſtances in Geometrical progreſſion. Erect the

the perpendiculars AH, BI, CK, &c. which ſhall be as the denſities of the fluid in the places A, B, C, D, E, &c. and the ſpecific gravities thereof in thoſe places will be as $\frac{AH}{SA^2}$, $\frac{BI}{SB^2}$, $\frac{CK}{SC^2}$, &c. Suppoſe theſe gravities to be uniformly continued, the firſt from A to B, the ſecond from B to C, the third from C to D, &c. And theſe drawn into the altitudes AB, BC, CD, DE, &c. or, which is the ſame thing, into the diſtances SA, SB, SC, &c. proportional to thoſe altitudes, will give $\frac{AH}{SA}$, $\frac{BI}{SB}$, $\frac{CK}{SC}$, &c. the exponents of the preſſures. Therefore ſince the denſities are as the ſums of thoſe preſſures, the differences $AH-BI$, $BI-CK$, &c. of the denſities will be as the differences of thoſe ſums $\frac{AH}{SA}$, $\frac{BI}{SB}$, $\frac{CK}{SC}$, &c. With the centre S, and the aſymptotes SA, Sx, deſcribe any Hyperbola, cutting the perpendiculars AH, BI, CK, &c. in a, b, c, &c. and the perpendiculars Ht, Iu, Kw let fall upon the aſymptote Sx, in h, i, k; and the differences of the denſities tu, uw, &c. will be as $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. And the rectangles $tu \times th$, $uw \times ui$, &c. or tp, uq, &c. as $\frac{AH \times th}{SA}$, $\frac{BI \times ui}{SB}$, &c. that is, as Aa, Bb, &c. For, by the nature of the Hyperbola, SA is to AH or St, as th to Aa, and therefore $\frac{AH \times th}{SA}$ is equal to Aa. And, by a like reaſoning, $\frac{BI \times ui}{SB}$ is equal to Bb, &c. But Aa, Bb, Cc, &c. are continually proportional, and therefore proportional to their differences $Aa-Bb$, $Bb-Cc$, &c. and therefore the rectangles tp, uq, &c. are proportional

to thoſe differences; as alſo the ſums of the rectangles $tp + uq$ or $tp + uq + wr$ to the ſums of the differences $Aa - Cc$ or $Aa - Dd$. Suppoſe ſeveral of theſe terms, and the ſum of all the differences, as $Aa - Ff$, will be proportional to the ſum of all the rectangles, as $zthn$. Increaſe the number of terms, and diminiſh the diſtances of the points A, B, C, &c. *in infinitum*, and thoſe rectangles will become equal to the hyperbolic area $zthn$, and therefore the difference $Aa - Ff$ is proportional to this area. Take now any diſtances as SA, SD, SF in harmonic progreſſion, and the differences $Aa - Dd$, $Dd - Ff$ will be equal; and therefore the areas $thlx$, $xlnz$ proportional to thoſe differences will be equal among themſelves, and the denſities St, Sx, Sz, that is, AH, DL, FN continually proportional. *Q. E. D.*

Cor. 2. Hence if any two denſities of the fluid, as AH and BI be given, the area $thiu$, anſwering to their difference tu will be given; and thence the denſity FN will be found at any height SF, by taking the area $thnz$ to that given area $thiu$ as the difference $Aa - Ff$ to the difference $Aa - Bb$.

Scholium.

By a like reaſoning, it may be proved, that if the gravity of the particles of a fluid be diminiſhed in a triplicate ratio of the diſtances from the centre; and the reciprocals of the ſquares of the diſtances SA, SB, SC, &c. (namely $\frac{SA^3}{SA^2}, \frac{SA^3}{SB^2}, \frac{SA^3}{SC^2}$) be taken in an Arithmetical progreſſion, the denſities AH, BI, CK, &c. will be in a Geometrical progreſſion. And if the gravity be diminiſhed in a quadruplicate ratio of the diſtances, and the reciprocals of the cubes of the diſtances (as $\frac{SA^4}{SA^3}, \frac{SA^4}{SB^3}, \frac{SA^4}{SC^3}$ &c.) be taken in

Arith-

Arithmetical progreſſion, the denſities *AH*, *BI*, *CK*, &c. will be in Geometrical progreſſion. And ſo *in infinitum*. Again, if the gravity of the particles of the fluid be the ſame at all diſtances, and the diſtances be in Arithmetical progreſſion, the denſities will be in a Geometrical progreſſion, as Dr. *Halley* has found. If the gravity be as the diſtance, and the ſquares of the diſtances be in Arithmetical progreſſion, the denſities will be in Geometrical progreſſion. And ſo *in infinitum*. Theſe things will be ſo, when the denſity of the fluid condenſed by compreſſion is as the force of compreſſion, or, which is the ſame thing, when the ſpace poſſeſſed by the fluid is reciprocally as this force. Other laws of condenſation may be ſuppoſed, as that the cube of the compreſſing force may be as the biquadrate of the denſity; or the triplicate ratio of the force the ſame with the quadruplicate ratio of the denſity: In which caſe, if the gravity be reciprocally as the ſquare of the diſtance from the centre, the denſity will be reciprocally as the cube of the diſtance. Suppoſe that the cube of the compreſſing force be as the quadrato-cube of the denſity; and if the gravity be reciprocally as the ſquare of the diſtance, the denſity will be reciprocally in a ſeſquiplicate ratio of the diſtance. Suppoſe the compreſſing force to be in a duplicate ratio of the denſity, and the gravity reciprocally in a duplicate ratio of the diſtance, and the denſity will be reciprocally as the diſtance. To run over all the caſes that might be offer'd, would be tedious. But as to our own air, this is certain from experiment, that its denſity is either accurately or very nearly at leaſt as the compreſſing force; and therefore the denſity of the air in the atmoſphere of the earth is as the weight of the whole incumbent air, that is, as the height of the mercury in the barometer.

PRO-

Proposition XXIII. Theorem XVIII.

If a fluid be compoſed of particles mutually flying each other, and the denſity be as the compreſſion, the centrifugal forces of the particles will be reciprocally proportional to the diſtances of their centres. And vice verſa, *particles flying each other with forces that are reciprocally proportional to the diſtances of their centres, compoſe an elaſtic fluid, whoſe denſity is as the compreſſion.* Pl. 5. Fig. 4.

Let the fluid be ſuppoſed to be included in a cubic ſpace ACE, and then to be reduced by compreſſion into a leſſer cubic ſpace ace; and the diſtances of the particles retaining a like ſituation with reſpect to each other in both the ſpaces, will be as the ſides AB, ab of the cubes; and the denſities of the mediums will be reciprocally as the containing ſpaces AB^3, ab^3. In the plane ſide of the greater cube $ABCD$ take the ſquare DP equal to the plane ſide db of the leſſer cube: and, by the ſuppoſition, the preſſure with which the ſquare DP urges the incloſed fluid, will be to the preſſure with which that ſquare db urges the incloſed fluid, as the denſities of the mediums are to each other, that is, as ab^3 to AB^3. But the preſſure with which the ſquare DB urges the included fluid, is to the preſſure with which the ſquare DP urges the ſame fluid, as the ſquare DB to the ſquare DP, that is, as AB^2 to ab^2. Therefore, *ex æquo*, the preſſure with which the ſquare DB urges the fluid is to the preſſure with which the ſquare db urges the fluid; as ab to AB. Let the planes FGH, fgh, be drawn thro' the middles of the two cubes, and divide the fluid into two parts. Theſe parts will

preſs each other mutually with the ſame forces with which they are themſelves preſſed by the planes *AC*, *ac*, that is, in the proportion of *ab* to *AB*: and therefore the centrifugal forces by which theſe preſſures are ſuſtained, are in the ſame ratio. The number of the particles being equal, and the ſituation alike, in both cubes, the forces which all the particles exert, according to the planes *FGH*, *fgh*, upon all, are as the forces which each exerts on each. Therefore the forces which each exerts on each according to the plane *FGH* in the greater cube, are to the forces which each exerts on each according to the plane *fgh* in the leſſer cube, as *ab* to *AB*, that is, reciprocally as the diſtances of the particles from each other. *Q.E.D.*

And, *vice verſâ*, if the forces of the ſingle particles are reciprocally as the diſtances, that is, reciprocally as the ſides of the cubes *AB*, *ab*; the ſums of the forces will be in the ſame ratio, and the preſſures of the ſides *DB*, *db* as the ſums of the forces; and the preſſure of the ſquare *DP* to the preſſure of the ſide *DB* as ab^2 to AB^2. And, *ex æquo*, the preſſure of the ſquare *DP* to the preſſure of the ſide *db* as ab^3 to AB^3, that is, the force of compreſſion in the one to the force of compreſſion in the other, as the denſity in the former to the denſity in the latter. *Q.E.D.*

Scholium.

By a like reaſoning, if the centrifugal forces of the particles are reciprocally in the duplicate ratio of the diſtances between the centres, the cubes of the compreſſing forces will be as the biquadrates of the denſities. If the centrifugal forces be reciprocally in the triplicate or quadruplicate ratio of the diſtances, the cubes of the compreſſing forces will be as the quadrato-cubes, or cubo-cubes of the denſities. And univerſally, if D be put for the diſtance, and E for the denſity

ſity of the compreſſed fluid, and the centrifugal forces be reciprocally as any power D^n of the diſtance, whoſe index is the number n; the compreſſing forces will be as the cube roots of the power E^{n+2}, whoſe index is the number $n+2$: and the contrary. All theſe things are to be underſtood of particles whoſe centrifugal forces terminate in thoſe particles that are next them, or are diffuſed not much further. We have an example of this in magnetical bodies. Their attractive virtue is terminated nearly in bodies of their own kind that are next them. The virtue of the magnet is contracted by the interpoſition of an iron plate; and is almoſt terminated at it. For bodies further off are not attracted by the magnet ſo much as by the iron plate. If in this manner particles repel others of their own kind that lie next them, but do not exert their virtue on the more remote, particles of this kind will compoſe ſuch fluids as are treated of in this propoſition. If the virtue of any particle diffuſe itſelf every way *in infinitum*, there will be required a greater force to produce an equal condenſation of a greater quantity of the fluid. But whether elaſtic fluids do really conſiſt of particles ſo repelling each other, is a phyſical queſtion. We have here demonſtrated mathematically the property of fluids conſiſting of particles of this kind, that hence philoſophers may take occaſion to diſcuſs that queſtion.

SEC.

SECTION VI.

Of the motion and reſiſtance of funependulous bodies.

PROPOSITION XXIV. THEOREM XIX.

The quantities of matter in funependulous bodies, whoſe centres of oſcillation are equally diſtant from the centre of ſuſpenſion, are in a ratio compounded of the ratio of the weights and the duplicate ratio of the times of the oſcillations in vacuo.

For the velocity, which a given force can generate in a given matter in a given time, is as the force and the time directly, and the matter inverſely. The greater the force or the time is, or the leſs the matter, the greater velocity will be generated. This is manifeſt from the ſecond law of motion. Now if pendulums are of the ſame length, the motive forces in places equally diſtant from the perpendicular are as the weights; and therefore if two bodies by oſcillating deſcribe equal arcs, and thoſe arcs are divided into equal parts; ſince the times in which the bodies deſcribe each of the correſpondent parts of the arcs are as the times of the whole oſcillations, the velocities in the correſpondent parts of the

the oſcillations will be to each other, as the motive forces and the whole times of the oſcillations directly, and the quantities of matter reciprocally: and therefore the quantities of matter are as the forces and the times of the oſcillations directly and the velocities reciprocally. But the velocities reciprocally are as the times, and therefore the times directly and the velocities reciprocally are as the ſquares of the times; and therefore the quantities of matter are as the motive forces and the ſquares of the times, that is, as the weights and the ſquares of the times. *Q. E. D.*

Cor. 1. Therefore if the times are equal, the quantities of matter in each of the bodies are as the weights.

Cor. 2. If the weights are equal, the quantities of matter will be as the ſquares of the times.

Cor. 3. If the quantities of matter are equal, the weights will be reciprocally as the ſquares of the times.

Cor. 4. Whence ſince the ſquares of the times, *ceteris paribus*, are as the lengths of the pendulums; therefore if both the times and quantities of matter are equal, the weights will be as the lengths of the pendulums.

Cor. 5. And univerſally, the quantity of matter in the pendulous body is as the weight and the ſquare of the time directly, and the length of the pendulum inverſely.

Cor. 6. But in a non-reſiſting medium, the quantity of matter in the pendulous body is as the comparative weight and the ſquare of the time directly, and the length of the pendulum inverſely. For the comparative weight is the motive force of the body in any heavy medium, as was ſhewn above; and therefore does the ſame thing in ſuch a non-reſiſting medium, as the abſolute weight does in a vacuum.

Cor. 7. And hence appears a method both of comparing bodies one among another, as to the quantity of

matter in each; and of comparing the weights of the ſame body in different places, to know the variation of its gravity. And by experiments made with the greateſt accuracy, I have always found the quantity of matter in bodies to be proportional to their weight.

PROPOSITION XXV. THEOREM XX.

Funipendulous bodies that are, in any medium, reſiſted in the ratio of the moments of time, and funipendulous bodies that move in a non-reſiſting medium of the ſame ſpecific gravity, perform their oſcillations in a cycloid in the ſame time, and deſcribe proportional parts of arcs together. Pl. 5. Fig. 5.

Let *AB* be an arc of a cycloid, which a body *D*, by vibrating in a non-reſiſting medium ſhall deſcribe in any time. Biſect that arc in *C*, ſo that *C* may be the loweſt point thereof; and the accelerative force with which the body is urged in any place *D* or *d* or *E* will be as the length of the arc *CD* or *Cd* or *CE*. Let that force be expreſſed by that ſame arc; and ſince the reſiſtance is as the moment of the time, and therefore given, let it be expreſs'd by the given part *CO* of the cycloidal arc, and take the arc *Od* in the ſame ratio to the arc *CD* that the arc *OB* has to the arc *CB*: and the force with which the body in *d* is urged in a reſiſting medium, being the exceſs of the force *Cd* above the reſiſtance *CO*, will be expreſſed by the arc *Od*, and will therefore be to the force with which the body *D* is urged in a non-reſiſting medium in the place *D*, as the arc *Od* to the arc *CD*; and therefore alſo in the place *B*, as the arc *OB* to the arc *CB*. Therefore if two bodies *D*, *d* go from the place *B*, and are urged by theſe forces; ſince the forces at the beginning are as the

arcs *CB* and *OB*, the firſt velocities and arcs firſt deſcribed will be in the ſame ratio. Let thoſe arcs be *BD* and *Bd*, and the remaining arcs *CD*, *Od*, will be in the ſame ratio. Therefore the forces, being proportional to thoſe arcs *CD*, *Od*, will remain in the ſame ratio as at the beginning, and therefore the bodies will continue deſcribing together arcs in the ſame ratio. Therefore the forces and velocities and the remaining arcs *CD*, *Od*, will be always as the whole arcs *CB*, *OB*, and therefore thoſe remaining arcs will be deſcribed together. Therefore the two bodies *D* and *d* will arrive together at the places *C* and *O*; that which moves in the non-reſiſting medium, at the place *C*, and the other, in the reſiſting medium, at the place *O*. Now ſince the velocities in *C* and *O* are as the arcs *CB*, *OB*, the arcs which the bodies deſcribe when they go farther, will be in the ſame ratio. Let thoſe arcs be *CE* and *Oe*. The force with which the body *D* in a non-reſiſting medium is retarded in *E* is as *CE*, and the force with which the body *d* in the reſiſting medium is retarded in *e*, is as the ſum of the force *Ce* and the reſiſtance *CO*, that is, as *Oe*; and therefore the forces with which the bodies are retarded, are as the arcs *CB*, *OB*, proportional to the arcs *CE*, *Oe*; and therefore the velocities, retarded in that given ratio, remain in the ſame given ratio. Therefore the velocities and the arcs deſcribed with thoſe velocities, are always to each other in that given ratio of the arcs *CB* and *OB*; and therefore if the entire arcs *AB*, *aB* are taken in the ſame ratio, the bodies *D* and *d* will deſcribe thoſe arcs together, and in the places *A* and *a* will loſe all their motion together. Therefore the whole oſcillations are iſochronal, or are performed in equal times; and any parts of the arcs, as *BD*, *Bd*, or *BE*, *Be*, that are deſcribed together, are proportional to the whole arcs *BA*, *Ba*. *Q.E.D.*

Cor. Therefore the ſwifteſt motion in a reſiſting medium does not fall upon the loweſt point *C*, but is found in that point *O*, in which the whole arc deſcribed *Ba* is biſected. And the body proceeding from thence to *a*, is retarded at the ſame rate with which it was accelerated before in its deſcent from *B* to *O*.

Proposition XXVI. Theorem XXI.

Funipendulous bodies, that are reſiſted in the ratio of the velocity, have their oſcillations in a cycloid iſochronal.

For if two bodies, equally diſtant from their centres of ſuſpenſion, deſcribe, in oſcillating, unequal arcs, and the velocities in the correſpondent parts of the arcs be to each other as the whole arcs; the reſiſtances, proportional to the velocities, will be alſo to each other as the ſame arcs. Therefore if theſe reſiſtances be ſubducted from or added to the motive forces ariſing from gravity which are as the ſame arcs, the differences or ſums will be to each other in the ſame ratio of the arcs: and ſince the increments and decrements of the velocities are as theſe differences or ſums, the velocities will be always as the whole arcs: Therefore if the velocities are in any one caſe as the whole arcs, they will remain always in the ſame ratio. But at the beginning of the motion, when the bodies begin to deſcend and deſcribe thoſe arcs, the forces, which at that time are proportional to the arcs, will generate velocities proportional to the arcs. Therefore the velocities will be always as the whole arcs to be deſcribed, and therefore thoſe arcs will be deſcribed in the ſame time. *Q. E. D.*

Pro-

Proposition XXVII. Theorem XXII.

If funipendulous bodies are reſiſted in the duplicate ratio of their velocities, the differences between the times of the oſcillations in a reſiſting medium, and the times of the oſcillations in a non-reſiſting medium of the ſame ſpecific gravity, will be proportional to the arcs deſcribed in oſcillating nearly.

For let equal pendulums in a reſiſting medium deſcribe the unequal arcs A, B; and the reſiſtance of the body in the arc A will be to the reſiſtance of the body in the correſpondent part of the arc B in the duplicate ratio of the velocities, that is, as A A to B B nearly. If the reſiſtance in the arc B were to the reſiſtance in the arc A as A B to A A; the times in the arcs A and B would be equal (by the laſt Prop.) Therefore the reſiſtance A A in the arc A, or A B in the arc B, cauſes the exceſs of the time in the arc A above the time in a non-reſiſting medium; and the reſiſtance B B cauſes the exceſs of the time in the arc B above the time in a non-reſiſting medium. But thoſe exceſſes are as the efficient forces A B and B B nearly, that is, as the arcs A and B. *Q. E. D.*

Cor. 1. Hence from the times of the oſcillations in unequal arcs in a reſiſting medium, may be known the times of the oſcillations in a non-reſiſting medium of the ſame ſpecific gravity. For the difference of the times will be to the exceſs of the time in the leſſer arc above the time in a non-reſiſting medium, as the difference of the arcs to the leſſer arc.

Cor. 2. The ſhorter oſcillations are more iſochronal, and very ſhort ones are performed nearly in the

ſame times as in a non-reſiſting medium. But the times of thoſe which are performed in greater arcs are a little greater, becauſe the reſiſtance in the deſcent of the body, by which the time is prolonged, is greater, in proportion to the length deſcribed in the deſcent, than the reſiſtance in the ſubſequent aſcent, by which the time is contracted. But the time of the oſcillations, both ſhort and long, ſeems to be prolonged in ſome meaſure by the motion of the medium. For retarded bodies are reſiſted ſomewhat leſs, in proportion to the velocity, and accelerated bodies ſomewhat more, than thoſe that proceed uniformly forwards; becauſe the medium, by the motion it has received from the bodies, going forwards the ſame way with them, is more agitated in the former caſe, and leſs in the latter; and ſo conſpires more or leſs with the bodies moved. Therefore it reſiſts the pendulums in their deſcent more, and in their aſcent leſs, than in proportion to the velocity; and theſe two cauſes concurring prolong the time.

PROPOSITION XXVIII. THEOREM XXIII.

If a funipendulous body, oſcillating in a cycloid, be reſiſted in the ratio of the moments of the time, its reſiſtance will be to the force of gravity as the exceſs of the arc deſcribed in the whole deſcent above the arc deſcribed in the ſubſequent aſcent, to twice the length of the pendulum. Pl. 5. Fig. 5.

Let *BC* repreſent the arc deſcribed in the deſcent, *Ca* the arc deſcribed in the aſcent, and *Aa* the difference of the arcs: and things remaining as they were conſtructed and demonſtrated in Prop. 25. the force with which the oſcillating

oſcillating body is urged in any place *D*, will be to the force of reſiſtance as the arc *CD* to the arc *CO*, which is half of that difference *Aa*. Therefore the force with which the oſcillating body is urged at the beginning or the higheſt point of the cycloid, that is, the force of gravity, will be to the reſiſtance as the arc of the cycloid, between that higheſt point and loweſt point *C*, is to the arc *CO*; that is, (doubling thoſe arcs) as the whole cycloidal arc, or twice the length of the pendulum, to the arc *Aa*. *Q. E. D.*

Proposition XXIX. Problem VI.

Suppoſing that a body oſcillating in a cycloid is reſiſted in a duplicate ratio of the velocity: to find the reſiſtance in each place. Pl. 5. Fig. 6.

Let *Ba* be an arc deſcribed in one entire oſcillation, *C* the loweſt point of the cycloid, and *CZ* half the whole cycloidal arc, equal to the length of the pendulum; and let it be required to find the reſiſtance of the body in any place *D*. Cut the indefinite right line *OQ* in the points *O*, *S*, *P*, *Q*, ſo that (erecting the perpendiculars *OK*, *ST*, *PI*, *QE*, and with the centre *O*, and the aſymptotes *OK*, *OQ* deſcribing the hyperbola *TIGE* cutting the perpendiculars *ST*, *PI*, *QE* in *T*, *I* and *E*, and thro' the point *I* drawing *KF*, parallel to the aſymptote *OQ*, meeting the aſymptote *OK* in *K*, and the perpendiculars *ST* and *QE* in *L* and *F*) the hyperbolic area *PIEQ* may be to the hyperbolic area *PITS* as the arc *BC*, deſcribed in the deſcent of the body, to the arc *Ca* deſcribed in the aſcent; and that the area *IEF* may be to the area *ILT* as *OQ* to *OS*. Then with the perpendicular *MN* cut off the hyperbolic area *PINM*, and let that area be to the hyperbolic area *PIEQ* as the arc *CZ* to the arc

BC deſcribed in the deſcent. And if the perpendicular *RG* cut off the hyperbolic area *PIGR*, which ſhall be to the area *PIEQ* as any arc *CD* to the arc *BC* deſcribed in the whole deſcent; the reſiſtance in any place *D* will be to the force of gravity, as the area $\frac{OR}{OQ}$ *IEF* — *IGH* to the area *PINM*.

For ſince the forces ariſing from gravity with which the body is urged in the places *Z*, *B*, *D*, *a*, are as the arcs *CZ*, *CB*, *CD*, *Ca*, and thoſe arcs are as the areas *PINM*, *PIEQ*, *PIGR*, *PITS*; let thoſe areas be the exponents both of the arcs and of the forces reſpectively. Let *Dd* be a very ſmall ſpace deſcribed by the body in its deſcent; and let it be expreſſed by the very ſmall area *RGgr* comprehended between the parallels *RG*, *rg*; and produce *rg* to *h*, ſo that *GHhg*, and *RGgr* may be the contemporaneous decrements of the areas *IGH*, *PIGR*. And the increment *GHhg* — $\frac{Rr}{OQ}$*IEF*, or *Rr*×*HG* — $\frac{Rr}{OQ}$*IEF*, of the area $\frac{OR}{OQ}$*IEF* — *IGH* will be to the decrement *RGgr*, or *Rr*×*RG*, of the area *PIGR*, as *HG* — $\frac{IEF}{OQ}$ to *RG*; and therefore as *OR*×*HG* — $\frac{OR}{OQ}$*IEF* to *OR*×*GR* or *OP*×*PI*, that is (becauſe of the equal quantities *OR*×*HG*, *OR*×*HR* — *OR*×*GR*, *ORHK* — *OPIK*, *PIHR* and *PIGR* + *IGH*) as *PIGR* + *IGH* — $\frac{OR}{OQ}$*IEF* to *OPIK*. Therefore if the area $\frac{OR}{OQ}$*IEF* — *IGH* be called Y, and *RGgr* the decrement of the area *PIGR* be given, the increment of the area Y will be as *PIGR* — Y.

Then

Then if V repreſent the force ariſing from the gravity, proportional to the arc *CD* to be deſcribed, by which the body is acted upon in *D*, and R be put for the reſiſtance; V — R will be the whole force with which the body is urged in *D*. Therefore the increment of the velocity is as V — R and the particle of time in which it is generated conjunctly. But the velocity itſelf is as the contemporaneous increment of the ſpace deſcribed directly and the ſame particle of time inverſely. Therefore, ſince the reſiſtance is, by the ſuppoſition, as the ſquare of the velocity, the increment of the reſiſtance will (by Lem. 2.) be as the velocity and the increment of the velocity conjunctly, that is, as the moment of the ſpace and V — R conjunctly; and therefore, if the moment of the ſpace be given, as V — R; that is, if for the force V we put its exponent *PIGR*, and the reſiſtance R be expreſſed by any other area Z, as *PIGR* — Z.

Therefore the area *PIGR* uniformly decreaſing by the ſubduction of given moments, the area Y increaſes in proportion of *PIGR* — Y, and the area Z in proportion of *PIGR* — Z. And therefore if the areas Y and Z begin together, and at the beginning are equal, theſe, by the addition of equal moments, will continue to be equal; and in like manner decreaſing by equal moments will vaniſh together. And, *vice verſa*, if they together begin and vaniſh, they will have equal moments and be always equal: and that, becauſe if the reſiſtance Z be augmented, the velocity together with the arc *Ca*, deſcribed in the aſcent of the body, will be diminiſhed; and the point in which all the motion together with the reſiſtance ceaſes, coming nearer to the point *C*, the reſiſtance vaniſhes ſooner than the area Y. And the contrary will happen when the reſiſtance is diminiſhed.

Now the area Z begins and ends where the reſiſtance is nothing, that is, at the beginning of the motion where the arc *CD* is equal to the arc *CB*, and the right line

line RG falls upon the right line QE; and at the end of the motion where the arc CD is equal to the arc Ca, and RG falls upon the right line ST. And the area Y or $\frac{OR}{OQ}IEF - IGH$ begins and ends also where the reſiſtance is nothing, and therefore where $\frac{OR}{OQ}IEF$ and IGH are equal; that is, (by the conſtruction) where the right line RG falls ſucceſſively upon the right lines QE and ST. Therefore thoſe areas begin and vaniſh together, and are therefore always equal. Therefore the area $\frac{OR}{OQ}IEF - IGH$ is equal to the area Z, by which the reſiſtance is expreſſed, and therefore is to the area $PINM$ by which the gravity is expreſſed as the reſiſtance to the gravity. *Q.E.D.*

Cor. 1. Therefore the reſiſtance in the loweſt place C is to the force of gravity, as the area $\frac{OP}{OQ}IEF$ to the area $PINM$.

Cor. 2. But it becomes greateſt, where the area $PIHR$ is to the area IEF as OR to OQ. For in that caſe its moment (that is, $PIGR - Y$) becomes nothing.

Cor. 3. Hence alſo may be known the velocity in each place: as being in the ſubduplicate ratio of the reſiſtance, and at the beginning of the motion equal to the velocity of the body oſcillating in the ſame cycloid without any reſiſtance.

However, by reaſon of the difficulty of the calculation by which the reſiſtance and the velocity are found by this Propoſition, we have thought fit to ſubjoin the Propoſition following.

Pro-

Fig. 1. p. 71.

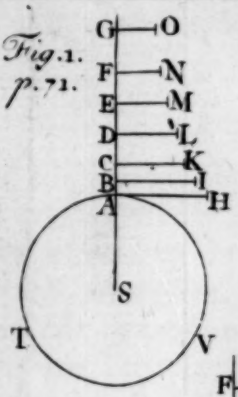


Fig. 2. p. 73.

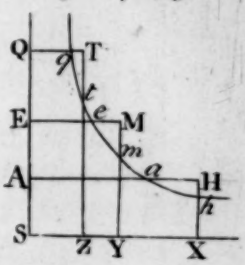


Fig. 3. ibid.

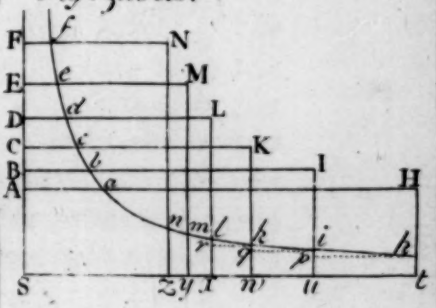
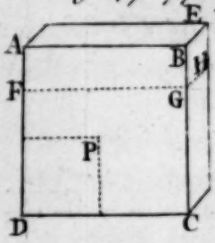


Fig. 4. p. 77.



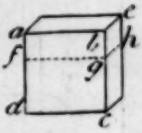


Fig. 5. p. 82. 86.

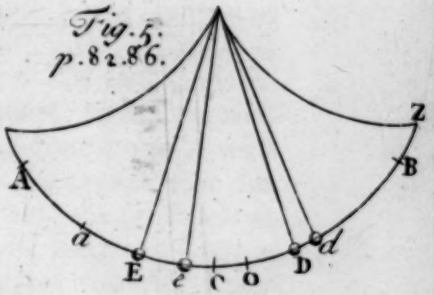


Fig. 6. p. 87.

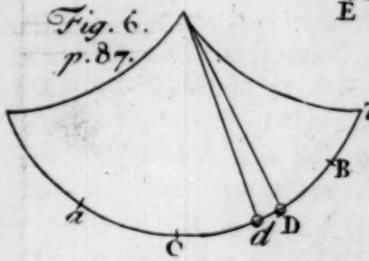
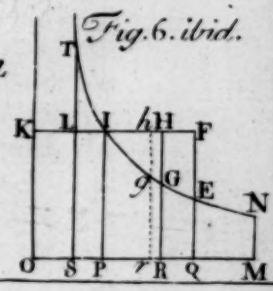


Fig. 6. ibid.



PROPOSITION XXX. THEOREM XXIV.

If a right line aB (Pl. 6. Fig. 1.) *be equal to the arc of a cycloid which an oſcillating body deſcribes, and at each of its points* D *the perpendiculars* DK *be erected, which ſhall be to the length of the pendulum as the reſiſtance of the body in the correſponding points of the arc to the force of gravity: I ſay, that the difference between the arc deſcribed in the whole deſcent and the arc deſcribed in the whole ſubſequent aſcent drawn into half the ſum of the ſame arcs, will be equal to the area* BKa *which all thoſe perpendiculars take up.*

Let the arc of the cycloid, deſcribed in one entire oſcillation, be expreſſed by the right line *aB*, equal to it, and the arc which would have been deſcribed *in vacuo*, by the length *AB*. Biſect *AB* in *C*, and the point *C* will repreſent the loweſt point of the cycloid, and *CD* will be as the force ariſing from gravity, with which the body in *D* is urged in the direction of the tangent of the cycloid, and will have the ſame ratio to the length of the pendulum as the force in *D* has to the force of gravity. Let that force therefore be expreſſed by that length *CD*, and the force of gravity by the length of the pendulum, and if in *DE* you take *DK* in the ſame ratio to the length of the pendulum as the reſiſtance has to the gravity, *DK* will be the exponent of the reſiſtance. From the centre *C* with the interval *CA* or *CB* deſcribe a ſemi-circle *BEeA*. Let the body deſcribe, in the leaſt time, the ſpace *Dd*, and erecting the perpendiculars *DE*, *de*, meeting the circumference in *E* and *e*, they will be as the velocities which

which the body deſcending *in vacuo* from the point *B* would acquire in the places *D* and *d*. This appears by Prop. 52. Book 1. Let therefore theſe velocities be expreſſed by thoſe perpendiculars *DE*, *de*; and let *DF* be the velocity which it acquires in *D* by falling from *B* in the reſiſting medium. And if from the centre *C* with the interval *CF* we deſcribe the circle *FfM* meeting the right lines *de* and *AB* in *f* and *M*, then *M* will be the place to which it would thenceforward, without farther reſiſtance, aſcend, and *df* the velocity it would acquire in *d*. Whence alſo if *Fg* repreſent the moment of the velocity which the body *D*, in deſcribing the leaſt ſpace *Dd*, loſes by the reſiſtance of the medium; and *CN* be taken equal to *Cg*: then will *N* be the place to which the body, if it met no farther reſiſtance, would thenceforward aſcend, and *MN* will be the decrement of the aſcent ariſing from the loſs of that velocity. Draw *Fm* perpendicular to *df*, and the decrement *Fg* of the velocity *DF* generated by the reſiſtance *DK* will be to the increment *fm* of the ſame velocity generated by the force *CD*, as the generating force *DK* to the generating force *CD*. But becauſe of the ſimilar triangles *Fmf*, *Fhg*, *FDC*, *fm* is to *Fm* or *Dd* as *CD* to *DF*; and, *ex æquo*, *Fg* to *Dd* as *DK* to *DF*. Alſo *Fh* is to *Fg* as *DF* to *CF*; and, *ex æquo perturbatè*, *Fh* or *MN* to *Dd* as *DK* to *CF* or *CM*; and therefore the ſum of all the $MN \times CM$ will be equal to the ſum of all the $Dd \times DK$. At the moveable point *M* ſuppoſe always a rectangular ordinate erected equal to the indeterminate *CM*, which by a continual motion is drawn into the whole length *Aa*; and the trapezium deſcribed by that motion, or its equal, the rectangle $Aa \times \frac{1}{2} aB$, will be equal to the ſum of all the $MN \times CM$, and therefore to the ſum of all the $Dd \times DK$, that is, to the area *BKVTa*. *Q.E.D.*

Cor.

Cor. Hence from the law of reſiſtance and the difference Aa of the arcs Ca, CB may be collected the proportion of the reſiſtance to the gravity nearly.

For if the reſiſtance DK be uniform, the figure $BKTa$ will be a rectangle under Ba and DK; and thence the rectangle under $\frac{1}{2}Ba$ and Aa will be equal to the rectangle under Ba and DK, and DK will be equal to $\frac{1}{2}Aa$. Wherefore ſince DK is the exponent of the reſiſtance, and the length of the pendulum the exponent of the gravity, the reſiſtance will be to the gravity as $\frac{1}{2}Aa$ to the length of the pendulum; altogether as in Prop. 28. is demonſtrated.

If the reſiſtance be as the velocity, the figure $BKTa$ will be nearly an ellipſis. For if a body, in a non-reſiſting medium, by one entire oſcillation, ſhould deſcribe the length BA, the velocity in any place D would be as the ordinate DE of the circle deſcribed on the diameter AB. Therefore ſince Ba in the reſiſting medium, and BA in the non-reſiſting one, are deſcribed nearly in the ſame times; and therefore the velocities in each of the points of Ba, are to the velocities in the correſpondent points of the length BA nearly as Ba is to BA; the velocity in the point D in the reſiſting medium will be as the ordinate of the circle or ellipſis deſcribed upon the diameter Ba; and therefore the figure $BKVTa$ will be nearly an ellipſis. Since the reſiſtance is ſuppoſed proportional to the velocity, let OV be the exponent of the reſiſtance in the middle point O; and an ellipſis $BRVSa$ deſcribed with the centre O, and the ſemiaxes OB, OV will be nearly equal to the figure $BKVTa$, and to its equal the rectangle $Aa \times BO$. Therefore $Aa \times BO$ is to $OV \times BO$ as the area of this ellipſis to $OV \times BO$; that is, Aa is to OV as the area of the ſemicircle to the ſquare of the radius, or as 11 to 7 nearly; and therefore $\frac{7}{11}Aa$ is to the length of the pendulum, as the reſiſtance of the oſcillating body in O to its gravity.

Now

Now if the reſiſtance DK be in the duplicate ratio of the velocity, the figure $BKVTa$ will be almoſt a Parabola having V for its vertex and OV for its axis, and therefore will be nearly equal to the rectangle under $\frac{2}{3}Ba$ and OV. Therefore the rectangle under $\frac{1}{2}Ba$ and Aa is equal to the rectangle $\frac{2}{3}Ba \times OV$, and therefore OV is equal to $\frac{1}{4}Aa$: and therefore the reſiſtance in O made to the oſcillating body is to its gravity as $\frac{1}{4}Aa$ to the length of the pendulum.

And I take theſe concluſions to be accurate enough for practical uſes. For ſince an Ellipſis or Parabola $BRVSa$ falls in with the figure $BKVTa$ in the middle point V, that figure, if greater towards the part BRV or VSa than the other, is leſs towards the contrary part, and is therefore nearly equal to it.

Proposition XXXI. Theorem XXV.

If the reſiſtance made to an oſcillating body in each of the proportional parts of the arcs deſcribed be augmented or diminiſhed in a given ratio; the difference between the arc deſcribed in the deſcent and the arc deſcribed in the ſubſequent aſcent, will be augmented or diminiſhed in the ſame ratio.

For that difference ariſes from the retardation of the pendulum by the reſiſtance of the medium, and therefore is as the whole retardation, and the retarding reſiſtance proportional thereto. In the foregoing Propoſition the rectangle under the right line $\frac{1}{2}aB$ and the difference Aa of the arcs CB, Ca was equal to the area $BKTa$. And that area, if the length aB remains, is augmented or diminiſhed in the ratio of the ordinates DK; that is, in the ratio of the reſiſtance, and is therefore as the length aB and the reſiſtance conjunctly.

junctly. And therefore the rectangle under Aa and $\frac{1}{2}aB$ is as aB and the reſiſtance conjunctly, and therefore Aa is as the reſiſtance. *Q. E. D.*

Cor. 1. Hence if the reſiſtance be as the velocity, the difference of the arcs in the ſame medium will be as the whole arc deſcribed: and the contrary.

Cor. 2. If the reſiſtance be in the duplicate ratio of the velocity, that difference will be in the duplicate ratio of the whole arc: and the contrary.

Cor. 3. And univerſally, if the reſiſtance be in the triplicate or any other ratio of the velocity, the difference will be in the ſame ratio of the whole arc: and the contrary.

Cor. 4. If the reſiſtance be partly in the ſimple ratio of the velocity, and partly in the duplicate ratio of the ſame, the difference will be partly in the ratio of the whole arc, and partly in the duplicate ratio of it: and the contrary. So that the law and ratio of the reſiſtance will be the ſame for the velocity, as the law and ratio of that difference for the length of the arc.

Cor. 5. And therefore if a pendulum deſcribe ſucceſſively unequal arcs, and we can find the ratio of the increment or decrement of this difference for the length of the arc deſcribed; there will be had alſo the ratio of the increment or decrement of the reſiſtance for a greater or leſs velocity.

General Scholium.

From theſe Propoſitions, we may find the reſiſtance of mediums by pendulums oſcillating therein. I found the reſiſtance of the air by the following experiments. I ſuſpended a wooden globe or ball weighing $57\frac{1}{2}$ ounces Averdupois, its diameter $6\frac{7}{8}$ *London* inches, by a fine thread on a firm hook, ſo that the diſtance between the hook and the centre of oſcillation of the globe was $10\frac{1}{2}$ foot. I marked on the thread a point 10 foot and

1 inch diſtant from the centre of ſuſpenſion; and even with that point I placed a ruler divided into inches, by the help whereof I obſerved the lengths of the arcs deſcribed by the pendulum. Then I number'd the oſcillations, in which the globe would loſe $\frac{1}{8}$ part of its motion. If the pendulum was drawn aſide from the perpendicular to the diſtance of 2 inches, and thence let go, ſo that in its whole deſcent it deſcribed an arc of two inches, and in the firſt whole oſcillation, compounded of the deſcent and ſubſequent aſcent, an arc of almoſt four inches: the ſame in 164 oſcillations loſt $\frac{1}{8}$ part of its motion, ſo as in its laſt aſcent to deſcribe an arc of $1\frac{1}{4}$ inches. If in the firſt deſcent it deſcribed an arc of 4 inches; it loſt $\frac{1}{8}$ part of its motion in 121 oſcillations, ſo as in its laſt aſcent to deſcribe an arc of $3\frac{1}{2}$ inches. If in the firſt deſcent it deſcribed an arc of 8, 16, 32, or 64 inches; it loſt $\frac{1}{8}$ part of its motion in 69, $35\frac{1}{2}$, $18\frac{1}{2}$, $9\frac{3}{4}$ oſcillations, reſpectively. Therefore the difference between the arcs deſcribed in the firſt deſcent and the laſt aſcent, was in the 1ſt, 2d, 3d, 4th, 5th, 6th caſe, $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 4, 8 inches, reſpectively. Divide thoſe differences by the number of oſcillations in each caſe, and in one mean oſcillation, wherein an arc of $3\frac{1}{4}$, $7\frac{1}{2}$, 15, 30, 60, 120 inches was deſcribed, the difference of the arcs deſcribed in the deſcent and ſubſequent aſcent will be $\frac{1}{650}$, $\frac{1}{242}$, $\frac{1}{69}$, $\frac{4}{71}$, $\frac{8}{37}$, $\frac{24}{29}$ parts of an inch, reſpectively. But theſe differences in the greater oſcillations are in the duplicate ratio of the arcs deſcribed nearly, but in leſſer oſcillations ſomething greater than in that ratio; and therefore (by Cor. 2. Prop. 31. of this Book) the reſiſtance of the globe, when it moves very ſwift, is in the duplicate ratio of the velocity, nearly; and when it moves ſlowly, ſomewhat greater than in that ratio.

Let

Now let V repreſent the greateſt velocity in any oſcillation, and let A, B, and C be given quantities, and let us ſuppoſe the difference of the arcs to be $AV + BV^{\frac{1}{2}} + CV^2$. Since the greateſt velocities are in the cycloid as $\frac{1}{2}$ the arcs deſcribed in oſcillating, and in the circle as $\frac{1}{2}$ the chords of thoſe arcs; and therefore in equal arcs are greater in the cycloid than in the circle, in the ratio of $\frac{1}{2}$ the arcs to their chords; but the times in the circle are greater than in the cycloid, in a reciprocal ratio of the velocity; it is plain that the differences of the arcs (which are as the reſiſtance and the ſquare of the time conjunctly) are nearly the ſame, in both curves: for in the cycloid thoſe differences muſt be on the one hand augmented, with the reſiſtance, in about the duplicate ratio of the arc to the chord, becauſe of the velocity augmented in the ſimple ratio of the ſame; and on the other hand diminiſhed, with the ſquare of the time, in the ſame duplicate ratio. Therefore to reduce theſe obſervations to the cycloid, we muſt take the ſame differences of the arcs as were obſerved in the circle, and ſuppoſe the greateſt velocities analogous to the half, or the whole arcs, that is, to the numbers $\frac{1}{2}$, 1, 2, 4, 8, 16. Therefore in the 2d, 4th, and 6th caſe, put 1, 4 and 16 for V; and the difference of the arcs in the 2d caſe will become $\frac{\frac{1}{2}}{121} = A + B + C$; in the 4th caſe $\frac{2}{35\frac{1}{2}} = 4A + 8B + 16C$; in the 6th caſe $\frac{8}{9\frac{4}{3}} = 16A + 64B + 256C$. Theſe equations reduced give A=0,0000916, B=0,0010847, and C=0,0029558. Therefore the difference of the arcs is as $0{,}0000916V + 0{,}0010847V^{\frac{1}{2}} + 0{,}0029558V^2$: and therefore ſince (by Cor. Prop. 30. applied to this caſe) the reſiſtance of the globe in the middle

middle of the arc deſcribed in oſcillating, where the velocity is V, is to its weight as $\frac{2}{11}AV + \frac{2}{10}BV^{\frac{1}{2}} + \frac{1}{4}CV^2$ to the length of the pendulum; if for A, B, and C you put the numbers found, the reſiſtance of the globe will be to its weight, as $0{,}0000583V + 0{,}0007593V^{\frac{1}{2}} + 0{,}0022169V^2$ to the length of the pendulum between the centre of ſuſpenſion and the ruler, that is, to 121 inches. Therefore ſince V in the 2d caſe repreſents 1, in the 4th caſe 4, and in the 6th caſe 16: the reſiſtance will be to the weight of the globe, in the 2d caſe as 0,0030345 to 121, in the 4th as 0,041748 to 121, in the 6th as 0,61705 to 121.

The arc which the point marked in the thread deſcribed in the 6th caſe, was of $120 - \frac{8}{9\frac{2}{3}}$ or $119\frac{1}{29}$ inches. And therefore ſince the radius was 121 inches, and the length of the pendulum between the point of ſuſpenſion and the centre of the globe was 126 inches, the arc which the centre of the globe deſcribed was $124\frac{1}{31}$ inches. Becauſe the greateſt velocity of the oſcillating body, by reaſon of the reſiſtance of the air, does not fall on the loweſt point of the arc deſcribed, but near the middle place of the whole arc: this velocity will be nearly the ſame as if the globe in its whole deſcent in a non-reſiſting medium ſhould deſcribe $62\frac{1}{62}$ inches the half of that arc, and that in a cycloid, to which we have above reduced the motion of the pendulum: and therefore that velocity will be equal to that which the globe would acquire by falling perpendicularly from a height equal to the verſed ſine of that arc. But that verſed ſine in the cycloid is to that arc $62\frac{1}{62}$ as the ſame arc to twice the length of the pendulum 252, and therefore equal to 15,278 inches. Therefore the velocity of the pendulum is the ſame which a body would acquire by falling, and in its fall de-

deſcribing a ſpace of 15,278 inches. Therefore with ſuch a velocity the globe meets with a reſiſtance, which is to its weight as 0,61705 to 121, or (if we take that part only of the reſiſtance which is in the duplicate ratio of the velocity) as 0,56752 to 121.

I found by an hydroſtatical experiment, that the weight of this wooden globe was to the weight of a globe of water of the ſame magnitude as 55 to 97: and therefore ſince 121 is to 213,4 in the ſame ratio, the reſiſtance made to this globe of water moving forwards with the abovementioned velocity, will be to its weight as 0,56752 to 213,4, that is, as 1 to $376\frac{4}{10}$. Whence ſince the weight of a globe of water, in the time in which the globe with a velocity uniformly continued deſcribes a length of 30,556 inches, will generate all that velocity in the falling globe; it is manifeſt that the force of reſiſtance uniformly continued in the ſame time will take away a velocity, which will be leſs than the other in the ratio of 1 to $376\frac{4}{10}$, that is, the $\frac{1}{376\frac{4}{10}}$ part of the whole velocity. And therefore in the time that the globe, with the ſame velocity uniformly continued, would deſcribe the length of its ſemi-diameter, or $3\frac{2}{16}$ inches, it would loſe the $\frac{1}{3342}$ part of its motion.

I alſo counted the oſcillations in which the pendulum loſt $\frac{1}{4}$ part of its motion. In the following table the upper numbers denote the length of the arc deſcribed in the firſt deſcent, expreſſed in inches and parts of an inch; the middle numbers denote the length of the arc deſcribed in the laſt aſcent; and in the loweſt place are the numbers of the oſcillations. I give an account of this experiment, as being more accurate than that in which only $\frac{1}{8}$ part of the motion was loſt. I leave the calculation to ſuch as are diſpoſed to make it.

Firſt deſcent	2	4	8	16	32	64
Laſt aſcent	$1\frac{1}{2}$	3	6	12	24	48
Numb. of oſcill.	374	272	$162\frac{1}{2}$	$83\frac{1}{3}$	$41\frac{2}{3}$	$22\frac{2}{3}$

I afterwards ſuſpended a leaden globe of 2 inches in diameter, weighing $26\frac{1}{4}$ ounces Averdupois by the ſame thread, ſo that between the centre of the globe and the point of ſuſpenſion there was an interval of $10\frac{1}{2}$ feet, and I counted the oſcillations in which a given part of the motion was loſt. The firſt of the following tables exhibits the number of oſcillations in which $\frac{1}{8}$ part of the whole motion was loſt; the ſecond the number of oſcillations in which there was loſt $\frac{1}{4}$ part of the ſame.

Firſt deſcent	1	2	4	8	16	32	64
Laſt aſcent	$\frac{7}{8}$	$\frac{7}{4}$	$3\frac{1}{2}$	7	14	28	56
Numb. of oſcill.	226	228	193	140	$90\frac{1}{2}$	53	30

Firſt deſcent	1	2	4	8	16	32	64
Laſt aſcent	$\frac{3}{4}$	$1\frac{1}{2}$	3	6	12	24	48
Numb. of oſcill.	510	518	420	318	204	121	70

Selecting in the firſt table the 3^{d}, 5th, and 7th obſervation, and expreſſing the greateſt velocities in theſe obſervations particularly by the numbers 1, 4, 16 reſpectively, and generally by the quantity V as above: there will come out in the 3^{d} obſervation $\frac{\frac{1}{2}}{193} = A + B + C$, in the 5th obſervation $\frac{2}{90\frac{1}{2}} = 4A + 8B + 16C$, in the 7^{h} obſervation $\frac{8}{30} = 16A + 64B + 256C$. Theſe equations reduced give $A = 0{,}001414$, $B = 0{,}000297$, $C = 0{,}000879$. And thence the reſiſtance of the globe moving with the velocity V will be to its weight $26\frac{1}{4}$ ounces, in the ſame ratio as $0{,}0009V + 0{,}000208V^{\frac{1}{2}} + 0{,}000659V^{2}$ to 121 inches

inches the length of the pendulum. And if we regard that part only of the reſiſtance which is in the duplicate ratio of the velocity, it will be to the weight of the globe as $0{,}000659V^2$ to 121 inches. But this part of the reſiſtance in the 1ſt experiment was to the weight of the wooden globe of $57\frac{1}{2}\frac{1}{2}$ ounces as $0{,}002217V^2$ to 121; and thence the reſiſtance of the wooden globe is to the reſiſtance of the leaden one (their velocities being equal) as $57\frac{1}{2}\frac{1}{2}$ into 0,002217 to $26\frac{1}{4}$ into 0,000659, that is, as $7\frac{1}{3}$ to 1. The diameters of the two globes were $6\frac{7}{8}$ and 2 inches, and the ſquares of theſe are to each other as $47\frac{1}{4}$ and 4, or $11\frac{1}{16}$ and 1, nearly. Therefore the reſiſtances of theſe equally ſwift globes were in leſs than a duplicate ratio of the diameters. But we have not yet conſider'd the reſiſtance of the thread, which was certainly very conſiderable, and ought to be ſubducted from the reſiſtance of the pendulums here found. I could not determine this accurately, but I found it greater than a third part of the whole reſiſtance of the leſſer pendulum; and thence I gathered that the reſiſtances of the globes, when the reſiſtance of the thread is ſubducted, are nearly in the duplicate ratio of their diameters. For the ratio of $7\frac{1}{3}-\frac{1}{3}$ to $1-\frac{1}{3}$, or $10\frac{1}{2}$ to 1 is not very different from the duplicate ratio of the diameters, $11\frac{1}{16}$ to 1.

Since the reſiſtance of the thread is of leſs moment in greater globes, I tried the experiment alſo with a globe whoſe diameter was $18\frac{1}{4}$ inches. The length of the pendulum between the point of ſuſpenſion and the centre of oſcillation was $122\frac{1}{2}$ inches, and between the point of ſuſpenſion and the knot in the thread $109\frac{1}{2}$ inches. The arc deſcribed by the knot at the firſt deſcent of the pendulum was 32 inches. The arc deſcribed by the ſame knot in the laſt aſcent after five oſcillations was 28 inches. The ſum of the arcs or the whole arc deſcribed in one mean oſcillation was 60

inches. The difference of the arcs 4 inches. The $\frac{1}{30}$ part of this, or the difference between the deſcent and aſcent in one mean oſcillation is $\frac{2}{15}$ of an inch. Then as the radius $109\frac{1}{2}$ to the radius $122\frac{1}{2}$ ſo is the whole arc of 60 inches deſcribed by the knot in one mean oſcillation to the whole arc of $67\frac{1}{8}$ inches deſcribed by the centre of the globe in one mean oſcillation; and ſo is the difference $\frac{2}{15}$ to a new difference 0,4475. If the length of the arc deſcribed were to remain, and the length of the pendulum ſhould be augmented in the ratio of 126 to $122\frac{1}{2}$; the time of the oſcillation would be augmented, and the velocity of the pendulum would be diminiſhed in the ſubduplicate of that ratio; ſo that the difference 0,4475 of the arcs deſcribed in the deſcent and ſubſequent aſcent would remain. Then if the arc deſcribed be augmented in the ratio of $124\frac{1}{31}$ to $67\frac{1}{8}$, that difference 0,4475 would be augmented in the duplicate of that ratio, and ſo would become 1,5295. Theſe things would be ſo upon the ſuppoſition, that the reſiſtance of the pendulum were in the duplicate ratio of the velocity. Therefore if the pendulum deſcribe the whole arc of $124\frac{1}{31}$ inches, and its length between the point of ſuſpenſion and the centre of oſcillation be 126 inches, the difference of the arcs deſcribed in the deſcent and ſubſequent aſcent would be 1,5295 inches. And this difference multiplied into the weight of the pendulous globe, which was 208 ounces, produces 318,136. Again in the pendulum abovementioned, made of a wooden globe, when its centre of oſcillation, being 126 inches from the point of ſuſpenſion, deſcribed the whole arc of $124\frac{1}{31}$ inches, the difference of the arcs deſcribed in the deſcent and aſcent was $\frac{126}{121}$ into $\frac{8}{9\frac{4}{5}}$. This multiplied into the weight of the globe, which was $57\frac{1}{2}$ ounces, produces 49,396. But I multiply theſe differences into the weights of the globes, in order to find their reſiſtances. For the

diffe-

differences ariſe from the reſiſtances, and are as the reſiſtances directly and the weights inverſely. Therefore the reſiſtances are as the numbers 318,136 and 49,396. But that part of the reſiſtance of the leſſer globe, which is in the duplicate ratio of the velocity, was to the whole reſiſtance as 0,56752 to 0,61675, that is, as 45,453 to 49,396; whereas that part of the reſiſtance of the greater globe is almoſt equal to its whole reſiſtance; and ſo thoſe parts are nearly as 318,136 and 45,453, that is, as 7 and 1. But the diameters of the globes are $18\frac{1}{4}$ and $6\frac{7}{8}$; and their ſquares $351\frac{2}{16}$ and $47\frac{17}{64}$ are as 7,438 and 1, that is, as the reſiſtances of the globes 7 and 1, nearly. The difference of theſe ratio's is ſcarce greater than may ariſe from the reſiſtance of the thread. Therefore thoſe parts of the reſiſtances which are, when the globes are equal, as the ſquares of the velocities; are alſo, when the velocities are equal, as the ſquares of the diameters of the globes.

But the greateſt of the globes, I uſed in theſe experiments, was not perfectly ſphærical, and therefore in this calculation I have, for brevity's ſake, neglected ſome little niceties; being not very ſollicitous for an accurate calculus, in an experiment that was not very accurate. So that I could wiſh, that theſe experiments were tried again with other globes, of a larger ſize, more in number, and more accurately formed; ſince the demonſtration of a vacuum depends thereon. If the globes be taken in a geometrical proportion, as ſuppoſe whoſe diameters are 4, 8, 16, 32 inches; one may collect from the progreſſion obſerved in the experiments what would happen if the globes were ſtill larger.

In order to compare the reſiſtances of different fluids with each other, I made the following trials. I procured a wooden veſſel 4 feet long, 1 foot broad, and 1 foot high. This veſſel, being uncover'd, I fill'd with ſpring-water, and having immerſed pendulums therein, I made them oſcillate in the water. And I

found that a leaden globe weighing 166 $\frac{1}{8}$ ounces, and in diameter 3 $\frac{1}{8}$ inches, moved therein as it is ſet down in the following table; the length of the pendulum from the point of ſuſpenſion to a certain point marked in the thread being 126 inches, and to the centre of oſcillation 134 $\frac{1}{8}$ inches.

The arc deſcribed in the firſt deſcent by a point marked in the thread, was inches	64 . 32 . 16 . 8 . 4 . 2 . 1 . $\frac{1}{2}$. $\frac{1}{4}$
The arc deſcribed in the laſt aſcent, was inches	48 . 24 . 12 . 6 . 3 . 1 $\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{8}$. $\frac{1}{16}$
The difference of the arcs proportional to the motion loſt, was inches	16 . 8 . 4 . 2 . 1 . $\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{8}$. $\frac{1}{16}$
The number of the oſcillations in water	$\frac{2}{60}$. 1 $\frac{1}{5}$. 3 . 7 . 11 $\frac{1}{4}$. 12 $\frac{2}{3}$. 13 $\frac{1}{3}$
The number of the oſcillations in air.	85 $\frac{1}{2}$. 287 . 535

In the experiments of the 4th column, there were equal motions loſt in 535 oſcillations made in the air, and 1 $\frac{1}{3}$ in water. The oſcillations in the air were indeed a little ſwifter than thoſe in the water. But if the oſcillations in the water were accelerated in ſuch a ratio that the motions of the pendulums might be equally ſwift in both mediums, there would be ſtill the ſame number 1 $\frac{1}{3}$ of oſcillations in the water, and by theſe the ſame quantity of motion would be loſt as before; becauſe the reſiſtance is increaſed and the ſquare of the time diminiſhed in the ſame duplicate ratio. The pendulums therefore being of equal velocities, there were equal

equal motions loſt in 535 oſcillations in the air, and $1\frac{1}{5}$ in the water; and therefore the reſiſtance of the pendulum in the water is to its reſiſtance in the air as 535 to $1\frac{1}{5}$. This is the proportion of the whole reſiſtances in the caſe of the 4th column.

Now let $AV + CV^2$ repreſent the difference of the arcs deſcribed in the deſcent and ſubſequent aſcent by the globe moving in air with the greateſt velocity V; and ſince the greateſt velocity is in the caſe of the 4th column to the greateſt velocity in the caſe of the 1ſt column as 1 to 8; and that difference of the arcs in the caſe of the 4th column to the difference in the caſe of the 1ſt column, as $\frac{2}{535}$ to $\frac{16}{85\frac{1}{2}}$, or as $85\frac{1}{2}$ to 4280: put in theſe caſes 1 and 8 for the velocities, and $85\frac{1}{2}$ and 4280 for the differences of the arcs, and $A + C$ will be $= 85\frac{1}{2}$, and $8A + 64C = 4280$ or $A + 8C = 535$; and then, by reducing theſe equations, there will come out $7C = 449\frac{1}{2}$ and $C = 64\frac{1}{14}$ and $A = 21\frac{3}{7}$: and therefore the reſiſtance, which is as $\frac{7}{11}AV + \frac{1}{4}CV^2$, will become as $13\frac{6}{11}V + 48\frac{2}{56}V^2$. Therefore in the caſe of the 4th column, where the velocity was 1, the whole reſiſtance is to its part proportional to the ſquare of the velocity, as $13\frac{6}{11} + 48\frac{2}{56}$ or $61\frac{4}{17}$ to $48\frac{2}{56}$; and therefore the reſiſtance of the pendulum in water is to that part of the reſiſtance in air, which is proportional to the ſquare of the velocity, and which in ſwift motions is the only part that deſerves conſideration, as $61\frac{4}{17}$ to $48\frac{2}{56}$ and 535 to $1\frac{1}{5}$ conjunctly, that is, as 571 to 1. If the whole thread of the pendulum oſcillating in the water had been immerſed, its reſiſtance would have been ſtill greater; ſo that the reſiſtance of the pendulum oſcillating in the water, that is, that part which is proportional to the ſquare of the velocity, and which only needs to be conſider'd in ſwift bodies, is to the reſiſtance of the ſame whole pendulum, oſcillating in air with

with the ſame velocity, as about 850 to 1, that is, as the denſity of water to the denſity of air, nearly.

In this calculation, we ought alſo to have taken in that part of the reſiſtance of the pendulum in the water, which was as the ſquare of the velocity, but I found (which will perhaps ſeem ſtrange) that the reſiſtance in the water was augmented in more than a duplicate ratio of the velocity. In ſearching after the cauſe, I thought upon this, that the veſſel was too narrow for the magnitude of the pendulous globe, and by its narrowneſs obſtructed the motion of the water as it yielded to the oſcillating globe. For when I immerſed a pendulous globe, whoſe diameter was one inch only; the reſiſtance was augmented nearly in a duplicate ratio of the velocity. I tried this by making a pendulum of two globes, of which the leſſer and lower oſcillated in the water, and the greater and higher was faſtened to the thread juſt above the water, and by oſcillating in the air, aſſiſted the motion of the pendulum, and continued it longer. The experiments made by this contrivance proved according to the following table.

Arc deſcr. in firſt deſcent	16 . 8 . 4 . 2 . 1 . $\frac{1}{2}$. $\frac{1}{4}$
Arc deſcr. in laſt aſcent	12 . 6 . 3 . $1\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{8}$. $\frac{1}{16}$
Diff. of arcs, proport. to mot. loſt	4 . 2 . 1 . $\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{8}$. $\frac{1}{16}$
Number of oſcillations	$3\frac{1}{8}$. $6\frac{1}{2}$. $12\frac{1}{12}$. $21\frac{1}{5}$. 34 . 53 . $62\frac{1}{5}$

In comparing the reſiſtances of the mediums with each other, I alſo cauſed iron pendulums to oſcillate in quickſilver. The length of the iron wire was about 3 feet, and the diameter of the pendulous globe about $\frac{1}{3}$ of an inch. To the wire, juſt above the quickſilver, there was fixed another leaden globe of a bigneſs ſufficient to continue the motion of the pendulum for ſome time. Then a veſſel, that would hold about 3 pounds of quickſilver, was filled by turns with quickſilver and common

common water, that by making the pendulum oſcillate ſucceſſively in theſe two different fluids, I might find the proportion of their reſiſtances: and the reſiſtance of the quickſilver proved to be to the reſiſtance of water as about 13 or 14 to 1; that is, as the denſity of quickſilver to the denſity of water. When I made uſe of a pendulous globe ſomething bigger, as of one whoſe diameter was about $\frac{1}{2}$ or $\frac{2}{3}$ of an inch, the reſiſtance of the quickſilver proved to be to the reſiſtance of the water as about 12 or 10 to 1. But the former experiment is more to be relied on, becauſe in the latter the veſſel was too narrow in proportion to the magnitude of the immerſed globe: For the veſſel ought to have been enlarged together with the globe. I intended to have repeated theſe experiments with larger veſſels, and in melted metals, and other liquors both cold and hot: but I had not leiſure to try all; and beſides, from what is already deſcribed, it appears ſufficiently that the reſiſtance of bodies moving ſwiftly is nearly proportional to the denſities of the fluids in which they move. I don't ſay accurately. For more tenacious fluids, of equal denſity, will undoubtedly reſiſt more than thoſe that are more liquid, as cold oil more than warm, warm oil more than rain-water, and water more than ſpirit of wine. But in liquors, which are ſenſibly fluid enough, as in air, in ſalt and freſh water, in ſpirit of wine, of turpentine and ſalts, in oil cleared of its fæces by diſtillation and warmed, in oil of vitriol and in mercury, and melted metals, and any other ſuch like, that are fluid enough to retain for ſome time the motion impreſſed upon them by the agitation of the veſſel, and which being poured out are eaſily reſolv'd into drops: I doubt not but the rule already laid down may be accurate enough, eſpecially if the experiments be made with larger pendulous bodies, and more ſwiftly moved.

Laſtly, ſince it is the opinion of ſome, that there is a certain æthereal medium extremely rare and ſubtile, which

which freely pervades the pores of all bodies; and from ſuch a medium ſo pervading the pores of bodies, ſome reſiſtance muſt needs ariſe: in order to try whether the reſiſtance, which we experience in bodies in motion, be made upon their outward ſuperficies only, or whether their internal parts meet with any conſiderable reſiſtance upon their ſuperficies; I thought of the following experiment. I ſuſpended a round deal box by a thread 11 feet long, on a ſteel hook by means of a ring of the ſame metal, ſo as to make a pendulum of the aforeſaid length. The hook had a ſharp hollow edge on its upper part, ſo that the upper arc of the ring preſſing on the edge might move the more freely: and the thread was faſtened to the lower arc of the ring. The pendulum being thus prepared, I drew it aſide from the perpendicular to the diſtance of about 6 feet, and that in a plane perpendicular to the edge of the hook, leſt the ring, while the pendulum oſcillated, ſhould ſlide to and fro on the edge of the hook: For the point of ſuſpenſion, in which the ring touches the hook, ought to remain immoveable. I therefore accurately noted the place, to which the pendulum was brought, and letting it go, I marked three other places, to which it returned at the end of the 1^{ſt}, 2^{d}, and 3^{d} oſcillation. This I often repeated, that I might find thoſe places as accurately as poſſible. Then I filled the box with lead and other heavy metals, that were near at hand. But firſt I weighed the box when empty, and that part of the thread that went round it, and half the remaining part extended between the hook and the ſuſpended box. For the thread ſo extended always acts upon the pendulum, when drawn aſide from the perpendicular, with half its weight. To this weight I added the weight of the air contained in the box. And this whole weight was about $\frac{1}{78}$ of the weight of the box when filled with the metals. Then becauſe the box when full of the metals, by extending the thread with its weight, in-

creaſed the length of the pendulum, I ſhortened the thread ſo as to make the length of the pendulum, when oſcillating, the ſame as before. Then drawing aſide the pendulum to the place firſt marked, and letting it go, I reckoned about 77 oſcillations, before the box returned to the ſecond mark, and as many afterwards before it came to the third mark, and as many after that, before it came to the fourth mark. From whence I conclude that the whole reſiſtance of the box, when full, had not a greater proportion to the reſiſtance of the box, when empty, than 78 to 77. For if their reſiſtances were equal, the box, when full, by reaſon of its *vis inſita*, which was 78 times greater than the *vis inſita* of the ſame when empty, ought to have continued its oſcillating motion ſo much the longer, and therefore to have returned to thoſe marks at the end of 78 oſcillations. But it returned to them at the end of 77 oſcillations.

Let therefore A repreſent the reſiſtance of the box upon its external ſuperficies, and B the reſiſtance of the empty box on its internal ſuperficies; and if the reſiſtances to the internal parts of bodies equally ſwift be as the matter, or the number of particles that are reſiſted: then 78B will be the reſiſtance made to the internal parts of the box, when full; and therefore the whole reſiſtance A + B of the empty box will be to the whole reſiſtance A + 78B of the full box as 77 to 78, and, by diviſion, A + B to 77B, as 77 to 1, and thence A + B to B as 77×77 to 1, and, by diviſion again, A to B as 5928 to 1. Therefore the reſiſtance of the empty box in its internal parts will be above 5000 times leſs than the reſiſtance on its external ſuperficies. This reaſoning depends upon the ſuppoſition that the greater reſiſtance of the full box ariſes, not from any other latent cauſe, but only from the action of ſome ſubtile fluid upon the included metal.

This

This experiment is related by memory, the paper being loſt, in which I had deſcribed it; ſo that I have been obliged to omit ſome fractional parts, which are ſlipt out of my memory. And I have no leiſure to try it again. The firſt time I made it, the hook being weak, the full box was retarded ſooner. The cauſe I found to be, that the hook was not ſtrong enough to bear the weight of the box; ſo that as it oſcillated to and fro, the hook was bent ſometimes this and ſometimes that way. I therefore procured a hook of ſufficient ſtrength, ſo that the point of ſuſpenſion might remain unmoved, and then all things happened as is above deſcribed.

SEC.

SECTION VII.

Of the motion of fluids and the reſiſtance made to projected bodies.

PROPOSITION XXXII. THEOREM XXVI.

Suppoſe two ſimilar ſyſtems of bodies conſiſting of an equal number of particles, and let the correſpondent particles be ſimilar and proportional, each in one ſyſtem to each in the other, and have a like ſituation among themſelves, and the ſame given ratio of denſity to each other; and let them begin to move among themſelves in proportional times, and with like motions, (that is, thoſe in one ſyſtem among one another, and thoſe in the other among one another.) And if the particles that are in the ſame ſyſtem do not touch one another, except in the moments of reflexion; nor attract, nor repel each other, except with accelerative forces that are as the diameters of the correſpondent particles inverſely, and the ſquares of the velocities directly: I ſay, that the particles of thoſe ſyſtems will continue to move among themſelves with like motions and in proportional times.

Like bodies in like ſituations are ſaid to be moved among themſelves with like motions and in proportional times,

times, when their ſituations at the end of thoſe times are always found alike in reſpect of each other: as ſuppoſe we compare the particles in one ſyſtem with the correſpondent particles in the other. Hence the times will be proportional, in which ſimilar and proportional parts of ſimilar figures will be deſcribed by correſpondent particles. Therefore if we ſuppoſe two ſyſtems of this kind, the correſpondent particles, by reaſon of the ſimilitude of the motions at their beginning, will continue to be moved with like motions, ſo long as they move without meeting one another. For if they are acted on by no forces, they will go on uniformly in right lines by the 1ſt law. But if they do agitate one another, with ſome certain forces, and thoſe forces are as the diameters of the correſpondent particles inverſely and the ſquares of the velocities directly; then becauſe the particles are in like ſituations, and their forces are proportional, the whole forces with which correſpondent particles are agitated, and which are compounded of each of the agitating forces, (by Corol. 2. of the Laws) will have like directions, and have the ſame effect as if they reſpected centres placed alike among the particles; and thoſe whole forces will be to each other as the ſeveral forces which compoſe them, that is, as the diameters of the correſpondent particles inverſely, and the ſquares of the velocities directly: and therefore will cauſe correſpondent particles to continue to deſcribe like figures. Theſe things will be ſo (by Cor. 1 and 8. Prop. 4. Book 1.) if thoſe centres are at reſt. But if they are moved, yet by reaſon of the ſimilitude of the tranſlations, their ſituations among the particles of the ſyſtem will remain ſimilar; ſo that the changes introduced into the figures deſcribed by the particles will ſtill be ſimilar. So that the motions of correſpondent and ſimilar particles will continue ſimilar till their firſt meeting with each other; and thence will ariſe ſimilar colliſions, and ſimilar reflexions; which will again beget

ſimilar

ſimilar motions of the particles among themſelves (by what was juſt now ſhewn) till they mutually fall upon one another again, and ſo on *ad infinitum*.

Cor. 1. Hence if any two bodies, which are ſimilar and in like ſituations to the correſpondent particles of the ſyſtems, begin to move amongſt them in like manner and in proportional times, and their magnitudes and denſities be to each other as the magnitudes and denſities of the correſponding particles: theſe bodies will continue to be moved in like manner and in proportional times. For the caſe of the greater parts of both ſyſtems and of the particles is the very ſame.

Cor. 2. And if all the ſimilar and ſimilarly ſituated parts of both ſyſtems be at reſt among themſelves: and two of them, which are greater than the reſt, and mutually correſpondent in both ſyſtems, begin to move in lines alike poſited, with any ſimilar motion whatſoever; they will excite ſimilar motions in the reſt of the parts of the ſyſtems, and will continue to move among thoſe parts in like manner and in proportional times; and will therefore deſcribe ſpaces proportional to their diameters.

Proposition XXXIII. Theorem XXVII.

The ſame things being ſuppoſed, I ſay that the greater parts of the ſyſtems are reſiſted in a ratio compounded of the duplicate ratio of their velocities, and the duplicate ratio of their diameters, and the ſimple ratio of the denſity of the parts of the ſyſtems.

For the reſiſtance ariſes partly from the centripetal or centrifugal forces with which the particles of the ſyſtem mutually act on each other, partly from the colliſions and reflexions of the particles and the greater parts.

The reſiſtances of the firſt kind are to each other as the whole motive forces from which they ariſe, that is, as the whole accelerative forces and the quantities of matter in correſponding parts; that is, (by the ſuppoſition) as the ſquares of the velocities directly, and the diſtances of the correſponding particles inverſely, and the quantities of matter in the correſpondent parts directly: and therefore ſince the diſtances of the particles in one ſyſtem are to the correſpondent diſtances of the particles of the other, as the diameter of one particle or part in the former ſyſtem to the diameter of the correſpondent particle or part in the other, and ſince the quantities of matter are as the denſities of the parts and the cubes of the diameters; the reſiſtances are to each other as the ſquares of the velocities and the ſquares of the diameters and the denſities of the parts of the ſyſtems. *Q. E. D.* The reſiſtances of the latter ſort are as the number of correſpondent reflexions and the forces of thoſe reflexions conjunctly. But the number of the reflexions are to each other as the velocities of the correſponding parts directly and the ſpaces between their reflexions inverſely. And the forces of the reflexions are as the velocities and the magnitudes and the denſities of the correſponding parts conjunctly; that is, as the velocities and the cubes of the diameters and the denſities of the parts. And joining all theſe ratio's, the reſiſtances of the correſponding parts are to each other as the ſquares of the velocities and the ſquares of the diameters and the denſities of the parts conjunctly. *Q. E. D.*

Cor. 1. Therefore if thoſe ſyſtems are two elaſtic fluids, like our air, and their parts are at reſt among themſelves; and two ſimilar bodies proportional in magnitude and denſity to the parts of the fluids and ſimilarly ſituated among thoſe parts, be any how projected in the direction of lines ſimilarly poſited; and the accelerative forces with which the particles of the fluids mutually,

mutually act upon each other, are as the diameters of the bodies projected inverſely and the ſquares of their velocities directly: thoſe bodies will excite ſimilar motions in the fluids in proportional times, and will deſcribe ſimilar ſpaces and proportional to their diameters.

Cor. 2. Therefore in the ſame fluid a projected body that moves ſwiftly meets with a reſiſtance that is in the duplicate ratio of its velocity, nearly. For if the forces, with which diſtant particles act mutually upon one another, ſhould be augmented in the duplicate ratio of the velocity, the projected body would be reſiſted in the ſame duplicate ratio accurately; and therefore in a medium, whoſe parts when at a diſtance do not act mutually with any force on one another, the reſiſtance is in the duplicate ratio of the velocity accurately. Let there be therefore three mediums *A*, *B*, *C*, conſiſting of ſimilar and equal parts regularly diſpoſed at equal diſtances. Let the parts of the mediums *A* and *B* recede from each other with forces that are among themſelves as *T* and *V*; and let the parts of the medium *C* be entirely deſtitute of any ſuch forces. And if four equal bodies *D*, *E*, *F*, *G* move in theſe mediums, the two firſt *D* and *E* in the two firſt *A* and *B*, and the other two *F* and *G* in the third *C*; and if the velocity of the body *D* be to the velocity of the body *E*, and the velocity of the body *F* to the velocity of the body *G* in the ſubduplicate ratio of the force *T* to the force *V*: the reſiſtance of the body *D* to the reſiſtance of the body *E*, and the reſiſtance of the body *F* to the reſiſtance of the body *G* will be in the duplicate ratio of the velocities; and therefore the reſiſtance of the body *D* will be to the reſiſtance of the body *F*, as the reſiſtance of the body *E* to the reſiſtance of the body *G*. Let the bodies *D* and *F* be equally ſwift, as alſo the bodies *E* and *G*; and augmenting the velocities of the bodies *D* and *F* in any ratio, and diminiſhing the forces of the particles of the medium *B* in the duplicate of the ſame

ratio, the medium *B* will approach to the form and condition of the medium *C* at pleaſure; and therefore the reſiſtances of the equal and equally ſwift bodies *E* and *G* in theſe mediums will perpetually approach to equality, ſo that their difference will at laſt become leſs than any given. Therefore ſince the reſiſtances of the bodies *D* and *F* are to each other as the reſiſtances of the bodies *E* and *G*, thoſe will alſo in like manner approach to the ratio of equality. Therefore the bodies *D* and *F*, when they move with very great ſwiftneſs, meet with reſiſtances very nearly equal; and therefore ſince the reſiſtance of the body *F* is in a duplicate ratio of the velocity, the reſiſtance of the body *D* will be nearly in the ſame ratio.

Cor. 3. The reſiſtance of a body moving very ſwift in an elaſtic fluid is almoſt the ſame as if the parts of the fluid were deſtitute of their centrifugal forces, and did not fly from each other: if ſo be that the elaſticity of the fluid ariſe from the centrifugal forces of the particles, and the velocity be ſo great as not to allow the particles time enough to act.

Cor. 4. Therefore ſince the reſiſtances of ſimilar and equally ſwift bodies, in a medium whoſe diſtant parts do not fly from each other, are as the ſquares of the diameters; the reſiſtances made to bodies moving with very great and equal velocities in an elaſtic fluid, will be as the ſquares of the diameters, nearly.

Cor. 5. And ſince ſimilar, equal, and equally ſwift bodies, moving thro' mediums of the ſame denſity, whoſe particles do not fly from each other mutually, will ſtrike againſt an equal quantity of matter in equal times, whether the particles of which the medium conſiſts be more and ſmaller, or fewer and greater, and therefore impreſs on that matter an equal quantity of motion, and in return (by the 3d law of motion) ſuffer an equal re-action from the ſame, that is, are equally reſiſted: it is manifeſt alſo, that in elaſtic fluids of the

the ſame denſity, when the bodies move with extreme ſwiftneſs, their reſiſtances are nearly equal; whether the fluids conſiſt of groſs parts, or of parts never ſo ſubtile. For the reſiſtance of projectiles moving with exceeding great celerities, is not much diminiſhed by the ſubtilty of the medium.

Cor. 6. All theſe things are ſo in fluids, whoſe elaſtic force takes its riſe from the centrifugal forces of the particles. But if that force ariſe from ſome other cauſe, as from the expanſion of the particles after the manner of wool, or the boughs of trees, or any other cauſe, by which the particles are hindered from moving freely among themſelves; the reſiſtance, by reaſon of the leſſer fluidity of the medium, will be greater than in the corollaries above.

Proposition XXXIV. Theorem XXVIII.

If in a rare medium, conſiſting of equal particles freely diſpoſed at equal diſtances from each other, a globe and a cylinder deſcribed on equal diameters move with equal velocities, in the direction of the axis of the cylinder: the reſiſtance of the globe will be but half ſo great as that of the cylinder.

For ſince the action of the medium upon the body is the ſame (by Cor. 5. of the laws) whether the body move in a quieſcent medium, or whether the particles of the medium impinge with the ſame velocity upon the quieſcent body: let us conſider the body as if it were quieſcent, and ſee with what force it would be impelled by the moving medium. Let therefore *ABKI* (*Pl. 6. Fig.* 2.) repreſent a ſphærical body deſcribed from the centre *C* with the ſemidiameter *CA*, and let the particles of the medium impinge with a gi-

ven velocity upon that ſphærical body, in the directions of right lines parallel to AC; and let FB be one of thoſe right lines. In FB take LB equal to the ſemidiameter CB, and draw BD touching the ſphere in B. Upon KC and BD let fall the perpendiculars BE, LD, and the force with which a particle of the medium, impinging on the globe obliquely in the direction FB, would ſtrike the globe in B, will be to the force with which the ſame particle, meeting the cylinder $ONGQ$ deſcribed about the globe with the axis ACI, would ſtrike it perpendicularly in b, as LD to LB or BE to BC. Again, the efficacy of this force to move the globe according to the direction of its incidence FB or AC, is to the efficacy of the ſame to move the globe according to the direction of its determination, that is, in the direction of the right line BC in which it impels the globe directly, as BE to BC. And joining theſe ratio's the efficacy of a particle, falling upon the globe obliquely in the direction of the right line FB, to move the globe in the direction of its incidence, is to the efficacy of the ſame particle falling in the ſame line perpendicularly on the cylinder, to move it in the ſame direction, as BE^2 to BC^2. Therefore if in bE, which is perpendicular to the circular baſe of the cylinder NAO, and equal to the radius AC, we take bH equal to $\frac{BE^2}{CB}$: then bH will be to bE as the effect of the particle upon the globe to the effect of the particle upon the cylinder. And therefore the ſolid which is formed by all the right lines bH will be to the ſolid formed by all the right lines bE as the effect of all the particles upon the globe to the effect of all the particles upon the cylinder. But the former of theſe ſolids is a paraboloid whoſe vertex is C, its axis CA and latus rectum CA; and the latter ſolid is a cylinder circumſcribing the paraboloid:

and

and it is known that a paraboloid is half its circumſcribed cylinder. Therefore the whole force of the medium upon the globe is half of the entire force of the ſame upon the cylinder. And therefore if the particles of the medium are at reſt, and the cylinder and globe move with equal velocities, the reſiſtance of the globe will be half the reſiſtance of the cylinder. *Q. E. D.*

Scholium.

By the ſame method other figures may be compared together as to their reſiſtance; and thoſe may be found which are moſt apt to continue their motions in reſiſting mediums. As if upon the circular baſe *CEBH* (*Pl. 6. Fig.* 3.) from the centre *O*, with the radius *OC*, and the altitude *OD*, one would conſtruct a fruſtum *CBGF* of a cone, which ſhould meet with leſs reſiſtance than any other fruſtum conſtructed with the ſame baſe and altitude, and going forwards towards *D* in the direction of its axis: biſect the altitude *OD* in *Q*, and produce *OQ* to *S*, ſo that *QS* may be equal to *QC*, and *S* will be the vertex of the cone whoſe fruſtum is ſought.

Whence by the bye, ſince the angle *CSB* is always acute, it follows, that if the ſolid *ADBE* (*Pl. 6. Fig.* 4.) be generated by the convolution of an elliptical or oval figure *ADBE* about its axe *AB*, and the generating figure be touched by three right lines *FG*, *GH*, *HI* in the points *F*, *B*, and *I*, ſo that *GH* ſhall be perpendicular to the axe in the point of contact *B*, and *FG*, *HI* may be inclined to *GH* in the angles *FGB*, *BHI* of 135 degrees; the ſolid ariſing from the convolution of the figure *ADFGHIE* about the ſame axe *AB*, will be leſs reſiſted than the former ſolid; if ſo be that both move forward in the direction of their axe *AB*, and that the extremity *B* of each go foremoſt. Which propoſition I conceive may be of uſe in the building of ſhips.

If the figure $DNFG$ be ſuch a curve, that if from any point thereof as N the perpendicular NM be let fall on the axe AB, and from the given point G there be drawn the right line GR parallel to a right line touching the figure in N, and cutting the axe produced in R, MN becomes to GR as GR^3 to $4BR \times GB^2$; the ſolid deſcribed by the revolution of this figure about its axe AB, moving in the beforementioned rare medium from A towards B, will be leſs reſiſted than any other circular ſolid whatſoever, deſcribed of the ſame length and breadth.

The demonſtration of theſe curious Theorems being omitted by the author, the analyſis thereof, communicated by a friend, is added at the end of this volume.

Proposition XXXV. Problem VII.

If a rare medium conſiſt of very ſmall quieſcent particles of equal magnitudes and freely diſpoſed at equal diſtances from one another: to find the reſiſtance of a globe moving uniformly forwards in this medium.

Case 1. Let a cylinder deſcribed with the ſame diameter and altitude be conceived to go forward with the ſame velocity in the direction of its axis, thro' the ſame medium. And let us ſuppoſe that the particles of the medium, on which the globe or cylinder falls, fly back with as great a force of reflexion as poſſible. Then ſince the reſiſtance of the globe (by the laſt Propoſition) is but half the reſiſtance of the cylinder, and ſince the globe is to the cylinder as 2 to 3, and ſince the cylinder by falling perpendicularly on the particles, and reflecting them with the utmoſt force communicates to them a velocity double to its own: it follows that the cylinder, in moving forward uniformly half the length of its axis, will communicate a motion to the particles, which is to the whole motion of the cylinder

as

Fig. 1. *p.* 91.

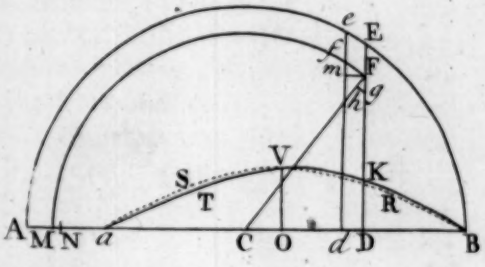


Fig. 2. *p.* 117.

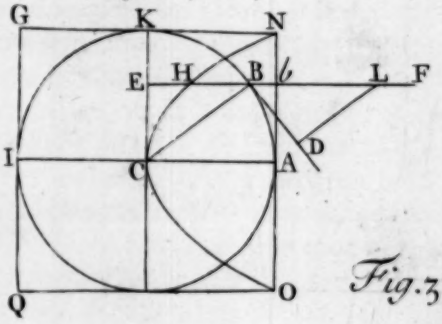


Fig. 3. *p.* 119.

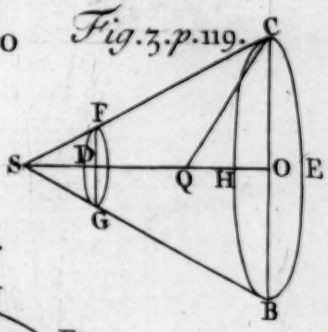
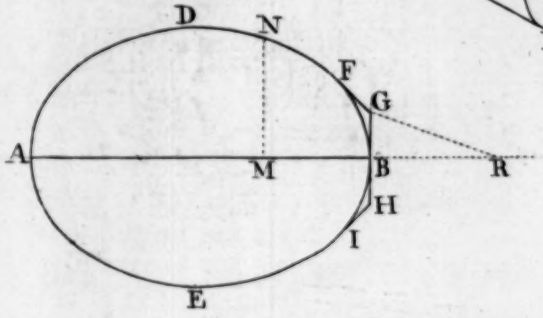


Fig. 4. *ibid.*



as the denſity of the medium to the denſity of the cylinder; and that the globe, in the time it deſcribes one length of its diameter in moving uniformly forwards, will communicate the ſame motion to the particles; and in the time that it deſcribes two thirds of its diameter, will communicate a motion to the particles, which is to the whole motion of the globe as the denſity of the medium to the denſity of the globe. And therefore the globe meets with a reſiſtance, which is to the force by which its whole motion may be either taken away or generated in the time in which it deſcribes two thirds of its diameter moving uniformly forwards, as the denſity of the medium to the denſity of the globe.

Case 2. Let us ſuppoſe that the particles of the medium incident on the globe or cylinder are not reflected; and then the cylinder falling perpendicularly on the particles will communicate its own ſimple velocity to them, and therefore meets a reſiſtance but half ſo great as in the former caſe, and the globe alſo meets with a reſiſtance but half ſo great.

Case 3. Let us ſuppoſe the particles of the medium to fly back from the globe with a force which is neither the greateſt nor yet none at all, but with a certain mean force; then the reſiſtance of the globe will be in the ſame mean ratio between the reſiſtance in the firſt caſe and the reſiſtance in the ſecond. *Q. E. I.*

Cor. 1. Hence if the globe and the particles are infinitely hard, and deſtitute of all elaſtic force, and therefore of all force of reflexion: the reſiſtance of the globe will be to the force by which its whole motion may be deſtroyed or generated, in the time that the globe deſcribes four third parts of its diameter, as the denſity of the medium to the denſity of the globe.

Cor. 2. The reſiſtance of the globe, *cæteris paribus*, is in the duplicate ratio of the velocity.

Cor. 3. The reſiſtance of the globe, *cæteris paribus*, is in the duplicate ratio of the diameter.

Cor.

Cor. 4. The reſiſtance of the globe is, *cæteris paribus*, as the denſity of the medium.

Cor. 5. The reſiſtance of the globe is in a ratio compounded of the duplicate ratio of the velocity, and the duplicate ratio of the diameter, and the ratio of the denſity of the medium.

Cor. 6. The motion of the globe and its reſiſtance may be thus expounded. Let *AB* (*Pl.* 7. *Fig.* 1.) be the time in which the globe may, by its reſiſtance uniformly continued, loſe its whole motion. Erect *AD*, *BC* perpendicular to *AB*. Let *BC* be that whole motion, and thro' the point *C*, the aſymptotes being *AD*, *AB*, deſcribe the hyperbola *CF*. Produce *AB* to any point *E*. Erect the perpendicular *EF* meeting the hyperbola in *F*. Compleat the parallelogram *CBEG*, and draw *AF* meeting *BC* in *H*. Then if the globe in any time *BE*, with its firſt motion *BC* uniformly continued, deſcribes in a non-reſiſting medium the ſpace *CBEG* expounded by the area of the parallelogram, the ſame in a reſiſting medium will deſcribe the ſpace *CBEF* expounded by the area of the hyperbola; and its motion at the end of that time will be expounded by *EF* the ordinate of the hyperbola; there being loſt of its motion the part *FG*. And its reſiſtance at the end of the ſame time will be expounded by the length *BH*; there being loſt of its reſiſtance the part *CH*. All theſe things appear by Cor. 1 and 3. Prop. 5. Book 2.

Cor. 7. Hence if the globe in the time T by the reſiſtance R uniformly continued, loſe its whole motion M: the ſame globe in the time *t* in a reſiſting medium, wherein the reſiſtance R decreaſes in a duplicate ratio of the velocity, will loſe out of its motion M the part $\frac{tM}{T+t}$, the part $\frac{TM}{T+t}$ remaining; and will deſcribe a ſpace which is to the ſpace deſcribed in the ſame

ſame time t with the uniform motion M, as the logarithm of the number $\frac{T+t}{T}$ multiplied by the number 2,302585092994 is to the number $\frac{t}{T}$, becauſe the hyperbolic area *BCFE* is to the rectangle *BCGE* in that proportion.

Scholium.

I have exhibited in this Propoſition the reſiſtance and retardation of ſphærical projectiles in mediums that are not continued, and ſhewn that this reſiſtance is to the force by which the whole motion of the globe may be deſtroyed or produced in the time in which the globe can deſcribe two thirds of its diameter, with a velocity uniformly continued, as the denſity of the medium to the denſity of the globe, if ſo be the globe and the particles of the medium be perfectly elaſtic, and are indued with the utmoſt force of reflexion: and that this force, where the globe and particles of the medium are infinitely hard and void of any reflecting force, is diminiſhed one half. But in continued mediums, as water, hot oil, and quickſilver, the globe as it paſſes thro them does not immediately ſtrike againſt all the particles of the fluid that generate the reſiſtance made to it, but preſſes only the particles that lie next to it, which preſs the particles beyond, which preſs other particles, and ſo on; and in theſe mediums the reſiſtance is diminiſhed one other half. A globe in theſe extremely fluid mediums meets with a reſiſtance that is to the force by which its whole motion may be deſtroyed or generated in the time wherein it can deſcribe, with that motion uniformly continued, eight third parts of its diameter, as the denſity of the medium to the denſity of the globe. This I ſhall endeavour to ſhew in what follows. Pro-

Proposition XXXVI. Problem VIII.

To define the motion of water running out of a cylindrical veſſel thro' a hole made at the bottom.

Let *ACDB* (*Pl.* 7. *Fig.* 2.) be a cylindrical veſſel, *AB* the mouth of it, *CD* the bottom parallel to the horizon, *EF* a circular hole in the middle of the bottom, *G* the centre of the hole, and *GH* the axis of the cylinder perpendicular to the horizon. And ſuppoſe a cylinder of ice *APQB* to be of the ſame breadth with the cavity of the veſſel, and to have the ſame axis, and to deſcend perpetually with an uniform motion, and that its parts as ſoon as they touch the ſuperficies *AB* diſſolve into water, and flow down by their weight into the veſſel, and in their fall compoſe the cataract or column of water *ABNFEM*, paſſing thro' the hole *EF*, and filling up the ſame exactly. Let the uniform velocity of the deſcending ice and of the contiguous water in the circle *AB* be that which the water would acquire by falling thro' the ſpace *IH*; and let *IH* and *HG* lie in the ſame right line, and thro' the point *I* let there be drawn the right line *KL* parallel to the horizon, and meeting the ice on both the ſides thereof in *K* and *L*. Then the velocity of the water running out at the hole *EF* will be the ſame that it would acquire by falling from *I* thro' the ſpace *IG*. Therefore, by *Galileo*'s Theorems, *IG* will be to *IH* in the duplicate ratio of the velocity of the water that runs out at the hole to the velocity of the water in the circle *AB*, that is, in the duplicate ratio of the circle *AB* to the circle *EF*; thoſe circles being reciprocally as the velocities of the water which in the ſame time and in equal quantities paſſes ſeverally thro'

each of them, and compleatly fills them both. We are now conſidering the velocity with which the water tends to the plane of the horizon. But the motion parallel to the ſame by which the parts of the falling water approach to each other, is not here taken notice of; ſince it is neither produced by gravity, nor at all changes the motion perpendicular to the horizon which the gravity produces. We ſuppoſe indeed that the parts of the water cohere a little, that by their coheſion they may in falling approach to each other with motions parallel to the horizon, in order to form one ſingle cataract, and to prevent their being divided into ſeveral: but the motion parallel to the horizon ariſing from this coheſion does not come under our preſent conſideration.

Case 1. Conceive now the whole cavity in the veſſel, which encompaſſes the falling water *ABNFEM*, to be full of ice, ſo that the water may paſs thro' the ice as thro' a funnel. Then if the water paſs very near to the ice only, without touching it; or, which is the ſame thing, if, by reaſon of the perfect ſmoothneſs of the ſurface of the ice, the water, tho' touching it, glides over it with the utmoſt freedom, and without the leaſt reſiſtance; the water will run thro' the hole *EF* with the ſame velocity as before, and the whole weight of the column of water *ABNFEM* will be all taken up as before in forcing out the water, and the bottom of the veſſel will ſuſtain the weight of the ice encompaſſing that column.

Let now the ice in the veſſel diſſolve into water; yet will the efflux of the water remain, as to its velocity, the ſame as before. It will not be leſs, becauſe the ice now diſſolved will endeavour to deſcend; it will not be greater, becauſe the ice now become water cannot deſcend without hindering the deſcent of other water equal to its own deſcent. The ſame force ought always to generate the ſame velocity in the effluent water.

But

But the hole at the bottom of the veſſel, by reaſon of the oblique motions of the particles of the effluent water, muſt be a little greater than before. For now the particles of the water do not all of them paſs thro' the hole perpendicularly; but flowing down on all parts from the ſides of the veſſel, and converging towards the hole, paſs thro' it with oblique motions; and in tending downwards meet in a ſtream whoſe diameter is a little ſmaller below the hole than at the hole itſelf, its diameter being to the diameter of the hole as 5 to 6, or as $5\frac{1}{2}$ to $6\frac{1}{2}$, very nearly, if I took the meaſures of thoſe diameters right. I procured a very thin flat plate having a hole pierced in the middle, the diameter of the circular hole being $\frac{1}{8}$ parts of an inch. And that the ſtream of running water might not be accelerated in falling, and by that acceleration become narrower, I fixed this plate, not to the bottom, but to the ſide of the veſſel, ſo as to make the water go out in the direction of a line parallel to the horizon. Then when the veſſel was full of water, I opened the hole to let it run out; and the diameter of the ſtream, meaſured with great accuracy at the diſtance of about half an inch from the hole, was $\frac{21}{40}$ of an inch. Therefore the diameter of this circular hole was to the diameter of the ſtream very nearly as 25 to 21. So that the water in paſſing thro' the hole, converges on all ſides, and after it has run out of the veſſel, becomes ſmaller by converging in that manner, and by becoming ſmaller is accelerated till it comes to the diſtance of half an inch from the hole, and at that diſtance flows in a ſmaller ſtream and with greater celerity than in the hole itſelf, and this in the ratio of 25×25 to 21×21 or 17 to 12 very nearly, that is, in about the ſubduplicate ratio of 2 to 1. Now it is certain from experiments, that the quantity of water, running out in a given time thro' a circular hole made in the bottom of a veſſel is equal to the quantity, which, flowing with the aforeſaid velocity,

city, would run out in the ſame time, thro' another circular hole, whoſe diameter is to the diameter of the former as 21 to 25. And therefore that running water in paſſing thro' the hole itſelf has a velocity downwards equal to that which a heavy body would acquire in falling thro' half the height of the ſtagnant water in the veſſel, nearly. But then after it has run out, it is ſtill accelerated by converging, till it arrives at a diſtance from the hole that is nearly equal to its diameter, and acquires a velocity greater than the other in about the ſubduplicate ratio of 2 to 1; which velocity a heavy body would nearly acquire, by falling thro' the whole height of the ſtagnant water in the veſſel.

Therefore in what follows let the diameter of the ſtream be repreſented by that leſſer hole which we called *EF*. And imagine another plane *VW* above the hole *EF*, (*Pl.* 7. *Fig.* 3.) and parallel to the plane thereof, to be placed at a diſtance equal to the diameter of the ſame hole, and to be pierced thro' with a greater hole *ST*, of ſuch a magnitude that a ſtream which will exactly fill the lower hole *EF* may paſs thro' it; the diameter of which hole will therefore be to the diameter of the lower hole as 25 to 21, nearly. By this means the water will run perpendicularly out at the lower hole; and the quantity of the water running out will be, according to the magnitude of this laſt hole, the ſame, very nearly, which the ſolution of the problem requires. The ſpace included between the two planes and the falling ſtream may be conſider'd as the bottom of the veſſel. But to make the ſolution more ſimple and mathematical, it is better to take the lower plane alone for the bottom of the veſſel, and to ſuppoſe that the water which flowed thro' the ice as thro' a funnel, and ran out of the veſſel thro' the hole *EF* made in the lower plane, preſerves its motion continually, and that the ice continues at reſt. Therefore in what follows let *ST* be the diameter of a circular hole deſcribed from the centre *Z*, and let the ſtream run out

of the veſſel thro' that hole when the water in the veſſel is all fluid. And let *EF* be the diameter of the hole which the ſtream, in falling thro', exactly fills up, whether the water runs out of the veſſel by that upper hole *ST*, or flows thro' the middle of the ice in the veſſel, as thro' a funnel. And let the diameter of the upper hole *ST* be to the diameter of the lower *EF* as about 25 to 21, and let the perpendicular diſtance between the planes of the holes be equal to the diameter of the leſſer hole *EF*. Then the velocity of the water downwards in running out of the veſſel thro' the hole *ST*, will be in that hole the ſame that a body may acquire by falling from half the height *IZ*: and the velocity of both the falling ſtreams will be, in the hole *EF*, the ſame which a body would acquire by falling from the whole height *IG*.

Case 2. If the hole *EF* be not in the middle of the bottom of the veſſel, but in ſome other part thereof, the water will ſtill run out with the ſame velocity as before, if the magnitude of the hole be the ſame. For tho an heavy body takes a longer time in deſcending to the ſame depth, by an oblique line, than by a perpendicular line; yet in both caſes it acquires in its deſcent the ſame velocity, as *Galileo* has demonſtrated.

Case 3. The velocity of the water is the ſame when it runs out thro' a hole in the ſide of the veſſel. For if the hole be ſmall, ſo that the interval between the ſuperficies *AB* and *KL* may vaniſh as to ſenſe, and the ſtream of water horizontally iſſuing out may form a parabolic figure: from the latus rectum of this parabola may be collected, that the velocity of the effluent water is that which a body may acquire by falling the height *IG* or *HG* of the ſtagnant water in the veſſel. For by making an experiment, I found that if the height of the ſtagnant water above the hole were 20 inches, and the height

height of the hole above a plane parallel to the horizon were alſo 20 inches, a ſtream of water ſpringing out from thence would fall upon the plane, at the diſtance of 37 inches, very nearly, from a perpendicular let fall upon that plane from the hole. For without reſiſtance the ſtream would have fallen upon the plane at the diſtance of 40 inches, the latus rectum of the parabolic ſtream being 80 inches.

Case 4. If the effluent water tend upwards, it will ſtill iſſue forth with the ſame velocity. For the ſmall ſtream of water ſpringing upwards, aſcends with a perpendicular motion to *GH* or *GI* the height of the ſtagnant water in the veſſel; excepting in ſo far as its aſcent is hindered a little by the reſiſtance of the air; and therefore it ſprings out with the ſame velocity that it would acquire in falling from that height. Every particle of the ſtagnant water is equally preſſed on all ſides, (by Prop. 19. Book 2.) and yielding to the preſſure, tends all ways with an equal force, whether it deſcends thro' the hole in the bottom of the veſſel, or guſhes out in an horizontal direction thro' an hole in the ſide, or paſſes into a canal, and ſprings up from thence thro' a little hole made in the upper part of the canal. And it may not only be collected from reaſoning, but is manifeſt alſo from the well-known experiments juſt mentioned, that the velocity with which the water runs out is the very ſame that is aſſigned in this Propoſition.

Case 5. The velocity of the effluent water is the ſame, whether the figure of the hole be circular, or ſquare, or triangular, or any other figure equal to the circular. For the velocity of the effluent water does not depend upon the figure of the hole, but ariſes from its depth below the plane *KL*.

Case 6. If the lower part of the veſſel *ABDC* be immerſed into ſtagnant water, and the height of the ſtagnant water above the bottom of the veſſel be *GR*; the velocity with which the water that is in the veſſel

will run out at the hole *EF* into the ſtagnant water, will be the ſame which the water would acquire by falling from the height *IR*. For the weight of all the water in the veſſel that is below the ſuperficies of the ſtagnant water will be ſuſtained in equilibrio by the weight of the ſtagnant water, and therefore does not at all accelerate the motion of the deſcending water in the veſſel. This caſe will alſo appear by experiments, meaſuring the times in which the water will run out.

Cor. 1. Hence if *CA* the depth of the water be produced to *K*, ſo that *AK* may be to *CK* in the duplicate ratio of the area of a hole made in any part of the bottom, to the area of the circle *AB*; the velocity of the effluent water will be equal to the velocity which the water would acquire by falling from the height *KC*.

Cor. 2. And the force with which the whole motion of the effluent water may be generated, is equal to the weight of a cylindric column of water, whoſe baſe is the hole *EF*, and its altitude 2*GI* or 2*CK*. For the effluent water, in the time it becomes equal to this column, may acquire, by falling by its own weight from the height *GI*, a velocity equal to that with which it runs out.

Cor. 3. The weight of all the water in the veſſel *ABDC* is to that part of the weight which is employed in forcing out the water as the ſum of the circles *AB* and *EF* to twice the circle *EF*. For let *IO* be a mean proportional between *IH* and *IG*, the water running out at the hole *EF*, will, in the time that a drop falling from *I* would deſcribe the altitude *IG*, become equal to a cylinder whoſe baſe is the circle *EF* and its altitude 2*IG*, that is, to a cylinder whoſe baſe is the circle *AB*, and whoſe altitude is 2*IO*. For the circle *EF* is to the circle *AB* in the ſubduplicate ratio of the altitude *IH* to the altitude *IG*; that is, in the ſimple ratio of the mean proportional *IO* to the altitude *IG*. Moreover, in the time that a drop falling from *I* can deſcribe

deſcribe the altitude IH, the water that runs out will have become equal to a cylinder whoſe baſe is the circle AB and its altitude $2IH$; and in the time that a drop falling from I thro' H to G deſcribes HG the difference of the altitudes, the effluent water, that is, the water contained within the ſolid $ABNFEM$ will be equal to the difference of the cylinders, that is, to a cylinder whoſe baſe is AB and its altitude $2HO$. And therefore all the water contained in the veſſel $ABDC$ is to the whole falling water contained in the ſaid ſolid $ABNFEM$ as HG to $2HO$, that is, as $HO + OG$ to $2HO$, or $IH + IO$ to $2IH$. But the weight of all the water in the ſolid $ABNFEM$ is employed in forcing out the water; and therefore the weight of all the water in the veſſel is to that part of the weight that is employed in forcing out the water, as $IH + IO$ to $2IH$, and therefore as the ſum of the circles EF and AB to twice the circle EF.

Cor. 4. And hence the weight of all the water in the veſſel $ABDC$ is to the other part of the weight which is ſuſtained by the bottom of the veſſel, as the ſum of the circles AB and EF to the difference of the ſame circles.

Cor. 5. And that part of the weight which the bottom of the veſſel ſuſtains, is to the other part of the weight employed in forcing out the water, as the difference of the circles AB and EF to twice the leſſer circle EF, or as the area of the bottom to twice the hole.

Cor. 6. That part of the weight which preſſes upon the bottom is to the whole weight of the water perpendicularly incumbent thereon as the circle AB to the ſum of the circles AB and EF, or as the circle AB to the exceſs of twice the circle AB above the area of the bottom. For that part of the weight which preſſes upon the bottom is to the weight of the whole water in the veſſel as the difference of the circles AB

and EF to the ſum of the ſame circles, (by Cor. 4.) and the weight of the whole water in the veſſel is to the weight of the whole water perpendicularly incumbent on the bottom as the circle AB to the difference of the circles AB and EF. Therefore, *ex æquo perturbatè*, that part of the weight which preſſes upon the bottom is to the weight of the whole water perpendicularly incumbent thereon as the circle AB to the ſums of the circles AB and EF, or the exceſs of twice the circle AB above the bottom.

Cor. 7. If in the middle of the hole EF there be placed the little circle PQ deſcribed about the centre G, and parallel to the horizon; the weight of water which that little circle ſuſtains is greater than the weight of a third part of a cylinder of water whoſe baſe is that little circle and its height GH. For let $ABNFEM$ (*Pl.* 7. *Fig.* 4.) be the cataract or column of falling water whoſe axis is GH as above, and let all the water, whoſe fluidity is not requiſite for the ready and quick deſcent of the water, be ſuppoſed to be congealed; as well round about the cataract, as above the little circle, And let PHQ be the column of water, congealed above the little circle, whoſe vertex is H, and its altitude GH. And ſuppoſe this cataract to fall with its whole weight downwards, and not in the leaſt to lie againſt or to preſs PHQ, but to glide freely by it without any friction, unleſs perhaps juſt at the very vertex of the ice where the cataract at the beginning of its fall may tends to a concave figure. And as the congealed water $AMEC$, $BNFD$ lying round the cataract, is convex in its internal ſuperficies AME, BNF towards the falling cataract, ſo this column PHQ will be convex towards the cataract alſo, and will therefore be greater than a cone whoſe baſe is that little circle PQ and its altitude GH, that is, greater than a third part of a cylinder deſcribed with the ſame baſe and altitude. Now that little circle ſuſtains the weight of this column,

that is, a weight greater than the weight of the cone or a third part of the cylinder.

Cor. 8. The weight of water which the circle $P\,Q$, when very ſmall, ſuſtains, ſeems to be leſs than the weight of two thirds of a cylinder of water whoſe baſe is that little circle, and its altitude HG. For, things ſtanding as above ſuppoſed, imagine the half of a ſphæroid deſcribed whoſe baſe is that little circle, and its ſemi-axis or altitude HG. This figure will be equal to two thirds of that cylinder, and will comprehend within it the column of congealed water $P\,H\,Q$, the weight of which is ſuſtained by that little circle. For tho' the motion of the water tends directly downwards, the external ſuperficies of that column muſt yet meet the baſe $P\,Q$ in an angle ſomewhat acute, becauſe the water in its fall is perpetually accelerated, and by reaſon of that acceleration becomes narrower. Therefore, ſince that angle is leſs than a right one, this column in the lower parts thereof will lie within the hemi-ſphæroid. In the upper parts alſo it will be acute or pointed; becauſe, to make it otherwiſe, the horizontal motion of the water muſt be at the vertex infinitely more ſwift than its motion towards the horizon. And the leſs this circle $P\,Q$ is, the more acute will the vertex of this column be; and the circle being diminiſhed in infinitum, the angle $P\,H\,Q$ will be diminiſhed in infinitum, and therefore the column will lie within the hemi-ſphæroid. Therefore that column is leſs than that hemi-ſphæroid, or than two third parts of the cylinder whoſe baſe is that little circle, and its altitude GH. Now the little circle ſuſtains a force of water equal to the weight of this column, the weight of the ambient water being employed in cauſing its efflux out at the hole.

Cor. 9. The weight of water which the little circle $P\,Q$ ſuſtains when it is very ſmall, is very nearly equal to the weight of a cylinder of water whoſe baſe is that little circle, and its altitude $\frac{1}{2}GH$. For this weight is

an arithmetical mean between the weights of the cone and the hemi-ſphæroid abovementioned. But if that little circle be not very ſmall, but on the contrary increaſed till it be equal to the hole EF; it will ſuſtain the weight of all the water lying perpendicularly above it, that is, the weight of a cylinder of water whoſe baſe is that little circle and its altitude GH.

Cor. 10. And (as far as I can judge) the weight which this little circle ſuſtains is always to the weight of a cylinder of water whoſe baſe is that little circle and its altitude $\frac{1}{4}GH$, as EF^2 to $EF^2 - \frac{1}{2}PQ^2$, or as the circle EF to the exceſs of this circle above half the little circle PQ, very nearly.

Lemma IV.

If a cylinder move uniformly forwards in the direction of its length, the reſiſtance made thereto is not at all changed by augmenting or diminiſhing that length; and is therefore the ſame with the reſiſtance of a circle, deſcribed with the ſame diameter, and moving forwards with the ſame velocity in the direction of a right line perpendicular to its plane.

For the ſides are not at all oppoſed to the motion; and a cylinder becomes a circle when its length is diminiſhed in infinitum.

Pro-

PROPOSITION XXXVII. THEOREM XXIX.

If a cylinder move uniformly forwards in a compreſſed, infinite, and non-elaſtic fluid, in the direction of its length; the reſiſtance ariſing from the magnitude of its tranſverſe ſection, is to the force by which its whole motion may be deſtroyed or generated, in the time that it moves four times its length, as the denſity of the medium to the denſity of the cylinder, nearly.

For let the veſſel $ABDC$ (*Pl.* 7. *Fig.* 5.) touch the ſurface of ſtagnant water with its bottom CD, and let the water run out of this veſſel into the ſtagnant water thro' the cylindric canal $EFTS$ perpendicular to the horizon; and let the little circle PQ be placed parallel to the horizon any where in the middle of the canal; and produce CA to K, ſo that AK may be to CK in the duplicate of the ratio, which the exceſs of the orifice of the canal EF above the little circle PQ, bears to the circle AB. Then 'tis manifeſt (by Caſe 5. Caſe 6. and Cor. 1. Prop. 36.) that the velocity of the water paſſing thro' the annular ſpace between the little circle and the ſides of the veſſel, will be the very ſame which the water would acquire by falling, and in its fall deſcribing the altitude KC or IG.

And (by Cor. 10. Prop. 36.) if the breadth of the veſſel be infinite, ſo that the lineola HI may vaniſh, and the altitudes IG, HG become equal; the force of the water that flows down, and preſſes upon the circle will be to the weight of a cylinder whoſe baſe is that little circle and the altitude $\frac{1}{2}IG$, as EF^2 to $EF^2 - \frac{1}{2}PQ^2$ very nearly. For the force of the water flowing downwards uniformly thro' the whole

canal will be the ſame upon the little circle PQ in whatſoever part of the canal it be placed.

Let now the orifices of the canal EF, ST be cloſed, and let the little circle aſcend in the fluid compreſſed on every ſide, and by its aſcent let it oblige the water that lies above it to deſcend thro' the annular ſpace between the little circle and the ſides of the canal. Then will the velocity of the aſcending little circle be to the velocity of the deſcending water as the difference of the circles EF and PQ is to the circle PQ; and the velocity of the aſcending little circle will be to the ſum of the velocities, that is, to the relative velocity of the deſcending water with which it paſſes by the little circle in its aſcent, as the difference of the circles EF and PQ to the circle EF, or as $EF^2 - PQ^2$ to EF^2. Let that relative velocity be equal to the velocity with which it was ſhewn above that the water would paſs thro' the annular ſpace if the circle were to remain unmoved, that is, to the velocity which the water would acquire by falling, and in its fall deſcribing the altitude IG; and the force of the water upon the aſcending circle will be the ſame as before, (by cor. 5. of the laws of motion) that is, the reſiſtance of the aſcending little circle will be to the weight of a cylinder of water whoſe baſe is that little circle and its altitude $\frac{1}{2}IG$, as EF^2 to $EF^2 - \frac{1}{2}PQ^2$ nearly. But the velocity of the little circle will be to the velocity which the water acquires by falling, and in its fall deſcribing the altitude IG, as $EF^2 - PQ^2$ to EF^2.

Let the breadth of the canal be increaſed in infinitum; and the ratio's between $EF^2 - PQ^2$ and EF^2, and between EF^2 and $EF^2 - \frac{1}{2}PQ^2$ will become at laſt ratio's of equality. And therefore the velocity of the little circle will now be the ſame which the water would acquire in falling, and in its fall deſcribing the altitude IG; and the reſiſtance will become equal to the weight of a cylinder whoſe baſe is that little circle, and

and its altitude half the altitude *IG*, from which the cylinder muſt fall to acquire the velocity of the aſcending circle. And with this velocity the cylinder in the time of its fall will deſcribe four times its length. But the reſiſtance of the cylinder moving forwards with this velocity in the direction of its length, is the ſame with the reſiſtance of the little circle, (by Lem. 4.) and is therefore nearly equal to the force by which its motion may be generated while it deſcribes four times its length.

If the length of the cylinder be augmented or diminiſhed, its motion, and the time in which it deſcribes four times its length, will be augmented or diminiſhed in the ſame ratio; and therefore the force by which the motion, ſo increaſed or diminiſhed, may be deſtroyed or generated, will continue the ſame; becauſe the time is increaſed or diminiſhed in the ſame proportion; and therefore that force remains ſtill equal to the reſiſtance of the cylinder, becauſe (by Lem. 4.) that reſiſtance will alſo remain the ſame.

If the denſity of the cylinder be augmented or diminiſhed, its motion, and the force by which its motion may be generated or deſtroyed in the ſame time, will be augmented or diminiſhed in the ſame ratio. Therefore the reſiſtance of any cylinder whatſoever will be to the force by which its whole motion may be generated or deſtroyed in the time during which it moves four times its length, as the denſity of the medium to the denſity of the cylinder, nearly. *Q. E. D.*

A fluid muſt be compreſſed to become continued; it muſt be continued and non-elaſtic, that all the preſſure ariſing from its compreſſion may be propagated in an inſtant; and ſo acting equally upon all parts of the body moved, may produce no change of the reſiſtance. The preſſure ariſing from the motion of the body is ſpent in generating a motion in the parts of the fluid, and this creates the reſiſtance. But the preſſure ariſing from

from the compreſſion of the fluid, be it never ſo forcible, if it be propagated in an inſtant, generates no motion in the parts of a continued fluid, produces no change at all of motion therein; and therefore neither augments nor leſſens the reſiſtance. This is certain, that the action of the fluid ariſing from the compreſſion cannot be ſtronger on the hinder parts of the body moved than on its fore parts, and therefore cannot leſſen the reſiſtance deſcribed in this Propoſition. And if its propagation be infinitely ſwifter than the motion of the body preſſed, it will not be ſtronger on the fore parts than on the hinder parts. But that action will be infinitely ſwifter and propagated in an inſtant, if the fluid be continued and non-elaſtic.

Cor. 1. The reſiſtances made to cylinders going uniformly forwards in the direction of their lengths thro' continued infinite mediums, are in a ratio compounded of the duplicate ratio of the velocities and the duplicate ratio of the diameters, and the ratio of the denſity of the mediums.

Cor. 2. If the breadth of the canal be not infinitely increaſed, but the cylinder go forwards in the direction of its length through an included quieſcent medium, its axis all the while coinciding with the axis of the canal; its reſiſtance will be to the force by which its whole motion in the time in which it deſcribes four times its length, may be generated or deſtroyed, in a ratio compounded of the ratio of EF^2 to $EF^2 - \frac{1}{2}PQ^2$ once, and the ratio of EF^2 to $EF^2 - PQ^2$ twice, and the ratio of the denſity of the medium to the denſity of the cylinder.

Cor. 3. The ſame things ſuppoſed, and that a length L is to the quadruple of the length of the cylinder in a ratio compounded of the ratio $EF^2 - \frac{1}{2}PQ^2$ to EF^2 once, and the ratio of $EF^2 - PQ^2$ to EF^2 twice; the reſiſtance of the cylinder will be to the force by which its whole motion, in the time during which it deſcribes the

the length L, may be deſtroyed or generated, as the denſity of the medium to the denſity of the cylinder.

Scholium.

In this propoſition we have inveſtigated that reſiſtance alone which ariſes from the magnitude of the tranſverſe ſection of the cylinder, neglecting that part of the ſame which may ariſe from the obliquity of the motions. For as in Caſe 1. of Prop. 36. the obliquity of the motions with which the parts of the water in the veſſel converged on every ſide to the hole *EF*, hindered the efflux of the water thro' the hole; ſo in this propoſition, the obliquity of the motions, with which the parts of the water, preſſed by the antecedent extremity of the cylinder, yield to the preſſure and diverge on all ſides, retards their paſſage, thro' the places that lie round that antecedent extremity, towards the hinder parts of the cylinder, and cauſes the fluid to be moved to a greater diſtance; which increaſes the reſiſtance, and that in the ſame ratio almoſt in which it diminiſhed the efflux of the water out of the veſſel, that is, in the duplicate ratio of 25 to 21, nearly. And as in Caſe 1. of that Propoſition, we made the parts of the water paſs thro' the hole *EF* perpendicularly and in the greateſt plenty, by ſuppoſing all the water in the veſſel lying round the cataract to be frozen, and that part of the water whoſe motion was oblique and uſeleſs to remain without motion; ſo in this propoſition, that the obliquity of the motions may be taken away, and the parts of the water may give the freeſt paſſage to the cylinder, by yielding to it with the moſt direct and quick motion poſſible, ſo that only ſo much reſiſtance may remain as ariſes from the magnitude of the tranſverſe ſection, and which is incapable of diminution, unleſs by diminiſhing the diameter of the cylinder; we muſt conceive thoſe parts of the fluid whoſe motions

are

are oblique and uſeleſs, and produce reſiſtance, to be at reſt among themſelves at both extremities of the cylinder, and there to cohere, and be joined to the cylinder. Let *ABCD* (*Pl. 7. Fig. 6.*) be a rectangle, and let *AE* and *BE* be two parabolic arcs, deſcribed with the axis *AB*, and with a latus rectum that is to the ſpace *HG*, which muſt be deſcribed by the cylinder in falling in order to acquire the velocity with which it moves, as *HG* to $\frac{1}{2}AB$. Let *CF* and *DF* be two other parabolic arcs deſcribed with the axis *CD*, and a latus rectum quadruple of the former; and by the convolution of the figure about the axis *EF* let there be generated a ſolid, whoſe middle part *ABDC* is the cylinder we are here ſpeaking of, and the extreme parts *ABE* and *CDF* contain the parts of the fluid, at reſt among themſelves, and concreted into two hard bodies, adhering to the cylinder at each end like a head and tail. Then if this ſolid *EACFDB* move in the direction of the length of its axis *FE* towards the parts beyond *E*, the reſiſtance will be the ſame which we have here determined in this propoſition, nearly; that is, it will have the ſame ratio to the force with which the whole motion of the cylinder may be deſtroyed or generated in the time that it is deſcribing the length $4AC$ with that motion uniformly continued, as the denſity of the fluid has to the denſity of the cylinder, nearly. And (by Cor. 7. Prop. 36.) the reſiſtance muſt be to this force in the ratio of 2 to 3, at the leaſt.

LEMMA

Plate VII. *Vol.* II. *P.* 140.

Fig. 1. *p.* 122.

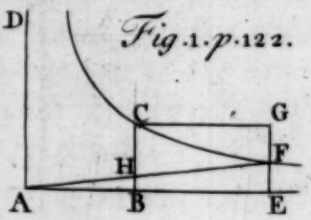


Fig. 2. *p.* 124.

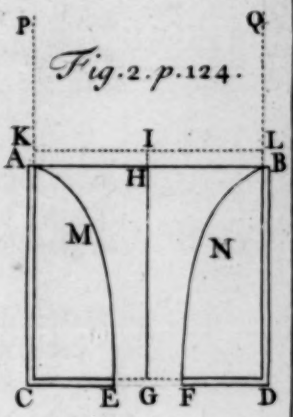


Fig. 3. *p.* 127.

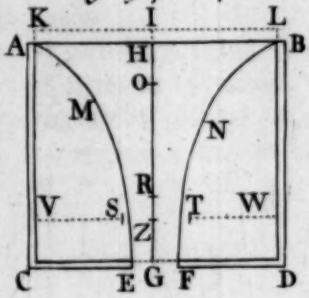


Fig. 4. *p.* 132.

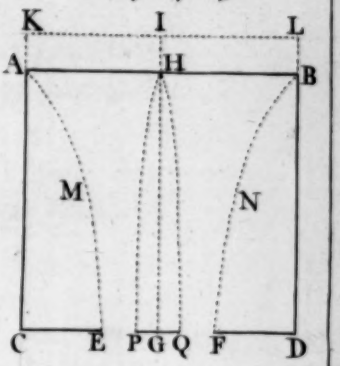


Fig. 5. *p.* 135.

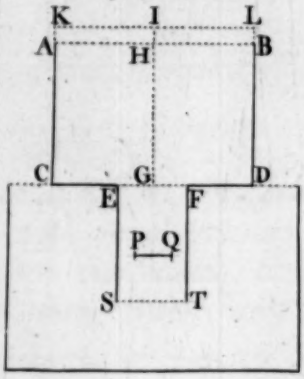
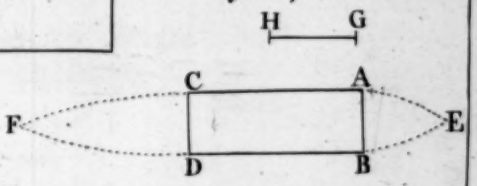


Fig. 6. *p.* 140.



Lemma V.

If a cylinder, a ſphere, and a ſphæroid, of equal breadths be placed ſucceſſively in the middle of a cylindric canal, ſo that their axes may coincide with the axis of the canal; theſe bodies will equally hinder the paſſage of the water thro' the canal.

For the ſpaces, lying between the ſides of the canal, and the cylinder, ſphere, and ſphæroid, thro' which the water paſſes, are equal; and the water will paſs equally thro' equal ſpaces.

This is true upon the ſuppoſition that all the water above the cylinder, ſphere, or ſphæroid, whoſe fluidity is not neceſſary to make the paſſage of the water the quickeſt poſſible, is congealed, as was explained above in Cor. 7. Prop. 36.

Lemma VI.

The ſame ſuppoſition remaining, the foremention-ed bodies are equally acted on by the water flowing thro' the canal.

This appears by Lem. 5. and the third law. For the water and the bodies act upon each other mutually and equally.

Lemma VII.

If the water be at reſt in the canal, and theſe bodies move with equal velocity and the contrary way thro' the canal, their reſiſtances will be equal among themſelves.

This appears from the laſt Lemma, for the relative motions remain the ſame among themſelves.

Scho-

SCHOLIUM.

The caſe is the ſame of all convex and round bodies, whoſe axes coincide with the axis of the canal. Some difference may ariſe from a greater or leſs friction; but in theſe *lemmata* we ſuppoſe the bodies to be perfectly ſmooth, and the medium to be void of all tenacity and friction; and that thoſe parts of the fluid which by their oblique and ſuperfluous motions may diſturb, hinder, and retard the flux of the water thro' the canal, are at reſt amongſt themſelves; being fixed like water by froſt, and adhering to the fore and hinder parts of the bodies in the manner explained in the Scholium of the laſt Propoſition. For in what follows, we conſider the very leaſt reſiſtance that round bodies deſcribed with the greateſt given tranſverſe ſections can poſſibly meet with.

Bodies ſwimming upon fluids, when they move ſtraight forwards, cauſe the fluid to aſcend at their fore parts and ſubſide at their hinder parts, eſpecially if they are of an obtuſe figure; and thence they meet with a little more reſiſtance than if they were acute at the head and tail. And bodies moving in elaſtic fluids, if they are obtuſe behind and before, condenſe the fluid a little more at their fore parts, and relax the ſame at their hinder parts; and therefore meet alſo with a little more reſiſtance than if they were acute at the head and tail. But in theſe lemma's and propoſitions we are not treating of elaſtic, but non-elaſtic fluids; not of bodies floating on the ſurface of the fluid, but deeply immerſed therein. And when the reſiſtance of bodies in non-elaſtic fluids is once known, we may then augment this reſiſtance a little in elaſtic fluids, as our air; and in the ſurfaces of ſtagnating fluids, as lakes and ſeas.

PRO-

PROPOSITION XXXVIII. THEOREM XXX.

If a globe move uniformly forward in a compreſſed, infinite, and non-elaſtic fluid, its reſiſtance is to the force by which its whole motion may be deſtroyed or generated in the time that it deſcribes eight third parts of its diameter, as the denſity of the fluid to the denſity of the globe, very nearly.

For the globe is to its circumſcribed cylinder as two to three; and therefore the force which can deſtroy all the motion of the cylinder while the ſame cylinder is deſcribing the length of four of its diameters, will deſtroy all the motion of the globe while the globe is deſcribing two thirds of this length, that is, eight third parts of its own diameter. Now the reſiſtance of the cylinder is to this force very nearly as the denſity of the fluid to the denſity of the cylinder or globe (by Prop. 37.) and the reſiſtance of the globe is equal to the reſiſtance of the cylinder (by Lem. 5, 6, 7.) *Q. E. D.*

COR. 1. The reſiſtances of globes in infinite compreſſed mediums are in a ratio compounded of the duplicate ratio of the velocity, and the duplicate ratio of the diameter, and the ratio of the denſity of the mediums.

COR. 2. The greateſt velocity with which a globe can deſcend by its comparative weight thro' a reſiſting fluid, is the ſame which it may acquire by falling with the ſame weight, and without any reſiſtance, and in its fall deſcribing a ſpace that is to four third parts of its diameter, as the denſity of the globe to the denſity of the fluid. For the globe in the time of its fall, moving with the velocity acquired in falling, will deſcribe a

ſpace

ſpace that will be to eight third parts of its diameter as the denſity of the globe to the denſity of the fluid; and the force of its weight which generates this motion, will be to the force that can generate the ſame motion in the time that the globe deſcribes eight third parts of its diameter, with the ſame velocity as the denſity of the fluid to the denſity of the globe; and therefore (by this Propoſition) the force of weight will be equal to the force of reſiſtance, and therefore cannot accelerate the globe.

Cor. 3. If there be given both the denſity of the globe and its velocity at the beginning of the motion, and the denſity of the compreſſed quieſcent fluid in which the globe moves; there is given at any time both the velocity of the globe and its reſiſtance, and the ſpace deſcribed by it, (by Cor. 7. Prop. 35.)

Cor. 4. A globe moving in a compreſſed quieſcent fluid of the ſame denſity with itſelf, will loſe half its motion before it can deſcribe the length of two of its diameters, (by the ſame Cor. 7.)

Proposition XXXIX. Theorem XXXI.

If a globe move uniformly forward thro' a fluid incloſed and compreſſed in a cylindric canal, its reſiſtance is to the force by which its whole motion may be generated or deſtroyed in the time in which it deſcribes eight third parts of its diameter, in a ratio compounded of the ratio of the orifice of the canal, to the exceſs of that orifice above half the greateſt circle of the globe; and the duplicate ratio of the orifice of the canal, to the exceſs of that orifice above the greateſt circle of the globe; and the ratio of the denſity of the fluid to the denſity of the globe, nearly.

But

This appears by cor. 2. prop. 37. and the demonſtration proceeds in the ſame manner as in the foregoing propoſition.

Scholium.

In the two laſt propoſitions we ſuppoſe (as was done before in lem. 5.) that all the water which precedes the globe, and whoſe fluidity increaſes the reſiſtance of the ſame, is congealed. Now if that water becomes fluid, it will ſomewhat increaſe the reſiſtance. But in theſe propoſitions that increaſe is ſo ſmall, that it may be neglected, becauſe the convex ſuperficies of the globe produces the very ſame effect almoſt as the congelation of the water.

Proposition XL. Problem IX.

To find by phænomena the reſiſtance of a globe moving through a perfectly fluid compreſſed medium.

Let A be the weight of the globe in vacuo, B its weight in the reſiſting medium, D the diameter of the globe, F a ſpace which is to $\frac{4}{3}$ D as the denſity of the globe to the denſity of the medium, that is, as A to A—B, G the time in which the globe falling with the weight B without reſiſtance deſcribes the ſpace F, and H the velocity which the body acquires by that fall. Then H will be the greateſt velocity with which the globe can poſſibly deſcend with the weight B in the reſiſting medium, by cor. 2. prop. 38; and the reſiſtance which the globe meets with, when deſcending with that velocity, will be equal to its weight B: and the reſiſtance it meets with, in any other velocity, will be to the weight B in the duplicate ratio of that velocity to the greateſt velocity H, by cor. 1. prop. 38.

This is the reſiſtance that ariſes from the inactivity of the matter of the fluid. That reſiſtance which ariſes from the elaſticity, tenacity, and friction of its parts, may be thus inveſtigated.

Let the globe be let fall ſo that it may deſcend in the fluid by the weight B; and let P be the time of falling, and let that time be expreſſed in ſeconds, if the time G be given in ſeconds. Find the abſolute Number N agreeing to the logarithm $0{,}434294481\,9\frac{2P}{G}$, and let L be the logarithm of the number $\frac{N+1}{N}$: and the velocity acquir'd in falling will be $\frac{N-1}{N+1}H$, and the height deſcribed will be $\frac{2PF}{G} - 1{,}386294361\,1F + 4{,}605170186LF$. If the fluid be of a ſufficient depth, we may neglect the term $4{,}605170186LF$; and $\frac{2PF}{G} - 1{,}386294361\,1F$ will be the altitude deſcribed, nearly. Theſe things appear by prop. 9. book 2. and its corollaries, and are true upon this ſuppoſition, that the globe meets with no other reſiſtance but that which ariſes from the inactivity of matter. Now if it really meet with any reſiſtance of another kind, the deſcent will be ſlower, and from the quantity of that retardation will be known the quantity of this new reſiſtance.

That the velocity and deſcent of a body falling in a fluid might more eaſily be known, I have compoſed the following table; the firſt column of which denotes the times of deſcent, the ſecond ſhews the velocities acquir'd in falling, the greateſt velocity being 100000000, the third exhibits the ſpaces deſcribed by falling in thoſe times, 2F being the ſpace which the body deſcribes in the time G with the greateſt velocity,

and the fourth gives the ſpaces deſcribed with the greateſt velocity in the ſame times. The numbers in the fourth column are $\frac{2P}{G}$, and by ſubducting the number 1,3862944—4,6051702L, are found the numbers in the third column; and theſe numbers muſt be multiplied by the ſpace F to obtain the ſpaces deſcribed in falling. A fifth column is added to all theſe, containing the ſpaces deſcribed in the ſame times by a body falling in vacuo with the force of B its comparative weight.

The Times P	*Velocities of the body falling in the fluid.*	*The ſpaces deſcribed in falling in the fluid.*	*The ſpaces deſcribed with the greateſt motion.*	*The ſpaces deſcribed by falling in vacuo.*
0,001G	99999$\frac{82}{3}$	0,000001F	0,002F	0,000001F
0,01G	999967	0,0001F	0,02F	0,0001F
0,1G	9966799	0,0099834F	0,2F	0,01F
0,2G	19737532	0,0397361F	0,4F	0,04F
0,3G	29131261	0,0886815F	0,6F	0,09F
0,4G	37994896	0,1559070F	0,8F	0,16F
0,5G	46211716	0,2402290F	1,0F	0,25F
0,6G	53704957	0,3402706F	1,2F	0,36F
0,7G	60436778	0,4545405F	1,4F	0,49F
0,8G	66403677	0,5815071F	1,6F	0,64F
0,9G	71629787	0,7196609F	1,8F	0,81F
1G	76159416	0,8675617F	2F	1F
2G	96402758	2,6500055F	4F	4F
3G	99505475	4,6186570F	6F	9F
4G	99932930	6,6143765F	8F	16F
5G	99990920	8,6137964F	10F	25F
6G	99998791	10,6137179F	12F	36F
7G	99999834	12,6137073F	14F	49F
8G	99999980	14,6137059F	16F	64F
9G	99999997	16,6137057F	18F	81F
10G	99999999$\frac{1}{3}$	18,6137056F	20F	100F

Scholium.

In order to inveſtigate the reſiſtances of fluids from experiments, I procured a ſquare wooden veſſel, whoſe length and breadth on the inſide was 9 inches *Engliſh* meaſure, and its depth 9 foot $\frac{1}{2}$; this I filled with rain-water: and having provided globes made up of wax, and lead included therein, I noted the times of the deſcents of theſe globes, the height through which they deſcended being 112 inches. A ſolid cubic foot of *Engliſh* meaſure contains 76 pounds *Troy* weight of rain-water; and a ſolid inch contains $\frac{1}{3}\frac{2}{6}$ ounces *Troy* weight or 253 $\frac{1}{3}$ grains; and a globe of water of one inch in diameter contains 132,645 grains in air, or 132,8 grains in vacuo; and any other globe will be as the exceſs of its weight in vacuo above its weight in water.

Exper. 1. A globe whoſe weight was 156 $\frac{1}{4}$ grains in air, and 77 grains in water, deſcribed the whole height of 112 inches in 4 ſeconds. And, upon repeating the experiment, the globe ſpent again the very ſame time of 4 ſeconds in falling.

The weight of this globe in vacuo is 156 $\frac{1}{3}\frac{1}{8}$ grains; and exceſs of this weight above the weight of the globe in water is 79 $\frac{1}{3}\frac{1}{8}$ grains. Hence the diameter of the globe appears to be 0,84224 parts of an inch. Then it will be, as that exceſs to the weight of the globe in vacuo, ſo is the denſity of the water to the denſity of the globe; and ſo is $\frac{1}{3}$ parts of the diameter of the globe (*viz.* 2,24597 inches) to the ſpace 2F, which will be therefore 4,4256 inches. Now a globe falling in vacuo with its whole weight of 156 $\frac{1}{3}\frac{1}{8}$ grains in one ſecond of time will deſcribe 193 $\frac{1}{3}$ inches; and falling in water in the ſame time with the weight of 77 grains without reſiſtance, will deſcribe 95,219 inches; and in the time G which is to one ſecond of time

time in the ſubduplicate ratio of the ſpace F, or of 2,2128 inches to 95,219 inches, will deſcribe 2,2128 inches, and will acquire the greateſt velocity H with which it is capable of deſcending in water. Therefore the time G is 0,"15244. And in this time G with that greateſt velocity H, the globe will deſcribe the ſpace 2F, which is 4,4256 inches; and therefore in 4 ſeconds will deſcribe a ſpace of 116,1245 inches. Subduct the ſpace 1,3862944F or 3,0676 inches, and there will remain a ſpace of 113,0569 inches, which the globe falling thro' water in a very wide veſſel will deſcribe in 4 ſeconds. But this ſpace, by reaſon of the narrowneſs of the wooden veſſel beforementioned, ought to be diminiſhed in a ratio compounded of the ſubduplicate ratio of the orifice of the veſſel to the exceſs of this orifice above half a great circle of the globe, and of the ſimple ratio of the ſame orifice to its exceſs above a great circle of the globe, that is, in a ratio of 1 to 0,9914. This done, we have a ſpace of 112,08 inches, which a globe falling thro' the water in this wooden veſſel in 4 ſeconds of time ought nearly to deſcribe by this theory: but it deſcribed 112 inches by the experiment.

Exper. 2. Three equal globes, whoſe weights were ſeverally 76 $\frac{1}{3}$ grains in air, and 5 $\frac{1}{16}$ grains in water, were let fall ſucceſſively; and every one fell thro' the water in 15 ſeconds of time, deſcribing in its fall a height of 112 inches.

By computation, the weight of each globe in vacuo is 76 $\frac{1}{12}$ grains; the exceſs of this weight above the weight in water, is 71 grains $\frac{12}{48}$; the diameter of the globe 0,81296 of an inch: $\frac{1}{3}$ parts of this diameter 2,16789 inches; the ſpace 2F is 2,3217 inches; the ſpace which a globe of 5 $\frac{1}{16}$ grains in weight would deſcribe in one ſecond without reſiſtance, 12,808 inches, and the time G 0",301056. Therefore the globe with the greateſt velocity it is capable of receiving

from a weight of 5 $\frac{1}{16}$ grains in its deſcent thro' water, will deſcribe in the time 0″,301056 the ſpace of 2,3217 inches; and in 15 ſeconds the ſpace 115,678 inches. Subduct the ſpace 1,3862944F or 1,609 inches, and there remains the ſpace 114,069 inches; which therefore the falling globe ought to deſcribe in the ſame time, if the veſſel were very wide. But becauſe our veſſel was narrow, the ſpace ought to be diminiſhed by about 0,895 of an inch. And ſo the ſpace will remain 113,174 inches, which a globe falling in this veſſel ought nearly to deſcribe in 15 ſeconds by the theory. But by the experiment it deſcribed 112 inches. The difference is not ſenſible.

Exper. 3. Three equal globes, whoſe weights were ſeverally 121 grains in air, and 1 grain in water, were ſucceſſively let fall; and they fell thro' the water in the times 46″, 47″, and 50″, deſcribing a height of 112 inches.

By the theory theſe globes ought to have fallen in about 40″. Now whether their falling more ſlowly were occaſion'd from hence, that in ſlow motions the reſiſtance ariſing from the force of inactivity, does really bear a leſs proportion to the reſiſtance ariſing from other cauſes; or whether it is to be attributed to little bubbles that might chance to ſtick to the globes, or to the rarefaction of the wax by the warmth of the weather, or of the hand that let them fall; or, laſtly, whether it proceeded from ſome inſenſible errors in weighing the globes in the water, I am not certain. Therefore the weight of the globe in water ſhould be of ſeveral grains, that the experiment may be certain, and to be depended on.

Exper. 4. I began the foregoing experiments to inveſtigate the reſiſtances of fluids, before I was acquainted with the theory laid down in the propoſitions immediately preceding. Afterwards, in order to examine the theory after it was diſcovered, I procured a wooden

wooden veſſel, whoſe breadth on the inſide was 8 $\frac{1}{3}$ inches, and its depth 15 feet and $\frac{1}{3}$. Then I made four globes of wax, with lead included, each of which weighed 139 $\frac{1}{4}$ grains in air, and 7 $\frac{1}{8}$ grains in water. Theſe I let fall, meaſuring the times of their falling in the water with a pendulum oſcillating to half ſeconds. The globes were cold, and had remained ſo ſome time, both when they were weighed and when they were let fall; becauſe warmth rarefies the wax, and by rarefying it diminiſhes the weight of the globe in the water; and wax, when rarefied, is not inſtantly reduced by cold to its former denſity. Before they were let fall, they were totally immerſed under water, leſt, by the weight of any part of them that might chance to be above the water, their deſcent ſhould be accelerated in its beginning. Then, when after their immerſion they were perfectly at reſt, they were let go with the greateſt care, that they might not receive any impulſe from the hand that let them down. And they fell ſucceſſively in the times of 47$\frac{1}{2}$, 48$\frac{1}{2}$, 50 and 51 oſcillations, deſcribing a height of 15 feet and 2 inches. But the weather was now a little colder than when the globes were weighed, and therefore I repeated the experiment another day; and then the globes fell in the times of 49, 49$\frac{1}{2}$, 50 and 53; and at a third trial in the times of 49$\frac{1}{2}$, 50, 51 and 53 oſcillations. And by making the experiment ſeveral times over, I found that the globes fell moſtly in the times of 49$\frac{1}{2}$ and 50 oſcillations. When they fell ſlower, I ſuſpect them to have been retarded by ſtriking againſt the ſides of the veſſel.

Now, computing from the theory, the weight of the globe in vacuo is 139 $\frac{1}{3}$ grains. The exceſs of this weight above the weight of the globe in water 132 $\frac{11}{40}$ grains, the diameter of the globe 0,99868 of an inch, $\frac{4}{3}$ parts of the diameter 2,66315 inches, the ſpace 2F 2,8066 inches, the ſpace which a globe weighing 7 $\frac{1}{8}$ grains falling without reſiſtance deſcribes

in a ſecond of time 9,88164 inches, and the time G o",376843. Therefore the globe with the greateſt velocity with which it is capable of deſcending thro' the water by the force of a weight of 7 $\frac{1}{8}$ grains will in the time o",376843 deſcribe a ſpace of 2,8066 inches, and in one ſecond of time a ſpace of 7,44766 inches, and in the time 25", or in 50 oſcillations the ſpace 186,1915 inches. Subduct the ſpace 1,386294 F or 1,9454 inches, and there will remain the ſpace 184,2461 inches, which the globe will deſcribe in that time in a very wide veſſel. Becauſe our veſſel was narrow, let this ſpace be diminiſhed in a ratio compounded of the ſubduplicate ratio of the orifice of the veſſel to the exceſs of this orifice above half a great circle of the globe, and of the ſimple ratio of the ſame orifice to its exceſs above a great circle of the globe; and we ſhall have the ſpace of 181,86 inches, which the globe ought by the theory to deſcribe in this veſſel in the time of 50 oſcillations, nearly. But it deſcribed the ſpace of 182 inches, by experiment, in 49$\frac{1}{2}$ or 50 oſcillations.

Exper. 5. Four globes, weighing 154 $\frac{1}{8}$ grains in air, and 21 $\frac{1}{2}$ grains in water, being let fall ſeveral times, fell in the times of 28 $\frac{1}{2}$, 29, 29 $\frac{1}{2}$, and 30, and ſometimes of 31, 32, and 33 oſcillations, deſcribing a height of 15 feet and 2 inches.

They ought by the theory to have fallen in the time of 29 oſcillations, nearly.

Exper. 6. Five globes, weighing 212 $\frac{1}{8}$ grains in air, and 79 $\frac{1}{2}$ in water, being ſeveral times let fall, fell in the times of 15, 15 $\frac{1}{2}$, 16, 17, and 18 oſcillations, deſcribing a height of 15 feet and 2 inches.

By the theory they ought to have fallen in the time of 15 oſcillations, nearly.

Exper. 7. Four globes weighing 293 $\frac{1}{8}$ grains in air, and 35 grains $\frac{7}{8}$ in water, being let fall ſeveral times, fell in the times of 29 $\frac{1}{2}$, 30, 30 $\frac{1}{2}$, 31, 32, and 33 oſcillations, deſcribing a height of 15 feet and 1 inch and $\frac{1}{2}$.

By

By the theory they ought to have fallen in the time of 28 oſcillations, nearly.

In ſearching for the cauſe that occaſioned theſe globes of the ſame weight and magnitude to fall, ſome ſwifter and ſome ſlower, I hit upon this; that the globes, when they were firſt let go and began to fall, oſcillated about their centres, that ſide which chanced to be the heavier deſcending firſt, and producing an oſcillating motion. Now by oſcillating thus, the globe communicates a greater motion to the water, than if it deſcended without any oſcillations; and by this communication loſes part of its own motion with which it ſhould deſcend; and therefore as this oſcillation is greater or leſs it will be more or leſs retarded. Beſides the globe always recedes from that ſide of itſelf which is deſcending in the oſcillation, and by ſo receding comes nearer to the ſides of the veſſel ſo as even to ſtrike againſt them ſometimes. And the heavier the globes are, the ſtronger this oſcillation is; and the greater they are, the more is the water agitated by it. Therefore to diminiſh this oſcillation of the globes, I made new ones of lead and wax, ſticking the lead in one ſide of the globe very near its ſurface; and I let fall the globe in ſuch a manner, that as near as poſſible, the heavier ſide might be loweſt at the beginning of the deſcent. By this means the oſcillations became much leſs than before, and the times in which the globes fell were not ſo unequal: as in the following experiments.

Exper. 8. Four globes weighing 139 grains in air and $6\frac{1}{2}$ in water, were let fall ſeveral times, and fell moſtly in the time of 51 oſcillations, never in more than 52, or in fewer than 50; deſcribing a height of 182 inches.

By the theory they ought to fall in about the time of 52 oſcillations.

Exper.

EXPER. 9. Four globes weighing 273 $\frac{1}{4}$ grains in air, and 140 $\frac{1}{4}$ in water, being ſeveral times let fall, fell in never fewer than 12, and never more than 13 oſcillations, deſcribing a height of 182 inches.

Theſe globes by the theory ought to have fallen in the time of 11 $\frac{1}{3}$ oſcillations, nearly.

EXPER. 10. Four globes, weighing 384 grains in air and 119 $\frac{1}{2}$ in water, being let fall ſeveral times, fell in the times of 17 $\frac{1}{4}$, 18, 18 $\frac{1}{2}$, and 19 oſcillations, deſcribing a height of 181 $\frac{1}{2}$ inches. And when they fell in the time of 19 oſcillations, I ſometimes heard them hit againſt the ſides of the veſſel before they reached the bottom.

By the theory they ought to have fallen in the time of 15 $\frac{1}{9}$ oſcillations, nearly.

EXPER. 11. Three equal globes, weighing 48 grains in the air, and 3 $\frac{2}{3}\frac{2}{1}$ in water, being ſeveral times let fall, fell in the times of 43 $\frac{1}{2}$, 44, 44 $\frac{1}{2}$, 45 and 46 oſcillations, and moſtly in 44 and 45, deſcribing a height of 182 inches $\frac{1}{2}$, nearly.

By the theory they ought to have fallen in the time of 46 oſcillations and $\frac{1}{9}$, nearly.

EXPER. 12. Three equal globes, weighing 141 grains in air and 4 $\frac{1}{8}$ in water, being let fall ſeveral times, fell in the times of 61, 62, 63, 64 and 65 oſcillations, deſcribing a ſpace of 182 inches.

And by the theory they ought to have fallen in 64 $\frac{1}{2}$ oſcillations, nearly.

From theſe experiments it is manifeſt, that when the globes fell ſlowly, as in the ſecond, fourth, fifth, eighth, eleventh, and twelfth experiments, the times of falling are rightly exhibited by the theory; but when the globes fell more ſwiftly as in the ſixth, ninth, and tenth experiments, the reſiſtance was ſomewhat greater than in the duplicate ratio of the velocity. For the globes in falling oſcillate a little; and this oſcillation, in thoſe globes that are light and fall ſlowly,

ſlowly, ſoon ceaſes by the weakneſs of the motion; but in greater and heavier globes, the motion being ſtrong, it continues longer; and is not to be checked by the ambient water, till after ſeveral oſcillations. Beſides, the more ſwiftly the globes move, the leſs are they preſſed by the fluid at their hinder parts; and if the velocity be perpetually increaſed, they will at laſt leave an empty ſpace behind them, unleſs the compreſſion of the fluid be increaſed at the ſame time. For the compreſſion of the fluid ought to be increaſed (by Prop. 32 and 33.) in the duplicate ratio of the velocity, in order to preſerve the reſiſtance in the ſame duplicate ratio. But becauſe this is not done, the globes that move ſwiftly are not ſo much preſſed at their hinder parts as the others; and by the defect of this preſſure it comes to paſs that their reſiſtance is a little greater than in a duplicate ratio of their velocity.

So that the theory agrees with the phænomena of bodies falling in water; it remains that we examine the phænomena of bodies falling in air.

EXPER. 13. From the top of St. *Paul*'s Church in *London* in *June* 1710. there were let fall together two glaſs globes, one full of quickſilver, the other of air; and in their fall they deſcribed a height of 220 *Engliſh* feet. A wooden table was ſuſpended upon iron hinges on one ſide, and the other ſide of the ſame was ſupported by a wooden pin. The two globes lying upon this table were let fall together by pulling out the pin by means of an iron wire reaching from thence quite down to the ground; ſo that, the pin being removed, the table, which had then no ſupport but the iron hinges, fell downwards; and turning round upon the hinges, gave leave to the globes to drop off from it. At the ſame inſtant, with the ſame pull of the iron wire that took out the pin, a pendulum oſcillating to ſeconds was let go, and began to oſcillate. The diameters and

weights

of the globes, and their times of falling, are exhibited in the following table.

The globes filled with mercury.			*The globes full of air.*		
Weights.	*Diameters.*	*Times in falling.*	*Weights.*	*Diameters.*	*Times in falling.*
908 *Grains*	0,8 *of an inch*	4″	510 *Grains*	5,1 *inches*	8″ 1/2
983	0,8	4—	642	5,2	8
866	0,8	4	599	5,1	8
747	0,75	4+	515	5,0	8 1/2
808	0,75	4	483	5,0	8 1/2
784	0,75	4+	641	5,2	8

But the times obſerved muſt be corrected; for the globes of mercury (by *Galileo*'s theory) in 4 ſeconds of time, will deſcribe 257 *Engliſh* feet, and 220 feet in only 3″ 42‴. So that the wooden table, when the pin was taken out, did not turn upon its hinges ſo quickly as it ought to have done; and the ſlowneſs of that revolution hindered the deſcent of the globes at the beginning. For the globes lay about the middle of the table, and indeed were rather nearer to the axis upon which it turned, than to the pin. And hence the times of falling were prolonged about 18‴; and therefore ought to be corrected by ſubducting that exceſs, eſpecially in the larger globes, which, by reaſon of the largeneſs of their diameters, lay longer upon the revolving table than the others. This being done, the times in which the ſix larger globes fell, will come forth 8″ 12‴, 7″ 42‴, 7″ 42‴, 7″ 57‴, 8″ 12‴, and 7″ 42‴.

Therefore the fifth in order among the globes that were full of air, being 5 inches in diameter, and 483 grains in weight, fell in 8″ 12‴, deſcribing a ſpace of 220 feet. The weight of a bulk of water equal to this globe is 16600 grains; and the weight of an equal bulk of air is $\frac{16600}{860}$ grains, or 19 $\frac{3}{10}$ grains; and therefore the weight of the globe in vacuo is 502 $\frac{3}{10}$ grains; and

and this weight is to the weight of a bulk of air equal to the globe as 502 $\frac{1}{10}$ to 19 $\frac{1}{10}$, and ſo is 2F to $\frac{1}{3}$ of the diameter of the globe, that is, to 13 $\frac{1}{3}$ inches. Whence 2F becomes 28 feet 11 inches. A globe falling in vacuo with its whole weight of 502 $\frac{1}{10}$ grains, will in one ſecond of time deſcribe 193 $\frac{1}{3}$ inches as above; and with the weight of 483 grains will deſcribe 185,905 inches; and with that weight 483 grains in vacuo will deſcribe the ſpace F or 14 feet 5 $\frac{1}{2}$ inches, in the time of 57‴ 58⁗, and acquire the greateſt velocity it is capable of deſcending with in the air. With this velocity the globe in 8″ 12‴ of time will deſcribe 245 feet and 5 $\frac{1}{3}$ inches. Subduct 1,3863F or 20 feet and $\frac{1}{2}$ an inch, and there remain 225 feet 5 inches. This ſpace therefore the falling globe ought by the theory to deſcribe in 8″ 12‴. But by the experiment it deſcribed a ſpace of 220 feet. The difference is inſenſible.

By like calculations applied to the other globes full of air, I compoſed the following table.

The weights of the globes.	*The diameters.*	*The times of falling from a height of 220 feet.*	*The ſpaces which they would deſcribe by the theory.*	*The Exceſſes.*
510 *grains*	5,1 *inches*	8″ 12‴	226 *feet* 11 *inches*	6 *feet* 11 *inches*
642	5,2	7 42	230 9	10 9
599	5,1	7 42	227 10	7 10
515	5	7 57	224 5	4 5
483	5	8 12	225 5	5 5
641	5,2	7 42	230 7	10 7

Exper. 14. *Anno* 1719. in the month of *July*, Dr. *Deſaguliers* made ſome experiments of this kind again, by forming hogs bladders into ſphærical orbs; which was done by means of a concave wooden ſphere, which the bladders, being wetted well firſt, were put into. After that, being blown full of air, they were

obliged to fill up the ſphærical cavity that contained them; and then, when dry, were taken out. Theſe were let fall from the lantern on the top of the cupola of the ſame church; namely, from a height of 272 feet; and at the ſame moment of time there was let fall a leaden globe whoſe weight was about 2 pounds *Troy* weight. And in the mean time ſome perſons ſtanding in the upper part of the church where the globes were let fall, obſerved the whole times of falling; and others ſtanding on the ground obſerved the differences of the times between the fall of the leaden weight, and the fall of the bladder. The times were meaſured by pendulums oſcillating to half ſeconds. And one of thoſe that ſtood upon the ground had a machine vibrating four times in one ſecond; and another had another machine accurately made with a pendulum vibrating four times in a ſecond alſo. One of thoſe alſo who ſtood at the top of the church had a like machine. And theſe inſtruments were ſo contrived, that their motions could be ſtopped or renewed at pleaſure. Now the leaden globe fell in about four ſeconds and $\frac{1}{4}$ of time; and from the addition of this time to the difference of time above ſpoken of, was collected the whole time in which the bladder was falling. The times which the five bladders ſpent in falling after the leaden globe had reached the ground were the firſt time, $14\frac{1}{4}''$, $12\frac{1}{4}''$, $14\frac{1}{8}''$, $17\frac{1}{4}''$, and $16\frac{1}{8}''$; and the ſecond time $14\frac{1}{2}''$, $14\frac{1}{4}''$, $14''$, $19''$ and $16\frac{1}{4}''$. Add to theſe $4\frac{1}{4}''$, the time in which the leaden globe was falling, and the whole times in which the five bladders fell, were, the firſt time $19''$, $17''$, $18\frac{1}{8}''$, $22''$ and $21\frac{1}{8}''$; and the ſecond time, $18\frac{1}{4}''$, $18\frac{1}{2}''$, $18\frac{1}{4}''$, $23\frac{1}{4}''$ and $21''$. The times obſerved at the top of the church were, the firſt time, $19\frac{1}{8}''$, $17\frac{1}{4}''$, $18\frac{1}{4}''$, $22\frac{1}{8}''$ and $21\frac{1}{8}''$; and the ſecond time, $19''$, $18\frac{1}{8}''$, $18\frac{1}{8}''$, $24''$ and $21\frac{1}{4}''$. But the bladders did not always fall di-

directly down, but ſometimes fluttered a little in the air, and waved to and fro as they were deſcending. And by theſe motions the times of their falling were prolonged, and increaſed by half a ſecond ſometimes, and ſometimes by a whole ſecond. The ſecond and fourth bladder fell moſt directly the firſt time, and the firſt and third the ſecond time. The fifth bladder was wrinkled, and by its wrinkles was a little retarded. I found their diameters by their circumferences meaſured with a very fine thread wound about them twice. In the following table I have compared the experiments with the theory; making the denſity of air to be to the denſity of rain-water as 1 to 860, and computing the ſpaces which by the theory the globes ought to deſcribe in falling.

The weights of the bladders.	*The diameters.*	*The times of falling from a height of 272 foot.*	*The ſpaces which by the theory ought to have been deſcribed in thoſe times.*		*The difference between the theory and the experiments.*	
128 *grains*	5,28 *inches*	19″	271 *foot*	11 *inches*	— 0 *foot*	1 *inch*
156	5,19	17	272	$0\frac{1}{2}$	+ 0	$0\frac{1}{2}$
$137\frac{1}{2}$	5,3	$18\frac{1}{2}$	272	7	+ 0	7
$97\frac{1}{2}$	5,26	22	277	4	+ 5	4
$99\frac{1}{2}$	5	$21\frac{1}{2}$	282	0	+ 10	0

Our theory therefore exhibits rightly, within a very little, all the reſiſtance that globes moving either in air or in water meet with; which appears to be proportional to the denſities of the fluids in globes of equal velocities and magnitudes.

In the ſcholium ſubjoined to the ſixth ſection, we ſhewed by experiments of pendulums, that the reſiſtances of equal and equally ſwift globes moving in air, water, and quickſilver, are as the denſities of the fluids. We here prove the ſame more accurately by experiments of bodies falling in air and water. For pendulums at each oſcillation excite a motion in the fluid always

ways contrary to the motion of the pendulum in its return; and the reſiſtance ariſing from this motion, as alſo the reſiſtance of the thread by which the pendulum is ſuſpended, makes the whole reſiſtance of a pendulum greater than the reſiſtance deduced from the experiments of falling bodies. For by the experiments of pendulums deſcribed in that ſcholium, a globe of the ſame denſity as water in deſcribing the length of its ſemidiameter in air would loſe the $\frac{1}{3342}$ part of its motion. But by the theory delivered in this ſeventh ſection, and confirmed by experiments of falling bodies, the ſame globe in deſcribing the ſame length would loſe only a part of its motion equal to $\frac{1}{4586}$ ſuppoſing the denſity of water to be to the denſity of air as 860 to 1. Therefore the reſiſtances were found greater by the experiments of pendulums (for the reaſons juſt mentioned) than by the experiments of falling globes; and that in the ratio of about 4 to 3. But yet ſince the reſiſtances of pendulums oſcillating in air, water, and quickſilver, are alike increaſed by like cauſes, the proportion of the reſiſtances in theſe mediums will be rightly enough exhibited by the experiments of pendulums, as well as by the experiments of falling bodies. And from all this it may be concluded, that the reſiſtances of bodies, moving in any fluids whatſoever, tho' of the moſt extreme fluidity, are, *cæteris paribus*, as the denſities of the fluids.

Theſe things being thus eſtabliſhed, we may now determine what part of its motion any globe projected in any fluid whatſoever would nearly loſe in a given time. Let D be the diameter of the globe, and V its velocity at the beginning of its motion, and T the time in which a globe with the velocity V can deſcribe in vacuo a ſpace that is to the ſpace $\frac{2}{3}$ D as the denſity of the globe to the denſity of the fluid; and the globe projected in that fluid will, in any other time

time t, loſe the part $\frac{tV}{T+t}$, the part $\frac{TV}{T+t}$ remaining; and will deſcribe a ſpace, which may be to that deſcribed in the ſame time in vacuo with the uniform velocity V, as the logarithm of the number $\frac{T+t}{T}$ multiplied by the number 2,302585093 is to the number $\frac{t}{T}$, by cor. 7. prop. 35. In ſlow motions the reſiſtance may be a little leſs, becauſe the figure of a globe is more adapted to motion than the figure of a cylinder deſcribed with the ſame diameter. In ſwift motions the reſiſtance may be a little greater, becauſe the elaſticity and compreſſion of the fluid do not increaſe in the duplicate ratio of the velocity. But theſe little niceties I take no notice of.

And tho' air, water, quickſilver, and the like fluids, by the diviſion of their parts in infinitum, ſhould be ſubtilized and become mediums infinitely fluid; nevertheleſs, the reſiſtance they would make to projected globes would be the ſame. For the reſiſtance conſider'd in the preceding propoſitions, ariſes from the inactivity of the matter; and the inactivity of matter is eſſential to bodies, and always proportional to the quantity of matter. By the diviſion of the parts of the fluid, the reſiſtance ariſing from the tenacity and friction of the parts may be indeed diminiſhed; but the quantity of matter will not be at all diminiſhed by this diviſion; and if the quantity of matter be the ſame, its force of inactivity will be the ſame; and therefore the reſiſtance here ſpoken of will be the ſame, as being always proportional to that force. To diminiſh this reſiſtance, the quantity of matter in the ſpaces thro' which the bodies move muſt be diminiſhed. And therefore the celeſtial ſpaces, thro' which the globes of the Planets and Comets are perpetually paſſing towards all parts,

with the utmoſt freedom, and without the leaſt ſenſible diminution of their motion, muſt be utterly void of any corporeal fluid, excepting perhaps ſome extremely rare vapours, and the rays of light.

Projectiles excite a motion in fluids as they paſs thro' them; and this motion ariſes from the exceſs of the preſſure of the fluid at the fore-parts of the projectile above the preſſure of the ſame at the hinder parts; and cannot be leſs in mediums infinitely fluid, than it is in air, water, and quickſilver, in proportion to the denſity of matter in each. Now this exceſs of preſſure does, in proportion to its quantity, not only excite a motion in the fluid, but alſo acts upon the projectile ſo as to retard its motion: and therefore the reſiſtance in every fluid is as the motion excited by the projectile in the fluid; and cannot be leſs in the moſt ſubtile æther in proportion to the denſity of that æther, than it is in air, water, and quickſilver, in proportion to the denſities of thoſe fluids.

SEC-

SECTION VIII.

Of motion propagated thro' fluids.

PROPOSITION XLI. THEOREM XXXII.

A preſſure is not propagated thro' a fluid in rectilinear directions, unleſs where the particles of the fluid lie in a right line. Pl. 8. Fig. 1.

If the particles *a*, *b*, *c*, *d*, *e*, lie in a right line, the preſſure may be indeed directly propagated from *a* to *e*; but then the particle *e* will urge the obliquely poſited particles *f* and *g* obliquely, and thoſe particles *f* and *g* will not ſuſtain this preſſure, unleſs they be ſupported by the particles *h* and *k* lying beyond them; but the particles that ſupport them, are alſo preſſed by them; and thoſe particles cannot ſuſtain that preſſure, without being ſupported by, and preſſing upon, thoſe particles that lie ſtill farther, as *l* and *m*, and ſo on in infinitum. Therefore the preſſure, as ſoon as it is propagated to particles that lie out of right lines, begins to deflect towards one hand and t'other, and will be propagated obliquely in infinitum; and after it has begun to be propagated obliquely, if it reaches more diſtant particles lying out of the right line, it will deflect again on each hand; and this it will do as

often as it lights on particles that do not lie exactly in a right line. *Q. E. D.*

Cor. If any part of a preſſure, propagated thro' a fluid from a given point, be intercepted by any obſtacle; the remaining part, which is not intercepted, will deflect into the ſpaces behind the obſtacle. This may be demonſtrated alſo after the following manner. Let a preſſure be propagated from the point *A* (*Pl.* 8. *Fig.* 2.) towards any part, and, if it be poſſible, in rectilinear directions; and the obſtacle *NBCK* being perforated in *BC*, let all the preſſure be intercepted but the coniform part *APQ* paſſing thro' the circular hole *BC*. Let the cone *APQ* be divided into fruſtums by the tranſverſe planes *de*, *fg*, *hi*. Then while the cone *ABC*, propagating the preſſure, urges the conic fruſtum *degf* beyond it on the ſuperficies *de*, and this fruſtum urges the next fruſtum *fgih* on the ſuperficies *fg*, and that fruſtum urges a third fruſtum, and ſo in infinitum; it is manifeſt (by the third law) that the firſt fruſtum *defg* is, by the reaction of the ſecond fruſtum *fghi*, as much urged and preſſed on the ſuperficies *fg*, as it urges and preſſes that ſecond fruſtum. Therefore the fruſtum *degf* is compreſſed on both ſides, that is, between the cone *Ade* and the fruſtum *fhig*; and therefore (by caſe 6. prop. 19.) cannot preſerve its figure, unleſs it be compreſſed with the ſame force on all ſides. Therefore with the ſame force with which it is preſſed on the ſuperficies *de*, *fg*, it will endeavour to break forth at the ſides *df*, *eg*; and there (being not in the leaſt tenacious or hard, but perfectly fluid) it will run out, expanding itſelf, unleſs there be an ambient fluid oppoſing that endeavour. Therefore, by the effort it makes to run out, it will preſs the ambient fluid, at its ſides *df*, *eg*, with the ſame force that it does the fruſtum *fghi*; and therefore the preſſure will be propagated as much from the ſides *df*, *eg* into the ſpaces *NO*, *KL* this way and that way, as it

is

is propagated from the ſuperficies *fg* towards *PQ*. *Q.E.D.*

Proposition XLII. Theorem XXXIII.

All motion propagated thro' a fluid, diverges from a rectilinear progreſs into the unmoved ſpaces. Pl. 8. Fig. 3.

Case 1. Let a motion be propagated from the point *A* thro' the hole *BC*, and, if it be poſſible, let it proceed in the conic ſpace *BCQP* according to right lines diverging from the point *A*. And let us firſt ſuppoſe this motion to be that of waves in the ſurface of ſtanding water; and let *de*, *fg*, *hi*, *kl*, &c. be the tops of the ſeveral waves, divided from each other by as many intermediate valleys or hollows. Then, becauſe the water in the ridges of the waves is higher than in the unmoved parts of the fluid *KL*, *NO*, it will run down from off the tops of thoſe ridges *e*, *g*, *i*, *l*, &c. *d*, *f*, *h*, *k*, &c. this way and that way towards *KL* and *NO*; and becauſe the water is more depreſſed in the hollows of the waves than in the unmoved parts of the fluid *KL*, *NO*, it will run down into thoſe hollows out of thoſe unmoved parts. By the firſt deflux the ridges of the waves will dilate themſelves this way and that way, and be propagated towards *KL* and *NO*. And becauſe the motion of the waves from *A* towards *PQ* is carried on by a continual deflux from the ridges of the waves into the hollows next to them; and therefore cannot be ſwifter than in proportion to the celerity of the deſcent; and the deſcent of the water on each ſide towards *KL* and *NO* muſt be performed with the ſame velocity; it follows, that the dilatation of the waves on each ſide towards *KL* and *NO* will be propagated with the ſame velocity as the waves them-

ſelves go forward with, directly from *A* to *PQ*. And therefore the whole ſpace this way and that way towards *KL* and *NO* will be filled by the dilated waves *rfgr*, *shis*, *tklt*, *vmnv*, &c. *Q.E.D.* That theſe things are ſo, any one may find by making the experiment in ſtill water.

Case 2. Let us ſuppoſe that *de*, *fg*, *hi*, *kl*, *mn*, repreſent pulſes ſucceſſively propagated from the point *A* thro' an elaſtic medium. Conceive the pulſes to be propagated by ſucceſſive condenſations and rarefactions of the medium, ſo that the denſeſt part of every pulſe may occupy a ſphærical ſuperficies deſcribed about the centre *A*, and that equal intervals intervene between the ſucceſſive pulſes. Let the lines *de*, *fg*, *hi*, *kl*, &c. repreſent the denſeſt parts of the pulſes, propagated thro' the hole *BC*; and becauſe the medium is denſer there, than in the ſpaces on either ſide towards *KL* and *NO*, it will dilate itſelf as well towards thoſe ſpaces *KL*, *NO* on each hand, as towards the rare intervals between the pulſes; and thence the medium becoming always more rare next the intervals, and more denſe next the pulſes, will partake of their motion. And becauſe the progreſſive motion of the pulſes ariſes from the perpetual relaxation of the denſer parts towards the antecedent rare intervals; and ſince the pulſes will relax themſelves on each hand towards the quieſcent parts of the medium *KL*, *NO*, with very near the ſame celerity; therefore the pulſes will dilate themſelves on all ſides into the unmoved parts *KL*, *NO*, with almoſt the ſame celerity with which they are propagated directly from the centre *A*; and therefore will fill up the whole ſpace *KLON*. *Q.E.D.* And we find the ſame by experience alſo in ſounds, which are heard tho' a mountain interpoſe; and if they come into a chamber thro' the window, dilate themſelves into all the parts of the room, and are heard in every corner; and not as reflected from the oppoſite walls, but directly

rectly propagated from the window, as far as our ſenſe can judge.

Case 3. Let us ſuppoſe laſtly, that a motion of any kind is propagated from *A* thro' the hole *BC*. Then ſince the cauſe of this propagation is, that the parts of the medium that are near the centre *A* diſturb and agitate thoſe which lie farther from it; and ſince the parts which are urged are fluid, and therefore recede every way towards thoſe ſpaces where they are leſs preſſed, they will by conſequence recede towards all the parts of the quieſcent medium; as well to the parts on each hand, as *KL* and *NO*, as to thoſe right before as *PQ*: and by this means all the motion, as ſoon as it has paſſed thro' the hole *BC*, will begin to dilate itſelf, and from thence, as from its principle and centre, will be propagated directly every way. *Q.E.D.*

Proposition XLIII. Theorem XXXIV.

Every tremulous body in an elaſtic medium propagates the motion of the pulſes on every ſide right forward; but in a non-elaſtic medium excites a circular motion.

Case 1. The parts of the tremulous body alternately going and returning, do in going urge and drive before them thoſe parts of the medium that lie neareſt, and by that impulſe compreſs and condenſe them; and in returning ſuffer thoſe compreſſed parts to recede again and expand themſelves. Therefore the parts of the medium that lie neareſt to the tremulous body, move to and fro by turns, in like manner as the parts of the tremulous body itſelf do; and for the ſame cauſe that the parts of this body agitate theſe parts of the medium, theſe parts being agitated by like tremors,

will in their turn agitate others next to themſelves, and theſe others agitated in like manner, will agitate thoſe that lie beyond them, and ſo on in infinitum. And in the ſame manner as the firſt parts of the medium were condenſed in going, and relaxed in returning, ſo will the other parts be condenſed every time they go, and expand themſelves every time they return. And therefore they will not be all going and all returning at the ſame inſtant, (for in that caſe they would always preſerve determined diſtances from each other, and there could be no alternate condenſation and rarefaction;) but ſince in the places where they are condenſed, they approach to, and in the places where they are rarefied, recede from, each other; therefore ſome of them will be going while others are returning; and ſo on in infinitum. The parts ſo going, and in their going condenſed, are pulſes, by reaſon of the progreſſive motion with which they ſtrike obſtacles in their way; and therefore the ſucceſſive pulſes produced by a tremulous body, will be propagated in rectilinear directions; and that at nearly equal diſtances from each other, becauſe of the equal intervals of time in which the body, by its ſeveral tremors, produces the ſeveral pulſes. And tho' the parts of the tremulous body go and return in ſome certain and determinate direction, yet the pulſes propagated from thence thro' the medium, will dilate themſelves towards the ſides, by the foregoing propoſition; and will be propagated on all ſides from that tremulous body, as from a common centre, in ſuperficies nearly ſphærical and concentrical. An example of this we have in waves excited by ſhaking a finger in water, which proceed not only forwards and backwards agreeably to the motion of the finger, but ſpread themſelves in the manner of concentrical circles all round the finger, and are propagated on every ſide. For the gravity of the water ſupplies the place of elaſtic force.

CASE

Case 2. If the medium be not elaſtic, then, becauſe its parts cannot be condenſed by the preſſure ariſing from the vibrating parts of the tremulous body, the motion will be propagated in an inſtant towards the parts where the medium yields moſt eaſily, that is, to the parts which the tremulous body leaves for ſome time vacuous behind it. The caſe is the ſame with that of a body projected in any medium whatever. A medium yielding to projectiles does not recede in infinitum, but with a circular motion comes round to the ſpaces which the body leaves behind it. Therefore as often as a tremulous body tends to any part, the medium yielding to it comes round in a circle to the parts which the body leaves; and as often as the body returns to the firſt place, the medium will be driven from the place it came round to, and return to its original place. And tho' the tremulous body be not firm and hard, but every way flexible; yet if it continue of a given magnitude, ſince it cannot impel the medium by its tremors any where without yielding to it ſomewhere elſe; the medium receding from the parts where it is preſſed, will always come round in a circle to the parts that yield to it. *Q. E. D.*

Cor. 'Tis a miſtake therefore to think, as ſome have done, that the agitation of the parts of flame conduces to the propagation of a preſſure in rectilinear directions thro' an ambient medium. A preſſure of that kind muſt be derived, not from the agitation only of the parts of flame, but from the dilatation of the whole.

Pro-

Proposition XLIV. Theorem XXXV.

If water aſcend and deſcend alternately in the erected legs KL, MN *of a canal or pipe; and a pendulum be conſtructed, whoſe length between the point of ſuſpenſion and the centre of oſcillation is equal to half the length of the water in the canal: I ſay, that the water will aſcend and deſcend in the ſame times in which the pendulum oſcillates.* Pl. 8. Fig. 4.

I meaſure the length of the water along the axes of the canal and its legs, and make it equal to the ſum of thoſe axes; and take no notice of the reſiſtance of the water, ariſing from its attrition by the ſides of the canal. Let therefore *AB*, *CD* repreſent the mean height of the water in both legs; and when the water in the leg *KL* aſcends to the height *EF*, the water will deſcend in the leg *MN* to the height *GH*. Let *P* be a pendulous body, *VP* the thread, *V* the point of ſuſpenſion, *RPQS* the cycloid which the pendulum deſcribes, *P* its loweſt point, *PQ* an arc equal to the height *AE*. The force, with which the motion of the water is accelerated and retarded alternately, is the exceſs of the weight of the water in one leg above the weight in the other; and therefore, when the water in the leg *KL* aſcends to *EF*, and in the other leg deſcends to *GH*, that force is double the weight of the water *EABF*, and therefore is to the weight of the whole water as *AE* or *PQ* to *VP* or *PR*. The force alſo with which the body *P* is accelerated or retarded in any place as *Q* of a cycloid, is (by cor. prop. 51.) to its whole weight, as its diſtance *PQ* from the loweſt place *P* to the length *PR* of the cycloid. Therefore the motive forces of the water and pendulum, deſcribing

bing the equal ſpaces AE, PQ are as the weights to be moved; and therefore if the water and pendulum are quieſcent at firſt, thoſe forces will move them in equal times, and will cauſe them to go and return together with a reciprocal motion. *Q. E. D.*

Cor. 1. Therefore the reciprocations of the water in aſcending and deſcending, are all performed in equal times, whether the motion be more or leſs intenſe or remiſs.

Cor. 2. If the length of the whole water in the canal be of $6\frac{1}{9}$ feet of *French* meaſure, the water will deſcend in one ſecond of time, and will aſcend in another ſecond, and ſo on by turns in infinitum; for a pendulum of $3\frac{1}{18}$ ſuch feet in length will oſcillate in one ſecond of time.

Cor. 3. But if the length of the water be increaſed or diminiſhed, the time of the reciprocation will be increaſed or diminiſhed in the ſubduplicate ratio of the length.

Proposition XLV. Theorem XXXVI.

The velocity of waves is in the ſubduplicate ratio of the breadths.

This follows from the conſtruction of the following propoſition.

Proposition XLVI. Problem X.

To find the velocity of waves.

Let a pendulum be conſtructed, whoſe length between the point of ſuſpenſion and the centre of oſcillation is equal to the breadth of the waves; and in the time that the pendulum will perform one ſingle oſcillation,

lation, the waves will advance forward nearly a ſpace equal to their breadth.

That which I call the breadth of the waves is the tranſverſe meaſure lying between the deepeſt part of the hollows, or the tops of the ridges. Let *ABCDEF* (*Pl.* 8. *Fig.* 5.) repreſent the ſurface of ſtagnant water aſcending and deſcending in ſucceſſive waves; and let *A*, *C*, *E*, &c. be the tops of the waves; and let *B*, *D*, *F*, &c. be the intermediate hollows. Becauſe the motion of the waves is carried on by the ſucceſſive aſcent and deſcent of the water, ſo that the parts thereof, as *A*, *C*, *E*, &c. which are higheſt at one time, become loweſt immediately after; and becauſe the motive force, by which the higheſt parts deſcend and the loweſt aſcend, is the weight of the elevated water, that alternate aſcent and deſcent will be analogous to the reciprocal motion of the water in the canal, and obſerve the ſame laws as to the times of its aſcent and deſcent; and therefore (by prop. 44.) if the diſtances between the higheſt places of the waves *A*, *C*, *E*, and the loweſt *B*, *D*, *F*, be equal to twice the length of any pendulum, the higheſt parts *A*, *C*, *E*, will become the loweſt in the time of one oſcillation, and in the time of another oſcillation will aſcend again. Therefore between the paſſage of each wave, the time of two oſcillations will intervene; that is, the wave will deſcribe its breadth in the time that pendulum will oſcillate twice; but a pendulum of four times that length, and which therefore is equal to the breadth of the waves, will juſt oſcillate once in that time. *Q. E. I.*

Cor. 1. Therefore waves, whoſe breadth is equal to $3\frac{1}{1\,8}$ *French* feet, will advance thro' a ſpace equal to their breadth in one ſecond of time; and therefore in one minute will go over a ſpace of $183\frac{1}{3}$ feet; and in an hour a ſpace of 11000 feet, nearly.

Cor. 2. And the velocity of greater or leſs waves will be augmented or diminiſhed in the ſubduplicate ratio of their breadth. Theſe

Fig. 1. *p.* 163.

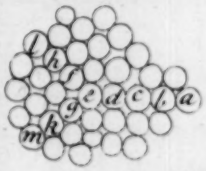


Fig. 2. *p.* 164.

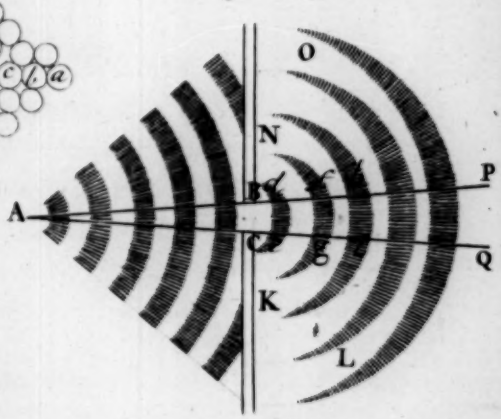


Fig. 3. *p.* 165.

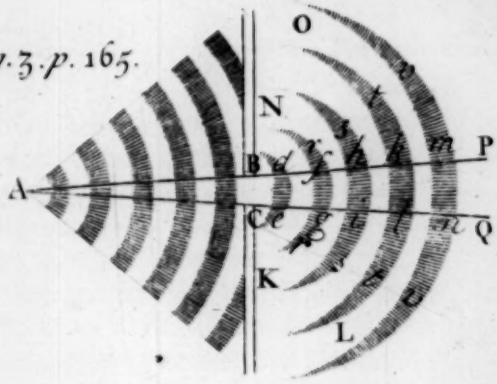
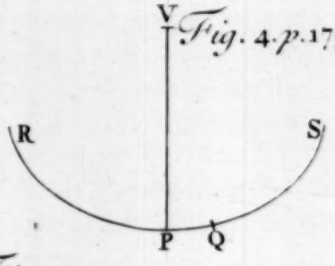


Fig. 4. *p.* 170.



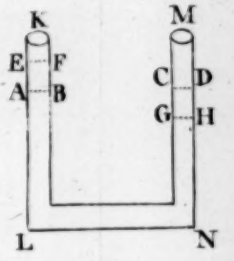
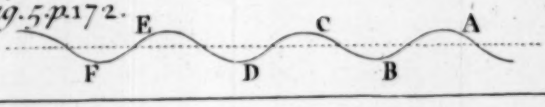


Fig. 5. *p.* 172.



Theſe things are true upon the ſuppoſition, that the parts of water aſcend or deſcend in a right line; but in truth, that aſcent and deſcent is rather performed in a circle; and therefore I propoſe the time defined by this propoſition as only near the truth.

PROPOSITION XLVII. THEOREM XXXVII.

If pulſes are propagated thro' a fluid, the ſeveral particles of the fluid, going and returning with the ſhorteſt reciprocal motion, are always accelerated or retarded according to the law of the oſcillating pendulum. Pl. 9. Fig. 1.

Let *AB*, *BC*, *CD*, &c. repreſent equal diſtances of ſucceſſive pulſes; *ABC* the line of direction of the motion of the ſucceſſive pulſes, propagated from *A* to *B*; *E*, *F*, *G* three phyſical points of the quieſcent medium ſituate in the right line *AC* at equal diſtances from each other; *Ee*, *Ff*, *Gg* equal ſpaces of extreme ſhortneſs, thro' which thoſe points go and return with a reciprocal motion in each vibration; ε, φ, γ any intermediate places of the ſame points; and *EF*, *FG* phyſical lineolæ, or linear parts of the medium lying between thoſe points, and ſucceſſively transfer'd into the places εφ, φγ, and *ef*, *fg*. Let there be drawn the right line *PS* equal to the right line *Ee*. Biſect the ſame in *O*, and from the centre *O*, with the interval *OP*, deſcribe the circle *SIPi*. Let the whole time of one vibration, with its proportional parts, be expounded by the whole circumference of this circle and its parts; in ſuch ſort, that when any time *PH* or *PHSh* is compleated, if there be let fall to *PS* the perpendicular *HL* or *hl*, and there be taken *E*ε equal to *PL* or *Pl*, the phyſical point *E* may be found in ε.

A

A point as *E* moving according to this law with a reciprocal motion, in its going from *E* thro' ε to *e*, and returning again thro' ε to *E*, will perform its ſeveral vibrations with the ſame degrees of acceleration and retardation with thoſe of an oſcillating pendulum. We are now to prove, that the ſeveral phyſical points of the medium will be agitated with ſuch a kind of motion. Let us ſuppoſe then, that a medium hath ſuch a motion excited in it from any cauſe whatſoever, and conſider what will follow from thence.

In the circumference *PHSh* let there be taken the equal arcs *HI*, *IK*, or *hi*, *ik*, having the ſame ratio to the whole circumference as the equal right lines *EF*, *FG* have to *BC* the whole interval of the pulſes. Let fall the perpendiculars *IM*, *KN* or *im*, *kn*; then becauſe the points *E*, *F*, *G* are ſucceſſively agitated with like motions, and perform their entire vibrations compoſed of their going and return, while the pulſe is transferr'd from *B* to *C*; if *PH* or *PHSh* be the time elapſed ſince the beginning of the motion of the point *E*, then will *PI* or *PHSi* be the time elapſed ſince the beginning of the motion of the point *F*, and *PK* or *PHSk* the time elapſed ſince the beginning of the motion of the point *G*; and therefore *Eε*, *Fφ*, *Gγ* will be reſpectively equal to *PL*, *PM*, *PN*, while the points are going, and to *Pl*, *Pm*, *Pn*, when the points are returning. Therefore εγ or *EG* + *Gγ* — *Eε* will, when the points are going, be equal to *EG* — *LN*, and in their return equal to *EG* + *ln*. But εγ is the breadth or expanſion of the part *EG* of the medium in the place εγ; and therefore the expanſion of that part in its going, is to its mean expanſion as *EG* — *LN* to *EG*; and in its return as *EG* + *ln* or *EG* + *LN* to *EG*. Therefore ſince *LN* is to *KH* as *IM* to the radius *OP*, and *KH* to *EG* as the circumference *PHShP* to *BC*; that is, if we put V for the radius of a circle whoſe circumference is equal to

BC

BC the interval of the pulſes, as OP to V; and, *ex æquo*, LN to EG as IM to V; the expanſion of the part EG or of the phyſical point F in the place $\epsilon\gamma$ to the mean expanſion of the ſame part in its firſt place EG, will be as $V-IM$ to V in going, and as $V+im$ to V in its return. Hence the elaſtic force of the point F in the place $\epsilon\gamma$ to its mean elaſtic force in the place EG, is as $\frac{1}{V-IM}$ to $\frac{1}{V}$ in its going, and as $\frac{1}{V+im}$ to $\frac{1}{V}$ in its return. And by the ſame reaſoning the elaſtic forces of the phyſical points E and G in going, are as $\frac{1}{V-HL}$ and $\frac{1}{V-KN}$ to $\frac{1}{V}$; and the difference of the forces to the mean elaſtic force of the medium, as $\frac{HL-KN}{VV-V\times HL-V\times KN+HL\times KN}$ to $\frac{1}{V}$; that is, as $\frac{HL-KN}{VV}$ to $\frac{1}{V}$, or as $HL-KN$ to V; if we ſuppoſe (by reaſon of the very ſhort extent of the vibrations) HL and KN to be indefinitely leſs than the quantity V. Therefore ſince the quantity V is given, the difference of the forces is as $HL-KN$; that is, (becauſe $HL-KN$ is proportional to HK, and OM to OI or OP; and becauſe HK and OP are given) as OM; that is, if Ff be biſected in Ω, as $\Omega\varphi$. And for the ſame reaſon the difference of the elaſtic forces of the phyſical points ϵ and γ in the return of the phyſical lineola $\epsilon\gamma$, is as $\Omega\varphi$. But that difference (that is, the exceſs of the elaſtic force of the point ϵ above the elaſtic force of the point γ) is the very force by which the intervening phyſical lineola $\epsilon\gamma$ of the medium is accelerated in going, and retarded in returning; and therefore the accelerative force of the phyſical lineola $\epsilon\gamma$ is

as its diſtance from Ω the middle place of the vibration. Therefore (by prop. 38. book 1.) the time is rightly expounded by the arc *PI*; and the linear part of the medium $s\gamma$ is moved according to the law abovementioned, that is, according to the law of a pendulum oſcillating; and the caſe is the ſame of all the linear parts of which the whole medium is compounded. *Q. E. D.*

Cor. Hence it appears that the number of the pulſes propagated is the ſame with the number of the vibrations of the tremulous body, and is not multiplied in their progreſs. For the phyſical lineola $s\gamma$ as ſoon as it returns to its firſt place is at reſt; neither will it move again, unleſs it receives a new motion, either from the impulſe of the tremulous body, or of the pulſes propagated from that body. As ſoon therefore as the pulſes ceaſe to be propagated from the tremulous body, it will return to a ſtate of reſt; and move no more.

Proposition XLVIII. Theorem XXXVIII.

The velocities of pulſes propagated in an elaſtic fluid, are in a ratio compounded of the ſubduplicate ratio of the elaſtic force directly, and the ſubduplicate ratio of the denſity inverſely; ſuppoſing the elaſtic force of the fluid to be proportional to its condenſation.

Case 1. If the mediums be homogeneous, and the diſtances of the pulſes in thoſe mediums be equal amongſt themſelves, but the motion in one medium is more intenſe than in the other: the contractions and dilatations of the correſpondent parts will be as thoſe motions. Not that this proportion is perfectly accurate. However, if the contractions and dilatations are not exceedingly intenſe, the error will not be ſenſible; and there-

therefore this proportion may be conſider'd as phyſically exact. Now the motive elaſtic forces are as the contractions and dilatations; and the velocities generated in the ſame time in equal parts are as the forces. Therefore equal and correſponding parts of correſponding pulſes will go and return together, thro' ſpaces proportional to their contractions and dilatations, with velocities that are as thoſe ſpaces: and therefore the pulſes, which in the time of one going and returning advance forwards a ſpace equal to their breadth, and are always ſucceeding into the places of the pulſes that immediately go before them, will, by reaſon of the equality of the diſtances, go forward in both mediums with equal velocity.

Case 2. If the diſtances of the pulſes or their lengths are greater in one medium than in another; let us ſuppoſe that the correſpondent parts deſcribe ſpaces, in going and returning, each time proportional to the breadths of the pulſes: then will their contractions and dilatations be equal. And therefore if the mediums are homogeneous, the motive elaſtic forces, which agitate them with a reciprocal motion, will be equal alſo. Now the matter to be moved by theſe forces is as the breadth of the pulſes; and the ſpace thro' which they move every time they go and return, is in the ſame ratio. And moreover, the time of one going and returning, is in a ratio compounded of the ſubduplicate ratio of the matter, and the ſubduplicate ratio of the ſpace; and therefore is as the ſpace. But the pulſes advance a ſpace equal to their breadths in the times of going once and returning once, that is, they go over ſpaces proportional to the times; and therefore are equally ſwift.

Case 3. And therefore in mediums of equal denſity and elaſtic force, all the pulſes are equally ſwift. Now if the denſity or the elaſtic force of the medium were augmented, then becauſe the motive force is increaſed

in the ratio of the elaſtic force, and the matter to be moved is increaſed in the ratio of the denſity; the time which is neceſſary for producing the ſame motion as before, will be increaſed in the ſubduplicate ratio of the denſity, and will be diminiſhed in the ſubduplicate ratio of the elaſtic force. And therefore the velocity of the pulſes will be in a ratio compounded of the ſubduplicate ratio of the denſity of the medium inverſely, and the ſubduplicate ratio of the elaſtic force directly. *Q. E. D.*

This propoſition will be made more clear from the conſtruction of the following problem.

Proposition XLIX. Problem XI.

The denſity and elaſtic force of a medium being given, to find the velocity of the pulſes.

Suppoſe the medium to be preſs'd by an incumbent weight after the manner of our air; and let A be the height of a homogeneous medium, whoſe weight is equal to the incumbent weight and whoſe denſity is the ſame with the denſity of the compreſſed medium in which the pulſes are propagated. Suppoſe a pendulum to be conſtructed, whoſe length between the point of ſuſpenſion and the centre of oſcillation is A: and in the time in which that pendulum will perform one entire oſcillation compoſed of its going and returning, the pulſe will be propagated right onwards, thro' a ſpace equal to the circumference of a circle deſcribed with the radius A.

For letting thoſe things ſtand which were conſtructed in Prop. 47. if any phyſical line as *EF* (*Pl.* 9. *Fig.* 1.) deſcribing the ſpace *PS* in each vibration, be acted on in the extremities *P* and *S* of every going and return that it makes by an elaſtic force that is equal to its weight; it will perform its ſeveral vibrations in the

the time in which the ſame might oſcillate in a cycloid, whoſe whole perimeter is equal to the length *PS*: and that becauſe equal forces will impel equal corpuſcles thro' equal ſpaces in the ſame or equal times. Therefore ſince the times of the oſcillations are in the ſubduplicate ratio of the lengths of the pendulums, and the length of the pendulum is equal to half the arc of the whole cycloid; the time of one vibration would be to the time of the oſcillation of a pendulum, whoſe length is A, in the ſubduplicate ratio of the length $\frac{1}{2}PS$ or *PO* to the length A. But the elaſtic force, with which the phyſical lineola *EG* is urged, when it is found in its extreme places *P*, *S*, was (in the demonſtration of prop. 47.) to its whole elaſtic force as $HL - KN$ to V, that is, (ſince the point *K* now falls upon *P*) as *HK* to V: and all that force, or, which is the ſame thing, the incumbent weight by which the lineola *EG* is compreſs'd, is to the weight of the lineola as the altitude A of the incumbent weight to *EG* the length of the lineola; and therefore, *ex æquo*, the force with which the lineola *EG* is urged in the places *P* and *S*, is to the weight of that lineola as $HK \times A$ to $V \times EG$; or as $PO \times A$ to VV; becauſe *HK* was to *EG* as *PO* to V. Therefore ſince the times, in which equal bodies are impelled thro' equal ſpaces, are reciprocally in the ſubduplicate ratio of the forces, the time of one vibration, produced by the action of that elaſtic force, will be to the time of a vibration, produced by the impulſe of the weight, in a ſubduplicate ratio of VV to $PO \times A$, and therefore to the time of the oſcillation of a pendulum whoſe length is A, in the ſubduplicate ratio of VV to $PO \times A$, and the ſubduplicate ratio of *PO* to A conjunctly; that is, in the entire ratio of V to A. But in the time of one vibration compoſed of the going and returning of the pendulum, the pulſe will be propagated right onwards thro' a ſpace equal to its breadth *BC*. Therefore the time in which a pulſe runs over the ſpace

BC, is to the time of one oſcillation compoſed of the going and returning of the pendulum, as V to A, that is, as *BC* to the circumference of a circle whoſe radius is A. But the time in which the pulſe will run over the ſpace *BC*, is to the time in which it will run over a length equal to that circumference, in the ſame ratio; and therefore in the time of ſuch an oſcillation, the pulſe will run over a length equal to that circumference. *Q. E. D.*

Cor. 1. The velocity of the pulſes is equal to that which heavy bodies acquire by falling with an equally accelerated motion, and in their fall deſcribing half the altitude A. For the pulſe will, in the time of this fall, ſuppoſing it to move with the velocity acquired by that fall, run over a ſpace that will be equal to the whole altitude A; and therefore in the time of one oſcillation compoſed of one going and return, will go over a ſpace equal to the circumference of a circle deſcribed with the radius A: for the time of the fall is to the time of oſcillation, as the radius of a circle to its circumference.

Cor. 2. Therefore ſince that altitude A is as the elaſtic force of the fluid directly, and the denſity of the ſame inverſely; the velocity of the pulſes will be in a ratio compounded of the ſubduplicate ratio of the denſity inverſely, and the ſubduplicate ratio of the elaſtic force directly.

Proposition L. Problem XII.

To find the diſtances of the pulſes.

Let the number of the vibrations of the body, by whoſe tremor the pulſes are produced, be found to any given time. By that number divide the ſpace which a pulſe can go over in the ſame time, and the part found will be the breadth of one pulſe. *Q. E. I.*

Scho-

Scholium.

The laſt propoſitions reſpect the motions of light and ſounds. For ſince light is propagated in right lines, it is certain that it cannot conſiſt in action alone, (by Prop. 41 and 42.) As to ſounds, ſince they ariſe from tremulous bodies, they can be nothing elſe b' t pulſes of the air propagated thro' it, (by Prop. 43.) And this is confirmed by the tremors, which ſounds, if they be loud and deep, excite in the bodies near them, as we experience in the ſound of drums. For quick and ſhort tremors are leſs eaſily excited. But it is well known, that any ſounds, falling upon ſtrings in uniſon with the ſonorous bodies, excite tremors in thoſe ſtrings. This is alſo confirmed from the velocity of ſounds. For ſince the ſpecific gravities of rain-water and quick-ſilver are to one another as about 1 to $13\frac{1}{3}$, and when the mercury in the barometer is at the height of 30 inches of our meaſure, the ſpecific gravities of the air and of rain-water are to one another as about 1 to 870: therefore the ſpecific gravity of air and quick-ſilver are to each other as 1 to 11890. Therefore when the height of the quick-ſilver is at 30 inches, a height of uniform air, whoſe weight would be ſufficient to compreſs our air to the denſity we find it to be of, muſt be equal to 356700 inches or 29725 feet of our meaſure. And this is that very height of the medium, which I have called A in the conſtruction of the foregoing propoſition. A circle whoſe radius is 29725 feet is 186768 feet in circumference. And ſince a pendulum $39\frac{1}{3}$ inches in length compleats one oſcillation, compoſed of its going and return, in two ſeconds of time, as is commonly known; it follows that a pendulum 29725 feet or 356700 inches in length will perform a like oſcillation in $190\frac{1}{4}$ ſeconds. Therefore

in that time a ſound will go right onwards 186768 feet, and therefore in one ſecond 979 feet.

But in this computation we have made no allowance for the craſſitude of the ſolid particles of the air, by which the ſound is propagated inſtantaneouſly. Becauſe the weight of air is to the weight of water as 1 to 870, and becauſe ſalts are almoſt twice as denſe as water; if the particles of air are ſuppoſed to be of near the ſame denſity as thoſe of water or ſalt, and the rarity of the air ariſes from the intervals of the particles; the diameter of one particle of air will be to the interval between the centres of the particles, as 1 to about 9 or 10, and to the interval between the particles themſelves as 1 to 8 or 9. Therefore to 979 feet, which, according to the above calculation, a ſound will advance forward in one ſecond of time, we may add $\frac{2}{9}\frac{1}{2}$, or about 109 feet, to compenſate for the craſſitude of the particles of the air: and then a ſound will go forward about 1088 feet in one ſecond of time.

Moreover, the vapors floating in the air, being of another ſpring, and a different tone, will hardly, if at all, partake of the motion of the true air in which the ſounds are propagated. Now if theſe vapors remain unmoved, that motion will be propagated the ſwifter thro' the true air alone, and that in the ſubduplicate ratio of the defect of the matter. So if the atmoſphere conſiſt of ten parts of true air and one part of vapors, the motion of ſounds will be ſwifter in the ſubduplicate ratio of 11 to 10, or very nearly in the entire ratio of 21 to 20, than if it were propagated thro' eleven parts of true air: and therefore the motion of ſounds above diſcovered muſt be encreaſed in that ratio. By this means the ſound will paſs thro' 1142 feet in one ſecond of time.

Theſe things will be found true in ſpring and autumn, when the air is rarefied by the gentle warmth of thoſe ſeaſons, and by that means its elaſtic force becomes

comes ſomewhat more intenſe. But in winter, when the air is condenſed by the cold, and its elaſtic force is ſomewhat remitted, the motion of ſounds will be ſlower in a ſubduplicate ratio of the denſity; and on the other hand, ſwifter in the ſummer.

Now by experiments it actually appears that ſounds do really advance in one ſecond of time about 1142 feet of *Engliſh* meaſure, or 1070 feet of *French* meaſure.

The velocity of ſounds being known, the intervals of the pulſes are known alſo. For M. *Sauveur*, by ſome experiments that he made, found that an open pipe about five *Paris* feet in length, gives a ſound of the ſame tone with a viol-ſtring that vibrates a hundred times in one ſecond. Therefore there are near 100 pulſes in a ſpace of 1070 *Paris* feet, which a ſound runs over in a ſecond of time; and therefore one pulſe fills up a ſpace of about 10 $\frac{7}{10}$ *Paris* feet, that is, about twice the length of the pipe. From whence it is probable, that the breadths of the pulſes, in all ſounds made in open pipes, are equal to twice the length of the pipes.

Moreover, from the corollary of prop. 47. appears the reaſon, why the ſounds immediately ceaſe with the motion of the ſonorous body, and why they are heard no longer when we are at a great diſtance from the ſonorous bodies, than when we are very near them. And beſides, from the foregoing principles it plainly appears how it comes to paſs that ſounds are ſo mightily encreaſed in ſpeaking-trumpets. For all reciprocal motion uſes to be encreaſed by the generating cauſe at each return. And in tubes hindering the dilatation of the ſounds, the motion decays more ſlowly, and recurs more forcibly; and therefore is the more encreaſed by the new motion impreſſed at each return. And theſe are the principal phænomena of ſounds.

SECTION IX.

Of the circular motion of fluids.

HYPOTHESIS.

The reſiſtance, ariſing from the want of lubricity in the parts of a fluid, is, cæteris paribus, *proportional to the velocity with which the parts of the fluid are ſeparated from each other.*

PROPOSITION LI. THEOREM XXXVIII.

If a ſolid cylinder infinitely long, in an uniform and infinite fluid, revolve with an uniform motion about an axis given in poſition, and the fluid be forced round by only this impulſe of the cylinder, and every part of the fluid perſevere uniformly in its motion; I ſay, that the periodic times of the parts of the fluid are as their diſtances from the axis of the cylinder.

Let *AFL* (*Pl.* 9. *Fig.* 2.) be a cylinder turning uniformly about the axis *S*, and let the concentric circles *BGM*, *CHN*, *DIO*, *EKP*, &c. divide the fluid into innumerable concentric cylindric ſolid orbs of the ſame

ſame thickneſs. Then, becauſe the fluid is homogeneous, the impreſſions which the contiguous orbs make upon each other mutually, will be (by the hypotheſis) as their tranſlations from each other, and as the contiguous ſuperficies upon which the impreſſions are made. If the impreſſion made upon any orb be greater or leſs on its concave, than on its convex ſide, the ſtronger impreſſion will prevail, and will either accelerate or retard the motion of the orb, according as it agrees with, or is contrary to the motion of the ſame. Therefore, that every orb may perſevere uniformly in its motion, the impreſſions made on both ſides muſt be equal, and their directions contrary. Therefore ſince the impreſſions are as the contiguous ſuperficies, and as their tranſlations from one another; the tranſlations will be inverſely as the ſuperficies, that is, inverſely as the diſtances of the ſuperficies from the axis. But the differences of the angular motions about the axis, are as thoſe tranſlations applied to the diſtances, or as the tranſlations directly and the diſtances inverſely; that is, joining theſe ratio's together, as the ſquares of the diſtances inverſely. Therefore if there be erected the lines *Aa*, *Bb*, *Cc*, *Dd*, *Ee*, &c. perpendicular to the ſeveral parts of the infinite right line *SABCDEQ* and reciprocally proportional to the ſquares of *SA*, *SB*, *SC*, *SD*, *SE*, &c. and thro' the extremities of thoſe perpendiculars there be ſuppoſed to paſs an hyperbolic curve; the ſums of the differences, that is, the whole angular motions, will be as the correſpondent ſums of the lines *Aa*, *Bb*, *Cc*, *Dd*, *Ee*, that is, (if to conſtitute a medium uniformly fluid, the number of the orbs be encreaſed and their breadth diminiſhed in infinitum) as the hyperbolic area's *AaQ*, *BbQ*, *CcQ*, *DdQ*, *EeQ*, &c. analogous to the ſums. And the times, reciprocally proportional to the angular motions, will be alſo reciprocally proportional to thoſe areas. Therefore the periodic time of any particle as *D*, is reciprocally as the area *DdQ*, that is, (as

appears

appears from the known methods of quadratures of curves) directly as the distance *SD*. *Q.E.D.*

Cor. 1. Hence the angular motions of the particles of the fluid are reciprocally as their distances from the axis of the cylinder, and the absolute velocities are equal.

Cor. 2. If a fluid be contained in a cylindric vessel of an infinite length, and contain another cylinder within, and both the cylinders revolve about one common axis, and the times of their revolutions be as their semidiameters, and every part of the fluid perseveres in its motion: the periodic times of the several parts will be as the distances from the axis of the cylinders.

Cor. 3. If there be added or taken away any common quantity of angular motion from the cylinder and fluid moving in this manner; yet because this new motion will not alter the mutual attrition of the parts of the fluid, the motion of the parts among themselves will not be changed. For the translations of the parts from one another depend upon the attrition. Any part will persevere in that motion, which, by the attrition made on both sides with contrary directions, is no more accelerated than it is retarded.

Cor. 4. Therefore if there be taken away from this whole system of the cylinders and the fluid, all the angular motion of the outward cylinder, we shall have the motion of the fluid in a quiescent cylinder.

Cor. 5. Therefore if the fluid and outward cylinder are at rest, and the inward cylinder revolve uniformly; there will be communicated a circular motion to the fluid, which will be propagated by degrees thro' the whole fluid; and will go on continually encreasing, till such time as the several parts of the fluid acquire the motion determined in cor. 4.

Cor. 6. And because the fluid endeavours to propagate its motion still farther, its impulse will carry the outmost cylinder also about with it, unless the cylinder

be violently detained; and accelerate its motion till the periodic times of both cylinders become equal among themſelves. But if the outward cylinder be violently detained, it will make an effort to retard the motion of the fluid; and unleſs the inward cylinder preſerve that motion by means of ſome external force impreſſed thereon, it will make it ceaſe by degrees.

All theſe things will be found true, by making the experiment in deep ſtanding water.

Proposition LII. Theorem XL.

If a ſolid ſphere, in an uniform and infinite fluid, revolves about an axis given in poſition with an uniform motion, and the fluid be forced round by only this impulſe of the ſphere; and every part of the fluid perſeveres uniformly in its motion: I ſay, that the periodic times of the parts of the fluid are as the ſquares of their diſtances from the centre of the ſphere.

Case 1. Let *AFL* be a ſphere turning uniformly about the axis *S*, and let the concentric circles *BGM*, *CHN*, *DIO*, *EKP*, &c. divide the fluid into innumerable concentric orbs of the ſame thickneſs. Suppoſe thoſe orbs to be ſolid; and becauſe the fluid is homogeneous, the impreſſions which the contiguous orbs make one upon another, will be (by the ſuppoſition) as their tranſlations from one another, and the contiguous ſuperficies upon which the impreſſions are made. If the impreſſion upon any orb be greater or leſs upon its concave than upon its convex ſide; the more forcible impreſſion will prevail, and will either accelerate or retard the velocity of the orb, according as it is directed

rected with a conſpiring or contrary motion to that of the orb. Therefore that every orb may perſevere uniformly in its motion, it is neceſſary that the impreſſions made upon both ſides of the orb ſhould be equal, and have contrary directions. Therefore ſince the impreſſions are as the contiguous ſuperficies, and as their tranſlations from one another; the tranſlations will be inverſly as the ſuperficies, that is, inverſly as the ſquares of the diſtances of the ſuperficies from the centre. But the differences of the angular motions about the axis are as thoſe tranſlations applied to the diſtances, or as the tranſlations directly and the diſtances inverſly; that is, by compounding thoſe ratio's, as the cubes of the diſtances inverſly. Therefore, if upon the ſeveral parts of the infinite right line $SABCDEQ$ there be erected the perpendiculars Aa, Bb, Cc, Dd, Ee, &c. reciprocally proportional to the cubes of SA, SB, SC, SD, SE, &c. the ſums of the differences, that is, the whole angular motions, will be as the correſponding ſums of the lines Aa, Bb, Cc, Dd, Ee, &c. that is, (if to conſtitute an uniformly fluid medium the number of the orbs be encreaſed and their thickneſs diminiſhed in infinitum) as the hyperbolic areas AaQ, BbQ, CcQ, DdQ, EeQ, &c. analogous to the ſums; and the periodic times being reciprocally proportional to the angular motions, will be alſo reciprocally proportional to thoſe areas. Therefore the periodic time of any orb DIO is reciprocally as the area DdQ, that is, (by the known methods of quadratures) directly as the ſquare of the diſtance SD. Which was firſt to be demonſtrated.

Case 2. From the centre of the ſphere let there be drawn a great number of indefinite right lines, making given angles with the axis, exceeding one another by equal differences; and, by theſe lines revolving about the axis, conceive the orbs to be cut into innumerable annuli: then will every annulus have four an-

nuli contiguous to it, that is, one on its inſide, one on its outſide, and two on each hand. Now each of theſe annuli cannot be impelled equally and with contrary directions by the attrition of the interior and exterior annuli unleſs the motion be communicated according to the law which we demonſtrated in caſe 1. This appears from that demonſtration. And therefore any ſeries of annuli, taken in any right line extending itſelf in infinitum from the globe, will move according to the law of caſe 1. except we ſhould imagine it hindered by the attrition of the annuli on each ſide of it. But now in a motion, according to this law, no ſuch attrition is, and therefore cannot be any obſtacle to the motion's perſevering according to that law. If annuli at equal diſtances from the centre revolve either more ſwiftly or more ſlowly near the poles than near the ecliptic; they will be accelerated if ſlow, and retarded if ſwift, by their mutual attrition; and ſo the periodic times will continually approach to equality, according to the law of caſe 1. Therefore this attrition will not at all hinder the motion from going on according to the law of caſe 1. and therefore that law will take place; that is, the periodic times of the ſeveral annuli will be as the ſquares of their diſtances from the centre of the globe. Which was to be demonſtrated in the ſecond place.

Case 3. Let now every annulus be divided by tranſverſe ſections into innumerable particles conſtituting a ſubſtance abſolutely and uniformly fluid; and becauſe theſe ſections do not at all reſpect the law of circular motion, but only ſerve to produce a fluid ſubſtance, the law of circular motion will continue the ſame as before. All the very ſmall annuli will either not at all change their aſperity and force of mutual attrition upon account of theſe ſections, or elſe they will change the ſame equally. Therefore the proportion of the cauſes remaining the ſame, the proportion of the effects will remain

remain the ſame alſo; that is, the proportion of the motions and the periodic times. *Q. E. D.* But now as the circular motion, and the centrifugal force thence ariſing, is greater at the ecliptic than at the poles, there muſt be ſome cauſe operating to retain the ſeveral particles in their circles; otherwiſe the matter that is at the ecliptic will always recede from the centre, and come round about to the poles by the outſide of the vortex, and from thence return by the axis to the ecliptic with a perpetual circulation.

Cor. 1. Hence the angular motions of the parts of the fluid about the axis of the globe, are reciprocally as the ſquares of the diſtances from the centre of the globe, and the abſolute velocities are reciprocally as the ſame ſquares applied to the diſtances from the axis.

Cor. 2. If a globe revolve with a uniform motion about an axis of a given poſition in a ſimilar and infinite quieſcent fluid with an uniform motion, it will communicate a whirling motion to the fluid like that of a vortex, and that motion will by degrees be propagated onwards in infinitum; and this motion will be encreaſed continually in every part of the fluid, till the periodical times of the ſeveral parts become as the ſquares of the diſtances from the centre of the globe.

Cor. 3. Becauſe the inward parts of the vortex are by reaſon of their greater velocity continually preſſing upon and driving forwards the external parts, and by that action are perpetually communicating motion to them, and at the ſame time thoſe exterior parts communicate the ſame quantity of motion to thoſe that lie ſtill beyond them, and by this action preſerve the quantity of their motion continually unchanged; it is plain that the motion is perpetually transferred from the centre to the circumference of the vortex, till it is quite ſwallowed up and loſt in the boundleſs extent of that circumference. The matter between any two ſphærical ſuperficies concentrical to the vortex will never be accelerated,

celerated; becauſe that matter will be always transferring the motion it receives from the matter nearer the centre to that matter which lies nearer the circumference.

Cor. 4. Therefore in order to continue a vortex in the ſame ſtate of motion, ſome active principle is required, from which the globe may receive continually the ſame quantity of motion which it is always communicating to the matter of the vortex. Without ſuch a principle it will undoubtedly come to paſs that the globe and the inward parts of the vortex, being always propagating their motion to the outward parts, and not receiving any new motion, will gradually move ſlower and ſlower, and at laſt be carried round no longer.

Cor. 5. If another globe ſhould be ſwimming in the ſame vortex at a certain diſtance from its centre, and in the mean time by ſome force revolve conſtantly about an axis of a given inclination; the motion of this globe will drive the fluid round after the manner of a vortex; and at firſt this new and ſmall vortex will revolve with its globe about the centre of the other; and in the mean time its motion will creep on, farther and farther, and by degrees be propagated in infinitum, after the manner of the firſt vortex. And for the ſame reaſon that the globe of the new vortex was carried about before by the motion of the other vortex, the globe of this other will be carried about by the motion of this new vortex, ſo that the two globes will revolve about ſome intermediate point, and by reaſon of that circular motion mutually fly from each other, unleſs ſome force reſtrains them. Afterwards, if the conſtantly impreſſed forces, by which the globes perſevere in their motions, ſhould ceaſe, and every thing be left to act according to the laws of mechanics, the motion of the globes will languiſh by degrees, (for the reaſon aſſigned in cor. 3 and 4.) and the vortices at laſt will quite ſtand ſtill.

Cor.

Cor. 6. If ſeveral globes in given places ſhould conſtantly revolve with determined velocities about axes given in poſition, there would ariſe from them as many vortices going on in infinitum. For upon the ſame account that any one globe propagates its motion in infinitum, each globe apart will propagate its own motion in infinitum alſo; ſo that every part of the infinite fluid will be agitated with a motion reſulting from the actions of all the globes. Therefore the vortices will not be confined by any certain limits, but by degrees run mutually into each other; and by the mutual actions of the vortices on each other, the globes will be perpetually moved from their places, as was ſhewn in the laſt corollary; neither can they poſſibly keep any certain poſition among themſelves, unleſs ſome force reſtrains them. But if thoſe forces, which are conſtantly impreſſed upon the globes to continue theſe motions, ſhould ceaſe; the matter (for the reaſon aſſigned in cor. 3 and 4.) will gradually ſtop, and ceaſe to move in vortices.

Cor. 7. If a ſimilar fluid be incloſed in a ſphærical veſſel, and by the uniform rotation of a globe in its centre, is driven round in a vortex; and the globe and veſſel revolve the ſame way about the ſame axis, and their periodical times be as the ſquares of the ſemidiameters; the parts of the fluid will not go on in their motions without acceleration or retardation, till their periodical times are as the ſquares of their diſtances from the centre of the vortex. No conſtitution of a vortex can be permanent but this.

Cor. 8. If the veſſel, the incloſed fluid, and the globe, retain this motion, and revolve beſides with a common angular motion about any given axis; becauſe the mutual attrition of the parts of the fluid is not changed by this motion, the motions of the parts among each other will not be changed. For the tranſlations of the parts among themſelves depend upon this attrition.

Any

Any part will perſevere in that motion, in which its attrition on one ſide retards it juſt as much as its attrition on the other ſide accelerates it.

Cor. 9. Therefore if the veſſel be quieſcent, and the motion of the globe be given, the motion of the fluid will be given. For conceive a plane to paſs thro' the axis of the globe, and to revolve with a contrary motion; and ſuppoſe the ſum of the time of this revolution and of the revolution of the globe to be to the time of the revolution of the globe, as the ſquare of the ſemidiameter of the veſſel to the ſquare of the ſemidiameter of the globe; and the periodic times of the parts of the fluid in reſpect of this plane will be as the ſquares of their diſtances from the centre of the globe.

Cor. 10. Therefore if the veſſel move about the ſame axis with the globe, or with a given velocity about a different one, the motion of the fluid will be given. For if from the whole ſyſtem we take away the angular motion of the veſſel, all the motions will remain the ſame among themſelves as before, by cor. 8. and thoſe motions will be given by cor. 9.

Cor. 11. If the veſſel and the fluid are quieſcent, and the globe revolves with an uniform motion, that motion will be propagated by degrees through the whole fluid to the veſſel, and the veſſel will be carried round by it, unleſs violently detained; and the fluid and the veſſel will be continually accelerated till their periodic times become equal to the periodic times of the globe. If the veſſel be either withheld by ſome force, or revolve with any conſtant and uniform motion, the medium will come by little and little to the ſtate of motion defined in cor. 8. 9. 10. nor will it ever perſevere in any other ſtate. But if then the forces, by which the globe and veſſel revolve with certain motions, ſhould ceaſe, and the whole ſyſtem be left to act according to the mechanical laws, the veſſel and globe, by means of the intervening fluid, will act upon

each other, and will continue to propagate their motions through the fluid to each other, till their periodic times become equal among themſelves, and the whole ſyſtem revolves together like one ſolid body.

Scholium.

In all theſe reaſonings, I ſuppoſe the fluid to conſiſt of matter of uniform denſity and fluidity. I mean that the fluid is ſuch, that a globe placed any where therein may propagate with the ſame motion of its own, at diſtances from it ſelf continually equal, ſimilar and equal motions in the fluid, in the ſame interval of time. The matter by its circular motion endeavours to recede from the axis of the vortex; and therefore preſſes all the matter that lies beyond. This preſſure makes the attrition greater, and the ſeparation of the parts more difficult; and by conſequence diminiſhes the fluidity of the matter. Again, if the parts of the fluid are in any one place denſer or larger than in the others, the fluidity will be leſs in that place, becauſe there are fewer ſuperficies where the parts can be ſeparated from each other. In theſe caſes I ſuppoſe the defect of the fluidity to be ſupplied by the ſmoothneſs or ſoftneſs of the parts, or ſome other condition; otherwiſe the matter where it is leſs fluid, will cohere more, and be more ſluggiſh, and therefore will receive the motion more ſlowly, and propagate it farther than agrees with the ratio above aſſigned. If the veſſel be not ſphærical, the particles will move in lines, not circular, but anſwering to the figure of the veſſel, and the periodic times will be nearly as the ſquares of the mean diſtances from the centre. In the parts between the centre and the circumference, the motions will be ſlower where the ſpaces are wide, and ſwifter where narrow; but yet the particles will not tend to the circumference

cumference at all the more for their greater ſwiftneſs. For they then deſcribe arcs of leſs curvity, and the conatus of receding from the centre is as much diminiſhed by the diminution of this curvature, as it is augmented by the increaſe of the velocity. As they go out of narrow into wide ſpaces they recede a little farther from the centre, but in doing ſo are retarded; and when they come out of wide into narrow ſpaces they are again accelerated; and ſo each particle is retarded and accelerated by turns for ever. Theſe things will come to paſs in a rigid veſſel. For the ſtate of vortices in an infinite fluid is known by cor. 6. of this propoſition.

I have endeavoured in this propoſition to inveſtigate the properties of vortices, that I might find whether the celeſtial phænomena can be explained by them. For the phænomenon is this, that the periodic times of the Planets revolving about Jupiter, are in the ſeſquiplicate ratio of their diſtances from Jupiter's centre; and the ſame rule obtains alſo among the Planets that revolve about the Sun. And theſe rules obtain alſo with the greateſt accuracy, as far as has been yet diſcovered by aſtronomical obſervation. Therefore, if thoſe Planets are carried round in vortices revolving about Jupiter and the Sun, the vortices muſt revolve according to that law. But here we found the periodic times of the parts of the vortex to be in the duplicate ratio of the diſtances from the centre of motion; and this ratio cannot be diminiſhed and reduced to the ſeſquiplicate, unleſs either the matter of the vortex be more fluid, the farther it is from the centre, or the reſiſtance ariſing from the want of lubricity in the parts of the fluid, ſhould, as the velocity with which the parts of the fluid are ſeparated goes on increaſing, be augmented with it in a greater ratio than that in which the velocity increaſes. But neither of theſe ſuppoſitions ſeem reaſonable. The more groſs and leſs fluid parts

will tend to the circumference, unleſs they are heavy towards the centre. And tho', for the ſake of demonſtration, I propoſed, at the beginning of this Section, an hypotheſis that the reſiſtance is proportional to the velocity, nevertheleſs, 'tis in truth probable that the reſiſtance is in a leſs ratio than that of the velocity. Which granted, the periodic times of the parts of the vortex will be in a greater than the duplicate ratio of the diſtances from its centre. If, as ſome think, the vortices move more ſwiftly near the centre, then ſlower to a certain limit, then again ſwifter near the circumference, certainly neither the ſeſquiplicate, nor any other certain and determinate ratio can obtain in them. Let philoſophers then ſee how that phænomenon of the ſeſquiplicate ratio can be accounted for by vortices.

PROPOSITION LIII. THEOREM XLI.

Bodies, carried about in a vortex and returning in the ſame orb, are of the ſame denſity with the vortex, and are moved according to the ſame law with the parts of the vortex, as to velocity and direction of motion.

For if any ſmall part of the vortex, whoſe particles or phyſical points preſerve a given ſituation among each other, be ſuppoſed to be congealed; this particle will move according to the ſame law as before, ſince no change is made either in its denſity, *vis inſita*, or figure. And again, if a congealed or ſolid part of the vortex be of the ſame denſity with the reſt of the vortex, and be reſolved into a fluid, this will move according to the ſame law as before, except in ſo far as its particles now become fluid, may be moved among themſelves. Neglect therefore the motion of the particles

ticles among themſelves, as not at all concerning the progreſſive motion of the whole, and the motion of the whole will be the ſame as before. But this motion will be the ſame with the motion of other parts of the vortex at equal diſtances from the centre; becauſe the ſolid, now reſolved into a fluid, is become perfectly like to the other parts of the vortex. Therefore a ſolid, if it be of the ſame denſity with the matter of the vortex, will move with the ſame motion as the parts thereof, being relatively at reſt in the matter that ſurrounds it. If it be more denſe, it will endeavour more than before to recede from the centre; and therefore overcoming that force of the vortex, by which, being as it were kept in equilibrio, it was retained in its orbit, it will recede from the centre, and in its revolution deſcribe a ſpiral, returning no longer into the ſame orbit. And by the ſame argument, if it be more rare it will approach to the centre. Therefore it can never continually go round in the ſame orbit, unleſs it be of the ſame denſity with the fluid. But we have ſhewn in that caſe, that it would revolve according to the ſame law with thoſe parts of the fluid that are at the ſame or equal diſtances from the centre of the vortex.

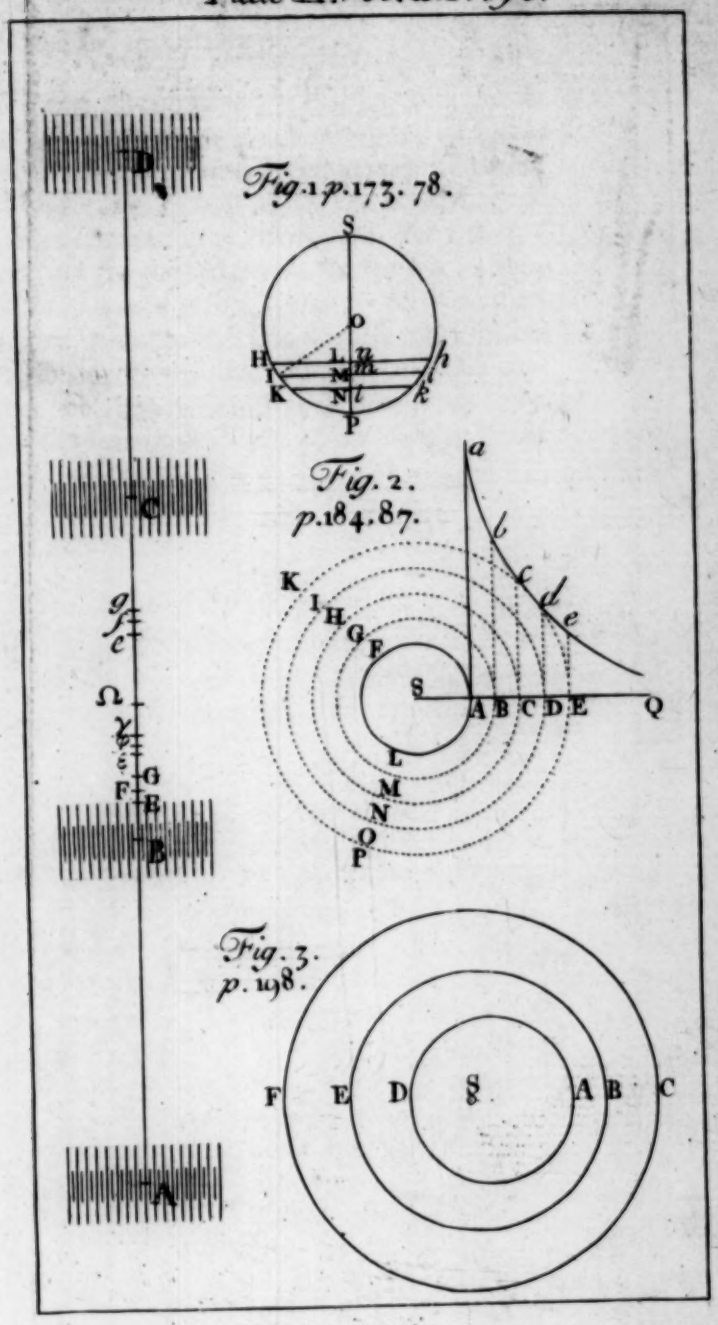
Cor. 1. Therefore a ſolid revolving in a vortex, and continually going round in the ſame orbit, is relatively quieſcent in the fluid that carries it.

Cor. 2. And if the vortex be of an uniform denſity, the ſame body may revolve at any diſtance from the centre of the vortex.

Scholium.

Hence it is manifeſt, that the Planets are not carried round in corporeal vortices. For according to the *Copernican* hypotheſis, the Planets going round the Sun,

revolve in ellipſes, having the Sun in their common focus; and by radii drawn to the ſun deſcribe areas proportional to the times. But now the parts of a vortex can never revolve with ſuch a motion. Let *AD*, *BE*, *CF*, (*Pl.* 9. *Fig.* 3.) repreſent three orbits deſcribed about the Sun *S*, of which let the utmoſt circle *CF* be concentric to the Sun; and let the aphelia of the two innermoſt be *A*, *B*; and their perihelia *D*, *E*. Therefore a body revolving in the orb *CF*, deſcribing, by a radius drawn to the Sun, areas proportional to the times, will move with an uniform motion. And according to the laws of aſtronomy, the body revolving in the orb *BE* will move ſlower in its aphelion *B*, and ſwifter in its perihelion *E*; whereas, according to the laws of mechanics, the matter of the vortex ought to move more ſwiftly in the narrow ſpace between *A* and *C*, than in the wide ſpace between *D* and *F*; that is, more ſwiftly in the aphelion than in the perihelion. Now theſe two concluſions contradict each other. So at the beginning of the ſign of Virgo, where the aphelion of Mars is at preſent, the diſtance between the orbits of Mars and Venus is to the diſtance between the ſame orbits at the beginning of the ſign of Piſces, as about 3 to 2; and therefore the matter of the vortex between thoſe orbits ought to be ſwifter at the beginning of Piſces, than at the beginning of Virgo, in the ratio of 3 to 2. For the narrower the ſpace is, thro' which the ſame quantity of matter paſſes in the ſame time of one revolution, the greater will be the velocity with which it paſſes thro' it. Therefore if the Earth being relatively at reſt in this celeſtial matter ſhould be carried round by it, and revolve together with it about the Sun, the velocity of the Earth at the beginning of Piſces would be to its velocity at the beginning of Virgo in a ſeſquialteral ratio. Therefore the Sun's apparent diurnal motion at the beginning of Virgo, ought to be above 70 minutes; and at the beginning of Piſces leſs than 48 minutes.


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Fig. 1 p. 173. 78.
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Fig. 2.
p. 184. 87.
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Fig. 3.
p. 198.
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minutes. Whereas on the contrary that apparent motion of the Sun is really greater at the beginning of Piſces than at the beginning of Virgo, as experience teſtifies; and therefore the earth is ſwifter at the beginning of Virgo than at the beginning of Piſces. So that the hypotheſis of vortices is utterly irreconcileable with aſtronomical phænomena, and rather ſerves to perplex than explain the heavenly motions. How theſe motions are performed in free ſpaces without vortices, may be underſtood by the firſt book; and I ſhall now more fully treat of it in the following book *of the Syſtem of the World.*



OF THE SYSTEM OF THE WORLD.

BOOK III.

IN the preceding books I have laid down the principles of philoſophy; principles, not philoſophical, but mathematical; ſuch, to wit, as we may build our reaſonings upon in philoſophical enquiries. Theſe principles are, the laws and conditions of certain motions, and powers or forces, which chiefly have reſpect to philoſophy. But leſt they ſhould have appeared of themſelves dry and barren, I have illuſtrated them here and there, with ſome philoſophical ſcholiums, giving an account of ſuch things, as are of more general nature, and which philoſophy ſeems chiefly to be founded on; ſuch as the denſity and the reſiſtance of bodies,

bodies, ſpaces void of all bodies, and the motion of light and ſounds. It remains, that from the ſame principles, I now demonſtrate the frame of the Syſtem of the World. Upon this ſubject, I had indeed compos'd the third book in a popular method, that it might be read by many. But afterwards conſidering that ſuch as had not ſufficiently enter'd into the principles, could not eaſily diſcern the ſtrength of the conſequences, nor lay aſide the prejudices to which they had been many years accuſtomed; therefore to prevent the diſputes which might be rais'd upon ſuch accounts, I choſe to reduce the ſubſtance of that book into the form of propoſitions (in the mathematical way) which ſhould be read by thoſe only, who had firſt made themſelves maſters of the principles eſtabliſh'd in the preceding books. Not that I would adviſe any one to the previous ſtudy of every propoſition of thoſe books. For they abound with ſuch as might coſt too much time, even to readers of good mathematical learning. It is enough if one carefully reads the definitions, the laws of motion, and the firſt three ſections of the firſt book. He may then paſs on to this book, of the Syſtem of the World, and conſult ſuch of the remaining propoſitions of the firſt two books, as the references in this, and his occaſions, ſhall require.

THE

THE RULES OF REASONING in PHILOSOPHY.

RULE I.

We are to admit no more cauſes of natural things, than ſuch as are both true and ſufficient to explain their appearances.

To this purpoſe the philoſophers ſay, that Nature do's nothing in vain, and more is in vain, when leſs will ſerve; For Nature is pleas'd with ſimplicity, and affects not the pomp of ſuperfluous cauſes.

RULE II.

Therefore to the ſame natural effects we muſt, as far as poſſible, aſſign the ſame cauſes.

As to reſpiration in a man, and in a beaſt; the deſcent of ſtones in *Europe* and in *America*; the light of our culinary fire and of the Sun; the reflection of light in the Earth, and in the Planets.

Rule III.

The qualities of bodies, which admit neither intenſion nor remiſſion of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be eſteemed the univerſal qualities of all bodies whatſoever.

For ſince the qualities of bodies are only known to us by experiments, we are to hold for univerſal, all ſuch as univerſally agree with experiments; and ſuch as are not liable to diminution, can never be quite taken away. We are certainly not to relinquiſh the evidence of experiments for the ſake of dreams and vain fictions of our own deviſing; nor are we to recede from the analogy of Nature, which uſes to be ſimple, and always conſonant to it ſelf. We no otherways know the extenſion of bodies, than by our ſenſes, nor do theſe reach it in all bodies; but becauſe we perceive extenſion in all that are ſenſible, therefore we aſcribe it univerſally to all others alſo. That abundance of bodies are hard we learn by experience. And becauſe the hardneſs of the whole ariſes from the hardneſs of the parts, we therefore juſtly infer the hardneſs of the undivided particles not only of the bodies we feel but of all others. That all bodies are impenetrable, we gather not from reaſon, but from ſenſation. The bodies which we handle we find impenetrable, and thence conclude impenetrability to be an univerſal property of all bodies whatſoever. That all bodies are moveable, and endow'd with certain powers (which we call the *vires inertiæ*) of perſevering in their motion or in their reſt, we only infer from the like properties obſerv'd in the bodies which we have ſeen. The extenſion, hardneſs, impenetrability, mobility, and *vis inertiæ* of the whole, reſult from the extenſion

ſion, hardneſs, impenetrability, mobility, and *vires inertiæ* of the parts: and thence we conclude the leaſt particles of all bodies to be alſo all extended, and hard, and impenetrable, and moveable, and endow'd with their proper *vires inertiæ*. And this is the foundation of all philoſophy. Moreover, that the divided but contiguous particles of bodies may be ſeparated from one another, is matter of obſervation; and, in the particles that remain undivided, our minds are able to diſtinguiſh yet leſſer parts, as is mathematically demonſtrated. But whether the parts ſo diſtinguiſh'd, and not yet divided, may, by the powers of nature, be actually divided and ſeparated from one another, we cannot certainly determine. Yet had we the proof of but one experiment, that any undivided particle, in breaking a hard and ſolid body, ſuffer'd a diviſion, we might by virtue of this rule, conclude, that the undivided as well as the divided particles, may be divided and actually ſeparated to infinity.

Laſtly, If it univerſally appears, by experiments and aſtronomical obſervations, that all bodies about the Earth, gravitate towards the Earth; and that in proportion to the quantity of matter which they ſeverally contain; that the Moon likewiſe, according to the quantity of its matter, gravitates towards the Earth; that on the other hand our Sea gravitates towards the Moon; and all the Planets mutually one towards another; and the Comets in like manner towards the Sun; we muſt, in conſequence of this rule, univerſally allow, that all bodies whatſoever are endow'd with a principle of mutual gravitation. For the argument from the appearances concludes with more force for the univerſal gravitation of all bodies, than for their impenetrability; of which among thoſe in the celeſtial regions, we have no experiments, nor any manner of obſervation. Not that I affirm gravity to be eſſential to bodies. By their *vis inſita* I mean nothing but their *vis inertiæ*. This

is immutable. Their gravity is diminiſhed as they recede from the Earth.

RULE IV.

In experimental philoſophy we are to look upon propoſitions collected by general induction from phænomena as accurately or very nearly true, notwithſtanding any contrary hypotheſes that may be imagined, till ſuch time as other phænomena occur, by which they may either be made more accurate, or liable to exceptions.

This rule we muſt follow that the argument of induction may not be evaded by hypotheſes.

THE

THE

PHÆNOMENA or APPEARANCES.

PHÆNOMENON I.

That the circumjovial planets, by radij drawn to Jupiter's center, deſcribe areas proportional to the times of deſcription, and that their periodic times, the fixed Stars being at reſt, are in the ſeſquiplicate proportion of their diſtances from its center.

THIS we know from aſtronomical obſervations. For the orbits of theſe planets differ but inſenſibly from circles concentric to Jupiter; and their motions in thoſe circles are found to be uniform. And all aſtronomers agree, that their periodic times are in the ſeſquiplicate proportion of the ſemidiameters of their orbits: and ſo it manifeſtly appears from the following table.

The periodic times of the Satellites of Jupiter.

$1^{d}.18^{h}.27'.34''$. $3^{d}.13^{h}.13'.42''$. $7^{d}.3^{h}.42'.36''$. $16^{d}.16^{h}.32'.9''$.

The

The diſtances of the Satellites from Jupiter's center.

From the obſervations of	1	2	3	4	
Borelli	$5\frac{2}{3}$	$8\frac{4}{7}$	14	$24\frac{2}{3}$	*ſemidiam. of Jupiter.*
Townley *by the Microm.*	5, 52	8,78	13,47	24,72	
Caſſini *by the Teleſcope.*	5	8	13	23	
Caſſini *by the eclip. of the ſatel.*	$5\frac{2}{3}$	9	$14\frac{21}{60}$	$25\frac{1}{10}$	
From the periodic times.	5,667	9,017	14,384	25,299	

Mr. *Pound* has determined by the help of excellent micrometers, the diameters of Jupiter and the elongation of its ſatellites after the following manner. The greateſt heliocentric elongation of the fourth ſatellite from Jupiter's centre was taken with a micrometer in a 15 foot teleſcope, and at the mean diſtance of Jupiter from the Earth was found about 8′. 16″. The elongation of the third ſatellite was taken with a micrometer in a teleſcope of 123 feet, and at the ſame diſtance of Jupiter from the Earth was found 4′. 42″. The greateſt elongations of the other ſatellites at the ſame diſtance of Jupiter from the Earth, are found from the periodic times to be 2′. 56″. 47‴. and 1′. 51″. 6‴.

The diameter of Jupiter taken with the micrometer in an 123 foot teleſcope ſeveral times, and reduced to Jupiter's mean diſtance from the Earth, proved always leſs than 40″, never leſs than 38″, generally 39″. This diameter in ſhorter teleſcopes is 40″, or 41″. For Jupiter's light is a little dilated by the unequal refrangibility of the rays, and this dilatation bears a leſs ratio to the diameter of Jupiter in the longer and more perfect teleſcopes, than in thoſe which are ſhorter and leſs perfect. The times in which two ſatellites, the firſt and the third, paſſed over Jupiter's body, were obſerved, from the beginning of the ingreſs to the beginning of the egreſs, and from the complete ingreſs to the complete egreſs, with the long teleſcope. And from the tranſit of the firſt ſatellite, the diameter of Jupiter at its mean diſtance

diſtance from the Earth, came forth $37\frac{1}{8}''$, and from the tranſit of the third $37\frac{1}{8}''$. There was obſerved alſo the time in which the ſhadow of the firſt ſatellite paſs'd over Jupiter's body, and thence the diameter of Jupiter at its mean diſtance from the Earth came out about $37''$. Let us ſuppoſe its diameter to be $37\frac{1}{4}''$ very nearly, and then the greateſt elongations of the firſt, ſecond, third and fourth ſatellite will be reſpectively equal to 5,965, 9,494, 15,141, and 26,63 ſemidiameters of Jupiter.

PHÆNOMENON. II.

That the circumſaturnal planets, by radij drawn to Saturn's center, deſcribe areas proportional to the times of deſcription, and that their periodic times, the fixed Stars being at reſt, are in the ſeſquiplicate proportion of their diſtances from its centre.

For as *Caſſini* from his own obſervations has determin'd, their diſtances from Saturn's centre, and their periodic times are as follow.

The periodic times of the ſatellites of Saturn.

$1^d.21^h.18'.27''.$ $2^d.17^h.41'.22''.$ $4^d.12^h.25'.12''.$ $15^d.22^h.41'.14''.$ $79^d.7^h.48'.00''.$

The diſtances of the ſatellites from Saturn's center, in ſemidiameters of its Ring.

From obſervations	$1\frac{12}{20}.$	$2\frac{1}{2}.$	$3\frac{1}{2}.$	8.	24.
From the periodic times.	1, 93.	2, 47.	3, 45.	8.	23, 35.

The greateſt elongation of the fourth ſatellite from Saturn's centre is commonly determined from the obſervations to be eight of thoſe ſemidiameters very nearly.

ly. But the greateſt elongation of this ſatellite from Saturn's centre, when taken with an excellent micrometer in M. *Huygens*'s teleſcope of 123 feet, appeared to be eight ſemidiameters and $\frac{2}{10}$ of a ſemidiameter. And from this obſervation and the periodic times, the diſtances of the ſatellites from Saturn's centre in ſemidiameters of the Ring are 2,1. 2,69. 3,75. 8,7. and 25,35. The diameter of Saturn obſerved in the ſame teleſcope was found to be to the diameter of the Ring as 3 to 7, and the diameter of the Ring, *May* 28, 29. 1719. was found to be 43″. And thence the diameter of the Ring when Saturn is at its mean diſtance from the Earth is 42″, and the diameter of Saturn 18″. Theſe things appear ſo in very long and excellent teleſcopes, becauſe in ſuch teleſcopes the apparent magnitudes of the heavenly bodies bear a greater proportion to the dilatation of light in the extremities of thoſe bodies, than in ſhorter teleſcopes. If we then reject all the ſpurious light, the diameter of Saturn will not amount to more than 16″.

PHÆNOMENON III.

That the five primary Planets, Mercury, Venus, Mars, Jupiter and Saturn, with their ſeveral orbits, encompaſs the Sun.

That Mercury and Venus revolve about the Sun, is evident from their moon-like appearances. When they ſhine out with a full face, they are in reſpect of us, beyond or above the Sun; when they appear half-full, they are about the ſame height on one ſide or other of the Sun; when horn'd, they are below or between, us and the Sun, and they are ſometimes, *when directly under*, ſeen like ſpots traverſing the Sun's diſk. That Mars ſurrounds the Sun, is as plain from its full face when near its conjunction with the Sun, and from the gibboſe figure which it ſhews in its quadratures. And the

ſame thing is demonſtrable of Jupiter and Saturn, from their appearing full in all ſituations; for the ſhadows of their ſatellites that appear ſometimes upon their disks make it plain that the light they ſhine with, is not their own, but borrowed from the Sun.

PHÆNOMENON IV.

That the fixed Stars being at reſt, the periodic times of the five primary Planets, and (whether of the Sun about the Earth, or) of the Earth about the Sun, are in the ſeſquiplicate proportion of their mean diſtances from the Sun.

This proportion, firſt obſerv'd by *Kepler*, is now receiv'd by all aſtronomers. For the periodic times are the ſame, and the dimenſions of the orbits are the ſame, whether the Sun revolves about the Earth, or the Earth about the Sun. And as to the meaſures of the periodic times, all aſtronomers are agreed about them. But for the dimenſions of the orbits, *Kepler* and *Bullialdus*, above all others, have determin'd them from obſervations with the greateſt accuracy: and the mean diſtances correſponding to the periodic times, differ but inſenſibly from thoſe which they have aſſign'd, and for the moſt part fall in between them; as we may ſee from the following Table.

The periodic times, with reſpect to the fixed Stars, of the Planets and Earth revolving *about the Sun, in days and decimal parts of a day.*

♄	♃	♂	☿	♀	☾
10759, 275.	4332, 514.	686, 9785.	365, 2565.	224, 6176.	27, 3216.

The

The mean diſtances of the Planets and of the Earth from the Sun.

	☿	♀	♂
According to *Kepler*	951000.	519650.	152350.
To *Bullialdus*	954198.	522520.	152350.
To the periodic Times	954006.	520096.	152369.

	♃	♀	☿
According to *Kepler*	100000.	72400.	38806.
To *Bullialdus*	100000.	72398.	38585.
To the periodic times	100000.	72333.	38710.

As to Mercury and Venus, there can be no doubt about their diſtances from the Sun; for they are determin'd by the elongations of thoſe Planets from the Sun. And for the diſtances of the ſuperior Planets, all diſpute is cut off by the eclipſes of the ſatellites of Jupiter. For, by thoſe eclipſes, the poſition of the ſhadow, which Jupiter projects, is determin'd; whence we have the heliocentric longitude of Jupiter. And from its heliocentric and geocentric longitudes compar'd together, we determine its diſtance.

PHÆNOMENON V.

Then the primary Planets, by radij drawn to the Earth, deſcribe areas no wiſe proportional to the times: But that the areas, which they deſcribe by radij drawn to the Sun, are proportional to the times of deſcription.

For to the Earth they appear ſometimes direct, ſometimes ſtationary, nay and ſometimes retrograde. But from the Sun they are always ſeen direct, and to proceed with a motion nearly uniform, that is to ſay, a little ſwifter in the perihelion and a little ſlower in the

aphelion diſtances, ſo as to maintain an equality in the deſcription of the areas. This is a noted propoſition among aſtronomers, and particularly demonſtrable in Jupiter, from the eclipſes of his ſatellites; by the help of which eclipſes, as we have ſaid, the heliocentric longitudes of that Planet, and its diſtances from the Sun are determined.

PHÆNOMENON VI.

That the Moon by a radius drawn to the Earth's centre, deſcribes an area proportional to the time of deſcription.

This we gather from the apparent motion of the Moon, compar'd with its apparent diameter. It is true that the motion of the Moon is a little diſturb'd by the action of the Sun. But in laying down theſe phænomena, I neglect thoſe ſmall and inconſiderable errors.

THE

THE PROPOSITIONS.

PROPOSITION I. THEOREM I.

That the forces by which the circumjovial Planets are continually drawn off from rectilinear motions, and retain'd in their proper orbits, tend to Jupiter's centre; and are reciprocally as the ſquares of the diſtances of the places of thoſe Planets *from that centre.*

THE former part of this propoſition appears from phæn. 1. and prop. 2. or 3. book 1. The latter from phæn. 1. and cor. 6. prop. 4. of the ſame book.

The ſame thing we are to underſtand of the Planets which encompaſs Saturn, by phæn. 2.

Proposition II. Theorem II.

That the forces by which the primary Planets are continually drawn off from rectilinear motions, and retain'd in their proper orbits, tend to the Sun; and are reciprocally as the ſquares of the diſtances of the places of thoſe Planets *from the Sun's centre.*

The former part of the propoſition is manifeſt from phæn. 5. and prop. 2. book 1. The latter from phæn. 4. and *cor.* 6. prop. 4. of the ſame book. But this part of the propoſition is, with great accuracy, demonſtrable from the quieſcence of the aphelion points. For a very ſmall aberration from the *reciprocal* duplicate proportion, would (by cor. 1. prop. 45. book 1.) produce a motion of the apſides, ſenſible enough in every ſingle revolution, and in many of them enormouſly great.

Proposition III. Theorem III.

That the force by which the Moon is retain'd in its orbit, tends to the Earth; and is reciprocally as the ſquare of the diſtance of its place from the Earth's centre.

The former part of the propoſition is evident from phæn. 6. and prop. 2. or 3. book 1. The latter from the very ſlow motion of the Moon's Apogee; which in every ſingle revolution amounting but to 3° 3'. *in conſequentia*, may be neglected. For (by cor. 1. prop. 45. book 1.) it appears, that if the diſtance of the Moon from the Earth's centre, is to the ſemidiameter of the Earth,

Earth, as D to 1; the force, from which ſuch a motion will reſult, is reciprocally as $D^{2\frac{4}{243}}$ *i, e.* reciprocally as the power of D, whoſe exponent is $2\frac{4}{243}$, that is to ſay, in the proportion of the diſtance ſomething greater than reciprocally duplicate, but which comes $59\frac{1}{4}$ times nearer to the duplicate than to the triplicate proportion. But in regard that this motion is owing to the action of the Sun, (as we ſhall afterwards ſhew) it is here to be neglected. The action of the Sun, attracting the Moon from the Earth, is nearly as the Moon's diſtance from the Earth; and therefore (by what we have ſhewed in cor. 2. pr. 45. book 1.) is to the centripetal force of the Moon, as 2 to 357,45, or nearly ſo; that is, as 1 to $178\frac{29}{40}$. And if we neglect ſo inconſiderable a force of the Sun, the remaining force, by which the Moon is retained in its orb, will be reciprocally as D^2. This will yet more fully appear from comparing this force with the force of gravity, as is done in the next propoſition.

Cor. If we augment the mean centripetal force by which the Moon is retained in its orb, firſt in the proportion of $177\frac{29}{40}$ to $178\frac{29}{40}$, and then in the duplicate proportion of the ſemidiameter of the Earth to the mean diſtance of the centres of the Moon and Earth; we ſhall have the centripetal force of the Moon at the ſurface of the Earth; ſuppoſing this force, in deſcending to the Earth's ſurface, continually to increaſe in the reciprocal duplicate proportion of the height.

Proposition IV. Theorem IV.

That the Moon gravitates towards the Earth; and, by the force of gravity is continually drawn off from a rectilinear motion, and retained in its orbit.

The mean diſtance of the Moon from the Earth in the ſyzygies in ſemidiameters of the Earth, is, accor-

ding to *Ptolomy* and moſt Aſtronomers, 59, according to *Vendelin* and *Huygens* 60, to *Copernicus* 60 $\frac{1}{3}$, to *Street* 60 $\frac{2}{3}$, and to *Tycho* 56 $\frac{1}{2}$. But *Tycho*, and all that follow his tables of refraction, making the refractions of the Sun and Moon (altogether againſt the nature of light) to exceed the refractions of the fixt Stars, and that by four or five minutes *near the Horizon*, did thereby increaſe the Moon's *horizontal* parallax by a like number of minutes, that is, by a twelfth, or fifteenth part of the whole parallax. Correct this error, and the diſtance will become about 60 $\frac{1}{2}$ ſemidiameters of the Earth, near to what others have aſſigned. Let us aſſume the mean diſtance of 60 diameters in the ſyzygies; and ſuppoſe one revolution of the Moon, in reſpect of the fixt ſtars, to be completed in $27^{d}. 7^{h}. 43'$, as Aſtronomers have determined; and the circumference of the Earth to amount to 123249600 *Paris* feet, as the *French* have found by menſuration. And now if we imagine the Moon, deprived of all motion, to be let go, ſo as to deſcend towards the Earth with the impulſe of all that force by which (by cor. prop. 3.) it is retained in its orb; it will, in the ſpace of one minute of time, deſcribe in its fall 15 $\frac{1}{12}$ *Paris* feet. This we gather by a calculus, founded either upon prop. 36. book 1. or (which comes to the ſame thing) upon cor. 9. prop. 4. of the ſame book. For the verſed ſine of that arc, which the Moon, in the ſpace of one minute of time, would by its mean motion deſcribe at the diſtance of 60 ſemidiameters of the Earth, is nearly 15 $\frac{1}{12}$ *Paris* feet, or more accurately 15 feet, 1 inch, and 1 line $\frac{4}{9}$. Whereſore, ſince that force, in approaching to the Earth, increaſes in the reciprocal duplicate proportion of the diſtance, and, upon that account, at the ſurface of the Earth, is 60×60 times greater, than at the Moon; a body in our regions, falling with that force, ought, in the ſpace of one minute of time, to deſcribe $60 \times 60 \times 15\frac{1}{12}$ *Paris* feet,

and,

and, in the ſpace of one ſecond of time, to deſcribe 15 $\frac{1}{13}$ of thoſe feet; or more accurately 15 feet, 1 inch, and 1 line $\frac{4}{9}$. And with this very force we actually find that bodies here upon Earth do really deſcend. For a pendulum oſcillating ſeconds in the latitude of *Paris*, will be 3 *Paris* feet, and 8 lines $\frac{1}{2}$ in length, as Mr. *Huygens* has obſerved. And the ſpace which a heavy body deſcribes by falling in one ſecond of time, is to half the length of this pendulum, in the duplicate ratio of the circumference of a circle to its diameter, (as Mr. *Huygens* has alſo ſhewn) and is therefore 15 *Paris* feet, 1 inch, 1 line $\frac{7}{9}$. And therefore the force by which the Moon is retained in its orbit becomes, at the very ſurface of the Earth, equal to the force of gravity which we obſerve in heavy bodies there. And therefore (by rule 1. & 2.) the force by which the Moon is retained in its orbit, is that very ſame force, which we commonly call gravity. For, were gravity another force different from that, then bodies deſcending to the Earth with the joint impulſe of both forces would fall with a double velocity, and in the ſpace of one ſecond of time would deſcribe 30 $\frac{1}{6}$ *Paris* feet; altogether againſt experience.

This calculus is founded on the hypotheſis of the Earth's ſtanding ſtill. For, if both Earth and Moon move about the Sun, and at the ſame time about their common centre of gravity; the diſtance of the centres of the Moon and Earth from one another, will be 60 $\frac{1}{2}$ ſemidiameters of the Earth; as may be found by a computation from prop. 60. book 1.

Scholium.

The demonſtration of this propoſition may be more diffuſely explained after the following manner. Suppoſe ſeveral moons to revolve about the Earth, as in the ſyſtem

ſtem of Jupiter or Saturn; the periodic times of theſe moons (by the argument of induction) would obſerve the ſame law which *Kepler* found to obtain among the Planets; and therefore their centripetal forces would be reciprocally as the ſquares of the diſtances from the centre of the Earth, by prop. 1. of this book. Now if the loweſt of theſe were very ſmall, and were ſo near the Earth as almoſt to touch the tops of the higheſt mountains; the centripetal force thereof, retaining it in its orb, would be very nearly equal to the weights of any *terreſtrial* bodies that ſhould be found upon the tops of thoſe mountains, as may be known by the foregoing computation. Therefore if the ſame little moon ſhould be deſerted by its centrifugal force that carries it through its orb, and ſo be diſabled from going onwards therein, it would deſcend to the Earth; and that with the ſame velocity as heavy bodies do actually fall with, upon the tops of thoſe very mountains; becauſe of the equality of the forces that oblige them both to deſcend. And if the force by which that loweſt moon would deſcend, were different from gravity, and if that moon were to gravitate towards the Earth, as we find terreſtrial bodies do upon the tops of mountains, it would then deſcend with twice the velocity, as being impelled by both theſe forces conſpiring together. Therefore ſince both theſe forces, that is, the gravity of heavy bodies, and the centripetal forces of the moons, reſpect the centre of the Earth, and are ſimilar and equal between themſelves, they will (by rule 1. and 2.) have one and the ſame cauſe. And therefore the force which retains the Moon in its orbit, is that very force which we commonly call gravity; becauſe otherwiſe this little moon at the top of a mountain, muſt either be without gravity, or fall twice as ſwiftly as heavy bodies uſe to do.

Proposition V. Theorem V.

That the circumjovial Planets gravitate towards Jupiter; the circumſaturnal towards Saturn; the circumſolar towards the Sun; and by the forces of their gravity are drawn off from rectilinear motions, and retained in curvilinear orbits.

For the revolutions of the circumjovial Planets about Jupiter, of the circumſaturnal about Saturn, and of Mercury and Venus, and the other circumſolar Planets about the Sun, are appearances of the ſame ſort with the revolution of the Moon about the Earth; and therefore by rule 2. muſt be owing to the ſame ſort of cauſes; eſpecially ſince it has been demonſtrated, that the forces, upon which thoſe revolutions depend, tend to the centres of Jupiter, of Saturn, and of the Sun; and that thoſe forces, in receding from Jupiter, from Saturn, and from the Sun, decreaſe in the ſame proportion, and according to the ſame law, as the force of gravity does in receding from the Earth.

Cor. 1. There is therefore a power of gravity tending to all the Planets. For doubtleſs Venus, Mercury, and the reſt, are bodies of the ſame ſort with Jupiter and Saturn. And ſince all attraction (by law 3.) is mutual, Jupiter will therefore gravitate towards all his own ſatellites, Saturn towards his, the Earth towards the Moon, and the Sun towards all the primary Planets.

Cor. 2. The force of gravity, which tends to any one Planet, is reciprocally as the ſquare of the diſtance of places from that Planet's centre.

Cor. 3. All the Planets do mutually gravitate towards one another, by cor. 1. and 2. And hence

it

it is, that Jupiter and Saturn, when near their conjunction, by their mutual attractions ſenſibly diſturb each other's motions. So the Sun diſturbs the motions of the Moon; and both Sun and Moon diſturb our Sea, as we ſhall hereafter explain.

Scholium.

The force which retains the celeſtial bodies in their orbits, has been hitherto called centripetal force. But it being now made plain, that it can be no other than a gravitating force, we ſhall hereafter call it gravity, For the cauſe of that centripetal force, which retains the Moon in its orbit, will extend it ſelf to all the Planets by rule 1. 2. and 4.

Proposition VI. Theorem VI.

That all bodies gravitate towards every Planet; and that the Weights of bodies towards any the ſame *Planet, at equal diſtances from the centre of the Planet, are proportional to the quantities of matter which they ſeverally contain.*

It has been, now of a long time, obſerved by others, that all ſorts of heavy bodies, (allowance being made for the inequality of retardation, which they ſuffer from a ſmall power of reſiſtance in the air) deſcend to the Earth *from equal heights* in equal times: and that equality of times we may diſtinguiſh to a great accuracy, by the help of pendulums. I tried the thing in gold, ſilver, lead, glaſs, ſand, common ſalt, wood, water, and wheat. I provided two wooden boxes, round and equal.

qual. I filled the one with wood, and ſuſpended an equal weight of gold (as exactly as I could) in the centre of oſcillation of the other. The boxes hanging by equal threads of 11 feet, made a couple of pendulums perfectly equal in weight and figure, and equally receiving the reſiſtance of the air. And placing the one by the other, I obſerved them to play together forwards and backwards, for a long time, with equal vibrations. And therefore the quantity of matter in the gold (by cor. 1. and 6. prop. 24. book 2.) was to the quantity of matter in the wood, as the action of the motive force (or *vis motrix)* upon all the gold, to the action of the ſame upon all the wood; that is, as the weight of the one to the weight of the other. And the like happened in the other bodies. By theſe experiments, in bodies of the ſame weight, I could manifeſtly have diſcovered a difference of matter leſs than the thouſandth part of the whole, had any ſuch been. But, without all doubt, the nature of gravity towards the Planets, is the ſame as towards the Earth. For, ſhould we imagine our terreſtrial bodies removed to the orb of the Moon, and there, together with the Moon, deprived of all motion, to be let go, ſo as to fall together towards the Earth: it is certain, from what we have demonſtrated before, that, in equal times, they would deſcribe equal ſpaces with the Moon, and of conſequence are to the Moon, in quantity of matter, as their weights to its weight. Moreover, ſince the ſatellites of Jupiter perform their revolutions in times which obſerve the ſeſquiplicate proportion of their diſtances from Jupiter's centre, their accelerative gravities towards Jupiter will be reciprocally as the ſquares of their diſtances from Jupiter's centre; that is, equal, at equal diſtances. And therefore, theſe ſatellites, if ſuppoſed to fall *towards Jupiter* from equal heights, would deſcribe equal ſpaces in equal times, in like manner as heavy bodies do on our Earth. And by the

ſame

ſame argument, if the circumſolar Planets were ſuppoſed to be let fall at equal diſtances from the Sun, they would, in their deſcent towards the Sun, deſcribe equal ſpaces in equal times. But forces, which equally accelerate unequal bodies, muſt be as thoſe bodies; that is to ſay, the weights of the Planets *towards the Sun* muſt be as their quantities of matter. Further, that the weights of Jupiter and of his ſatellites towards the Sun are proportional to the ſeveral quantities of their matter, appears from the exceeding regular motions of the ſatellites, (by cor. 3. prop. 65. book 1.) For if ſome of thoſe bodies were more ſtrongly attracted to the Sun in proportion to their quantity of matter, than others; the motions of the ſatellites would be diſturbed by that inequality of attraction (by cor. 2. prop. 65. book 1.) If, at equal diſtances from the Sun, any ſatellite in proportion to the quantity of its matter, did gravitate towards the Sun, with a force greater than Jupiter in proportion to his, according to any given proportion, ſuppoſe of *d* to *e*; then the diſtance between the centres of the Sun and of the ſatellite's orbit would be always greater than the diſtance between the centres of the Sun and of Jupiter, nearly in the ſubduplicate of that proportion; as by ſome computations I have found. And if the ſatellite did gravitate towards the Sun with a force, leſſer in the proportion of *e* to *d*, the diſtance of the centre of the ſatellite's orb from the Sun, would be leſs than the diſtance of the centre of Jupiter from the Sun, in the ſubduplicate of the ſame proportion. Therefore if, at equal diſtances from the Sun, the accelerative gravity of any ſatellite towards the Sun were greater or leſs than the accelerative gravity of Jupiter towards the Sun, but by one $\frac{1}{1000}$ part of the whole gravity; the diſtance of the centre of the ſatellite's orbit from the Sun would be greater or leſs than the diſtance of Jupiter from the Sun, by one $\frac{1}{2000}$ part of the whole diſtance; that is,

by a fifth part of the diſtance of the utmoſt ſatellite from the centre of Jupiter; an excentricity of the orbit, which would be very ſenſible. But the orbits of the ſatellites are concentric to Jupiter, and therefore the accelerative gravities of Jupiter, and of all its ſatellites towards the Sun, are equal among themſelves. And by the ſame argument, the weights of Saturn and of his ſatellites towards the Sun, at equal diſtances from the Sun, are as their ſeveral quantities of matter: and the weights of the Moon and of the Earth towards the Sun, are either none, or accurately proportional to the maſſes of matter which they contain. But ſome they are by cor. 1. and 3. prop. 5.

But further, the weights of all the parts of every Planet towards any other Planet, are one to another as the matter in the ſeveral parts. For if ſome parts did gravitate more, others leſs, than for the quantity of their matter; then the whole Planet, according to the ſort of parts with which it moſt abounds, would gravitate more or leſs, than in proportion to the quantity of matter in the whole. Nor is it of any moment, whether theſe parts are external or internal. For, if, for example, we ſhould imagine the terreſtrial bodies with us to be raiſed up to the orb of the Moon, to be there compared with its body: If the weights of ſuch bodies were to the weights of the external parts of the Moon, as the quantities of matter in the one and in the other reſpectively; but to the weights of the internal parts, in a greater or leſs proportion, then likewiſe the weights of thoſe bodies would be to the weight of the whole Moon, in a greater or leſs proportion; againſt what we have ſhewed above.

Cor. 1. Hence the weights of bodies do not depend upon their forms and textures. For if the weights could be altered with the forms, they would be greater or leſs, according to the variety of forms, in equal matter; altogether againſt experience.

Cor.

Cor. 2. Univerſally, all bodies about the Earth gravitate towards the Earth; and the weights of all, at equal diſtances from the Earth's centre, are as the quantities of matter which they ſeverally contain. This is the quality of all bodies, within the reach of our experiments; and therefore, (by rule 3.) to be affirmed of all bodies whatſoever. If the *æther*, or any other body, were either altogether void of gravity, or were to gravitate leſs in proportion to its quantity of matter; then, becauſe (according to *Ariſtotle*, *Des Cartes*, and others) there is no difference betwixt that and other bodies, but in *mere* form of matter, by a ſucceſſive change from form to form, it might be changed at laſt into a body of the ſame condition with thoſe which gravitate moſt in proportion to their quantity of matter; and, on the other hand, the heavieſt bodies, acquiring the firſt form of that body, might by degrees, quite loſe their gravity. And therefore the weights would depend upon the forms of bodies, and with thoſe forms might be changed, contrary to what was proved in the preceding corollary.

Cor. 3. All ſpaces are not equally Full. For if all ſpaces were equally full, then the ſpecific gravity of the fluid which fills the region of the air, on account of the extreme denſity of the matter, would fall nothing ſhort of the ſpecific gravity of quick-ſilver, or gold, or any other the moſt denſe body; and therefore, neither gold, nor any other body, could deſcend in air. For bodies do not deſcend in fluids, unleſs they are ſpecifically heavier than the fluids. And if the quantity of matter in a given ſpace, can, by any rarefaction, be diminiſhed, what ſhould hinder a diminution to infinity?

Cor. 4. If all the ſolid particles of all bodies are of the ſame denſity, nor can be rarified without pores a void ſpace or vacuum muſt be granted. By bodies

of

of the ſame denſity, I mean thoſe, whoſe *vires inertiæ* are in the proportion of their bulks.

Cor. 5. The power of gravity is of a different nature from the power of magnetiſm. For the magnetic attraction is not as the matter attracted. Some bodies are attracted more by the magnet, others leſs; moſt bodies not at all. The power of magnetiſm, in one and the ſame body, may be increaſed and diminiſhed; and is ſometimes far ſtronger, for the quantity of matter, than the power of gravity; and in receding from the magnet, decreaſes not in the duplicate, but almoſt in the triplicate proportion of the diſtance, as nearly as I could judge from ſome rude obſervations.

Proposition VII. Theorem VII.

That there is a power of gravity tending to all bodies, proportional to the ſeveral quantities of matter which they contain.

That all the Planets mutually gravitate one towards another, we have prov'd before; as well as that the force of gravity towards every one of them, conſider'd apart, is reciprocally as the ſquare of the diſtance of places from the centre of the planet. And thence (by prop. 69. book. 1. and its corollaries) it follows, that the gravity tending towards all the Planets, is proportional to the matter which they contain.

Moreover, ſince all the parts of any planet *A* gravitate towards any other planet *B*; and the gravity of every part is to the gravity of the whole, as the matter of the part to the matter of the whole; and (by law 3.) to every action correſponds an equal re-action: therefore the planet *B* will, on the other hand, gravitate towards all the parts of the planet *A*; and its gravity towards any one part will be to the gravity towards the

whole, as the matter of the part to the matter of the whole. *Q. E. D.*

Cor. 1. Therefore the force of gravity towards any whole planet, ariſes from, and is compounded of, the forces of gravity towards all its parts. Magnetic and electric attractions afford us examples of this. For all attraction towards the whole ariſes from the attractions towards the ſeveral parts. The thing may be eaſily underſtood in gravity, if we conſider a greater planet, as form'd of a number of leſſer planets, meeting together in one globe. For *hence it would appear that* the force of the whole muſt ariſe from the forces of the component parts. If it is objected, that, according to this law, all bodies with us muſt mutually gravitate one towards another, whereas no ſuch gravitation any where appears: I anſwer, that ſince the gravitation towards theſe bodies is to the gravitation towards the whole Earth, as theſe bodies are to the whole Earth, the gravitation towards them muſt be far leſs than to fall under the obſervation of our ſenſes.

Cor. 2. The force of gravity towards the ſeveral equal particles of any body, is reciprocally as the ſquare of the diſtance of places from the particles; as appears from cor. 3. prop. 74. book 1.

Proposition VIII. Theorem VIII.

In two ſpheres mutually gravitating each towards the other, if the matter in places on all ſides round about and equidiſtant from the centres, is ſimilar; the weight of either ſphere towards the other, will be reciprocally as the ſquare of the diſtance between their centres.

After I had found that the force of gravity towards a whole planet did ariſe from, and was compounded of the

the forces of gravity towards all its parts; and towards every one part, was in the reciprocal proportion of the ſquares of the diſtances from the part: I was yet in doubt, whether that reciprocal duplicate proportion did accurately hold, or but nearly ſo, in the total force compounded of ſo many partial ones. For it might be that the proportion which accurately enough took place in greater diſtances, ſhould be wide of the truth near the ſurface of the planet, where the diſtances of the particles are unequal, and their ſituation diſſimilar. But by the help of prop. 75. and 76. book 1, and their corollaries, I was at laſt ſatisfy'd of the truth of the propoſition, as it now lies before us.

Cor. 1. Hence we may find and compare together the weights of bodies towards different planets. For the weights of bodies revolving in circles about planets, are (by cor. 2. prop. 4. book 1.) as the diameters of the circles directly, and the ſquares of their periodic times reciprocally; and their weights at the ſurfaces of the planets, or at any other diſtances from their centres, are (by this prop.) greater or leſs, in the reciprocal duplicate proportion of the diſtances. Thus from the periodic times of Venus, revolving about the Sun, in $224^{d}. 16\frac{1}{4}^{h}$. of the utmoſt circumjovial ſatellite revolving about Jupiter, in $16^{d}. 16\frac{8}{15}^{h}$; of the Hugenian ſatellite about Saturn in $15^{d}. 22\frac{2}{3}^{h}$; and of the Moon about the Earth in $27^{d}. 7^{h}. 43'$; compared with the mean diſtance of Venus from the Sun, and with the greateſt heliocentric elongations of the outmoſt circumjovial ſatellite from Jupiter's centre, 8', 16". of the Hugenian ſatellite from the centre of Saturn, 3'. 4", and of the Moon from the Earth, 10'. 33"; by computation I found, that the weight of equal bodies, at equal diſtances from the centres of the Sun, of Jupiter, of Saturn, and of the Earth, towards the Sun, Jupiter, Saturn, and the Earth, were one to another, as 1, $\frac{1}{1067}$, $\frac{1}{3021}$, and $\frac{1}{169282}$ reſpectively. Then becauſe as

the diſtances are increaſed or diminiſhed, the weights are diminiſhed or increaſed in a duplicate ratio; the weights of equal bodies towards the Sun, Jupiter, Saturn, and the Earth, at the diſtances 10000, 997, 791 and 109 from their centres, that is, at their very ſuperficies, will be as 10000, 943, 529 and 435 reſpectively. How much the weights of bodies are at the ſuperficies of the Moon, will be ſhewn hereafter.

Cor. 2. Hence likewiſe we diſcover the quantity of matter in the ſeveral Planets. For their quantities of matter are as the forces of gravity at equal diſtances from their centres, that is, in the Sun, Jupiter, Saturn, and the Earth, as 1, $\frac{1}{1067}$, $\frac{1}{3021}$, and $\frac{1}{169282}$ reſpectively. If the parallax of the Sun be taken greater or leſs than 10″, 30‴, the quantity of matter in the Earth muſt be augmented or diminiſhed in the triplicate of that proportion.

Cor. 3. Hence alſo we find the denſities of the Planets. For (by prop. 72. book 1.) the weights of equal and ſimilar bodies towards ſimilar ſpheres, are, at the ſurfaces of thoſe ſpheres, as the diameters of the ſpheres. And therefore the denſities of diſſimilar ſpheres are as thoſe weights applied to the diameters of the ſpheres. But the true diameters of the Sun, Jupiter, Saturn, and the Earth, were one to another as 10000, 997, 791 and 109; and the weights towards the ſame, as 10000, 943, 529, and 435 reſpectively; and therefore their denſities are as 100, 94½, 67 and 400. The denſity of the Earth, which comes out by this computation, does not depend upon the parallax of the Sun, but is determined by the parallax of the Moon, and therefore is here truly defin'd. The Sun therefore is a little denſer than Jupiter, and Jupiter than Saturn, and the Earth four times denſer than the Sun; for the Sun, by its great heat, is kept in a ſort of a rareſy'd ſtate. The Moon is denſer than the Earth, as ſhall appear afterwards.

Cor.

Cor. 4. The ſmaller the Planets are, they are, *cæteris paribus*, of ſo much the greater denſity. For ſo the powers of gravity on their ſeveral ſurfaces, come nearer to equality. They are likewiſe, *cæteris paribus*, of the greater denſity, as they are nearer to the Sun. So Jupiter is more denſe than Saturn, and the Earth than Jupiter. For the Planets were to be placed at different diſtances from the Sun, that according to their degrees of denſity, they might enjoy a greater or leſs proportion of the Sun's heat. Our water, if it were remov'd as far as the orb of Saturn, would be converted into ice, and in the orb of Mercury would quickly fly away in vapour. For the light of the Sun, to which its heat is proportional, is ſeven times denſer in the orb of the Mercury than with us: and by the thermometer I have found, that a ſevenfold heat of our ſummer-ſun will make water boil. Nor are we to doubt, that the matter of Mercury is adapted to its heat, and is therefore more denſe than the matter of our Earth; ſince, in a denſer matter, the operations of nature require a ſtronger heat.

Proposition IX. Theorem IX.

That the force of gravity, conſider'd downwards from the ſurface of the planets, decreaſes nearly in the proportion of the diſtances from their centres.

If the matter of the planet were of an uniform denſity, this propoſition would be accurately true, (by prop. 73. book 1.) The error therefore can be no greater than what may ariſe from the inequality of the denſity.

Proposition X. Theorem X.

That the motions of the Planets in the heavens may ſubſiſt an exceeding long time.

In the ſcholium of prop. 40. book 2. I have ſhew'd that a globe of water, frozen into ice, and moving freely in our air, in the time that it would deſcribe the length of its ſemidiameter, would loſe by the reſiſtance of the air $\frac{1}{4586}$ part of its motion. And the ſame proportion holds nearly in all globes, how great ſoever, and mov'd with whatever velocity. But that our globe of earth is of greater denſity than it would be if the whole conſiſted of water only, I thus make out. If the whole conſiſted of water only, whatever was of leſs denſity than water, becauſe of its leſs ſpecific gravity, wou'd emerge and float above. And upon this account, if a globe of terreſtrial matter, cover'd on all ſides with water, was leſs denſe than water, it would emerge ſomewhere; and the ſubſiding water falling back, would be gathered to the oppoſite ſide. And ſuch is the condition of our Earth, which in a great meaſure is covered with ſeas. The Earth, if it was not for its greater denſity, would emerge from the ſeas, and, according to its degree of levity, would be raiſed more or leſs above their ſurface, the water of the ſeas flowing backwards to the oppoſite ſide. By the ſame argument, the ſpots of the Sun, which float upon the lucid matter thereof, are lighter than that matter. And however the Planets have been form'd, while they were yet in flu d maſſes, all the heavier matter ſubſided to the centre. Since therefore the common matter of our Earth on the ſurface thereof, is about twice as heavy as water, and a little lower, in mines, is found about three or four, or even five times more heavy; it is probable, that the quantity of the whole matter of the Earth may be

be five or ſix times greater than if it conſiſted all of water; eſpecially ſince I have before ſhew'd, that the Earth is about four times more denſe than Jupiter. If therefore Jupiter is a little more denſe than water, in the ſpace of thirty days, in which that planet deſcribes the length of 459 of its ſemidiameters, it would, in a medium of the ſame denſity with our air, loſe almoſt a tenth part of its motion. But ſince the reſiſtance of mediums decreaſes in proportion to their weight or denſity, ſo that water, which is $13\frac{4}{5}$ times lighter than quickſilver, reſiſts leſs in that proportion; and air, which is 860 times lighter than water, reſiſts leſs in the ſame proportion: Therefore in the heavens, where the weight of the medium, in which the Planets move, is immenſely diminiſhed, the reſiſtance will almoſt vaniſh.

It is ſhewn in the ſcholium of prop. 22. book 2. that at the height of 200 miles above the Earth, the air is more rare than it is at the ſuperficies of the Earth, in the ratio of 30 to 0,00000000000003998, or as 7500000000000 to 1 nearly. And hence the planet Jupiter, revolving in a medium of the ſame denſity with that ſuperior air, would not loſe by the reſiſtance of the medium the 10000000th part of its motion in 1000000 years. In the ſpaces near the Earth, the reſiſtance is produced only by the air, exhalations and vapours. When theſe are carefully exhauſted by the air pump from under the receiver, heavy bodies fall within the receiver with perfect freedom, and without the leaſt ſenſible reſiſtance; gold itſelf and the lighteſt down, let fall together, will deſcend with equal velocity; and though they fall through a ſpace of four, ſix, and eight feet, they will come to the bottom at the ſame time; as appears from experiments. And therefore the celeſtial regions being perfectly void of air and exhalations, the Planets and Comets meeting no ſenſible reſiſtance in thoſe

ſpaces, will continue their motions through them for an immenſe tract of time.

Hypothesis I.

That the centre of the ſyſtem of the world is immoveable.

This is acknowledg'd by all, while ſome contend that the Earth, others, that the Sun is fix'd in that centre. Let us ſee what may from hence follow.

Proposition XI. Theorem XI.

That the common centre of gravity of the Earth, the Sun, and all the Planets is immoveable.

For (by cor. 4. of the laws) that centre either is at reſt, or moves uniformly forward in a right line. But if that centre mov'd, the centre of the world would move alſo, againſt the hypotheſis.

Proposition XII. Theorem XII.

That the Sun is agitated by a perpetual motion, but never recedes far from the common centre of gravity of all the Planets.

For ſince (by cor. 2. prop. 8.) the quantity of matter in the Sun, is to the quantity of matter in Jupiter, as 1067 to 1: and the diſtance of Jupiter from the Sun, is to the ſemidiameter of the Sun, in a proportion but a ſmall matter greater; the common centre of gravity of Jupiter and the Sun, will fall upon a point a little without the ſurface of the Sun. By the ſame argument, ſince the quantity of matter in the Sun is to the quantity of matter in Saturn as 3021 to 1; and the

the diſtance of Saturn from the Sun is to the ſemidiameter of the Sun in a proportion but a ſmall matter leſs; the common centre of gravity of Saturn and the Sun will fall upon a point a little within the ſurface of the Sun. And purſuing the principles of this computation, we ſhould find that tho' the Earth and all the Planets were plac'd on one ſide of the Sun, the diſtance of the common centre of gravity of all from the centre of the Sun would ſcarcely amount to one diameter of the Sun. In other caſes, the diſtances of thoſe centres is always leſs. And therefore, ſince that centre of gravity is in perpetual reſt, the Sun, according to the various poſitions of the Planets, muſt perpetually be moved every way, but will never recede far from that centre.

Cor. Hence the common centre of gravity of the Earth, the Sun, and all the Planets is to be eſteem'd the Centre of the World. For ſince the Earth, the Sun and all the Planets, mutually gravitate one towards another, and are therefore, according to their powers of gravity, in perpetual agitation, as the laws of motion require; it is plain that their moveable centres cannot be taken for the immoveable centre of the world. If that body were to be plac'd in the centre, towards which other bodies gravitate moſt, (according to common opinion) that privilege ought to be allow'd to the Sun. But ſince the Sun it ſelf is mov'd, a fixt point is to be choſen, from which the centre of the Sun recedes leaſt, and from which it would recede yet leſs, if the body of the Sun were denſer and greater, and therefore leſs apt to be mov'd.

Proposition XIII. Theorem XIII.

The Planets move in ellipſes which have their common focus in the centre of the Sun; and, by radij drawn to that centre, they deſcribe areas proportional to the times of deſcription.

We have diſcours'd above of theſe motions from the phænomena. Now that we know the principles on which they depend, from thoſe principles we deduce the motions of the heavens *a priori*. Becauſe the weights of the Planets towards the Sun, are reciprocally as the ſquares of their diſtances from the Sun's centre; if the Sun was at reſt, and the other Planets did not mutually act one upon another, their orbits would be ellipſes, having the Sun in their common focus; and they would deſcribe areas proportional to the times *of deſcription* by prop. 1 & 11. and cor. 1. prop. 13. book 1. But the mutual actions of the Planets one upon another, are ſo very ſmall, that they may be neglected. And by prop. 66. book 1. they leſs diſturb the motions of the Planets around the Sun in motion, than if thoſe motions were perform'd about the Sun at reſt.

It is true, that the action of Jupiter upon Saturn is not to be neglected. For the force of gravity towards Jupiter is to the force of gravity towards the Sun as 1 to 1067; and therefore in the conjunction of Jupiter and Saturn, becauſe the diſtance of Saturn from Jupiter is to the diſtance of Saturn from the Sun, almoſt as 4 to 9; the gravity of Saturn towards Jupiter, will be to the gravity of Saturn towards the Sun, as 81 to 16 × 1067; or, as 1 to about 211. And hence ariſes a perturbation of the orb of Saturn in every conjunction of this Planet with Jupiter, ſo ſenſible that aſtrono-

mers are puzled with it. As the Planet is differently ſituated in theſe conjunctions, its excentricity is ſometimes augmented, ſometimes diminiſh'd; its aphelion is ſometimes carry'd forwards, ſometimes backwards, and its mean motion is by turns accelerated and retarded. Yet the whole error in its motion about the Sun, tho' ariſing from ſo great a force, may be almoſt avoided (except in the mean motion) by placing the lower focus of its orbit in the common centre of gravity of Jupiter and the Sun, (according to prop. 67. book 1.) and therefore that error when it is greateſt, ſcarcely exceeds two minutes. And the greateſt error in the mean motion, ſcarcely exceeds two minutes yearly. But in the conjunction of Jupiter and Saturn, the accelerative forces of gravity of the Sun towards Saturn, of Jupiter towards Saturn, and of Jupiter toward the Sun, are almoſt as 16, 81 and $\frac{16\times 81\times 3021}{25}$ or 156609; and therefore the difference of the forces of gravity of the Sun towards Saturn, and of Jupiter towards Saturn, is to the force of gravity of Jupiter towards the Sun, as 65 to 156609, or as 1 to 2409. But the greateſt power of Saturn to diſturb the motion of Jupiter is proportional to this difference; and therefore the perturbation of the orbit of Jupiter is much leſs than that of Saturn's. The perturbations of the other orbits are yet far leſs, except that the orbit of the Earth is ſenſibly diſturb'd by the Moon. The common centre of gravity of the Earth and Moon moves in an ellipſe about the Sun in the focus thereof, and by a radius drawn to the Sun, deſcribes areas proportional to the times of deſcription. But the Earth in the mean time by a menſtrual motion is revolv'd about this common centre.

PRO-

Proposition XIV. Theorem XIV.

The aphelions and nodes of the orbits of the Planets are fixt.

The aphelions are immoveable, by prop. 11. book 1. and ſo are the planes of the orbits by prop. 1. of the ſame book. And if the planes are fixt, the nodes muſt be ſo too. It is true, that ſome inequalities may ariſe from the mutual actions of the Planets and Comets in their revolutions. But theſe will be ſo ſmall that they may be here paſs'd by.

Cor. 1. The fixt Stars are immoveable, ſeeing they keep the ſame poſition to the aphelions and nodes of the Planets.

Cor. 2. And ſince theſe Stars are liable to no ſenſible parallax from the annual motion of the Earth, they can have no force, becauſe of their immenſe diſtance, to produce any ſenſible effect in our ſyſtem. Not to mention, that the fixt Stars, every where promiſcuouſly diſpers'd in the heavens, by their contrary attractions deſtroy their mutual actions, by prop. 70. book 1.

Scholium.

Since the Planets near the Sun (*viz.* Mercury, Venus, the Earth and Mars) are ſo ſmall that they can act but with little force upon each other; therefore their aphelions and nodes muſt be fixt, excepting in ſo far as they are diſturb'd by the actions of Jupiter and Saturn, and other higher bodies. And hence we may find, by the theory of gravity, that their aphelions move a little *in conſequentia*, in reſpect of the fixed Stars, and that in the ſeſquiplicate proportion of their ſeveral

ſeveral diſtances from the Sun. So that if the aphelion of Mars, in the ſpace of an hundred years, is carried 33′. 20″. *in conſequentia*, in reſpect of the fixed Stars; the aphelions of the Earth, of Venus, and of Mercury, will in an hundred years be carried forwards 17′. 40″. 10′. 53″. and 4′. 16″. reſpectively. But theſe motions are ſo inconſiderable, that we have neglected them in this propoſition.

Proposition XV. Theorem I.

To find the principal diameters of the orbits of the Planets.

They are to be taken in the ſubſeſquiplicate proportion of the periodic times by prop. 15. book 1. and then to be ſeverally augmented in the proportion of the ſum of the maſſes of matter in the Sun and each Planet to the firſt of two mean proportionals betwixt that ſum and the quantity of matter in the Sun, by prop. 60. book 1.

Proposition XVI. Problem II.

To find the eccentricities and aphelions of the Planets.

This problem is reſolved by prop. 18. book 1.

Proposition XVII. Theorem XV.

That the diurnal motions of the Planets are uniform, and that the libration of the Moon ariſes from its diurnal motion.

The propoſition is prov'd from the firſt law of motion, and cor. 22. prop. 66. book 1. Jupiter, with reſpect to the fixed Stars, revolves in 9^h. 56'. Mars in 24^h. 39'. Venus in about 23^h. the Earth in 23^h. 56'. the Sun in 25 ½ days, and the Moon in 27 days 7 hours 43'. Theſe things appear by the phænomena. The ſpots in the Sun's body return to the ſame ſituation on the Sun's disk, with reſpect to the Earth in 27 ½ days; and therefore with reſpect to the fixed Stars the Sun revolves in about 25 ½ days. But becauſe the lunar day, ariſing from its uniform revolution about its axe, is menſtrual, *that is, equal to the time of its periodic revolution in its orb*, therefore the ſame face of the Moon will be always nearly turned to the upper focus of its orb; but, as the ſituation of that focus requires, will deviate a little, to one ſide and to the other, from the Earth in the lower focus; and this is the libration in longitude. For the libration in latitude ariſes from the Moon's latitude, and the inclination of its axis to the plane of the ecliptic. This theory of the libration of the Moon, Mr. *N. Mercator* in his aſtronomy, publiſhed at the beginning of the Year 1676, explained more fully out of the letters I ſent him. The utmoſt ſatellite of Saturn ſeems to revolve about its axis with a motion like this of the Moon, reſpecting Saturn continually with the ſame face. For in its revolution round Saturn, as often as it comes to the eaſtern part of its orbit, it is ſcarcely viſible, and generally quite diſappears; which is like to be occaſioned by

by ſome ſpots in that part of its body, which is then turned toward the Earth, as M. *Caſſini* has obſerved. So alſo the utmoſt ſatellite of Jupiter ſeems to revolve about its axis with a like motion, becauſe in that part of its body which is turned from Jupiter, it has a ſpot, which always appears as if it were in Jupiter's own body, whenever the ſatellite paſſes between Jupiter and our eye.

PROPOSITION XVIII. THEOREM XVI.

That the axes of the Planets are leſs than the diameters drawn perpendicular to the axes.

The equal gravitation of the parts on all ſides would give a ſphærical figure to the Planets, if it was not for their diurnal revolution in a circle. By that circular motion it comes to paſs that the parts receding from the axe endeavour to aſcend about the equator. And therefore if the matter is in a fluid ſtate, by its aſcent towards the equator it will enlarge the diameters there, and by its deſcent towards the poles it will ſhorten the axe. So the diameter of Jupiter, (by the concurring obſervations of aſtronomers) is found ſhorter betwixt pole and pole, than from eaſt to weſt. And by the ſame argument, if our Earth was not higher about the equator than at the poles, the Seas would ſubſide about the poles, and riſing towards the equator, would lay all things there under water.

PROPOSITION XIX. PROBLEM III.

To find the proportion of the axe of a Planet to the diameters perpendicular thereto.

Our countryman Mr. *Norwood*, meaſuring a diſtance of 905751 feet of *London* meaſure between *London* and *York*,

York in 1635, and obſerving the difference of latitudes to be 2°. 28′, determined the meaſure of one degree to be 367196 feet of *London* meaſure, that is 57300 *Paris* toiſes. M. *Picart* meaſuring an arc of one degree, and 22′. 55″. of the meridian between *Amiens* and *Malvoiſine*, found an arc of one degree to be 57060 *Paris* toiſes. M. *Caſſini* the father meaſured the diſtance upon the meridian from the town of *Collioure* in *Rouſſillon* to the obſervatory of *Paris:* And his ſon added the diſtance from the obſervatory to the citadel of *Dunkirk*. The whole diſtance was 486156 $\frac{1}{2}$ toiſes, and the difference of the latitudes of *Collioure* and *Dunkirk* was 8 degrees, and 31′. 11 $\frac{1}{6}$″. Hence an arc of one degree appears to be 57061 *Paris* toiſes. And from theſe meaſures we conclude, that the circumference of the Earth is 123249600, and its ſemidiameter 19615800 *Paris* feet, upon the ſuppoſition that the Earth is of a ſphærical figure.

In the latitude of *Paris* a heavy body falling in a ſecond of time, deſcribes 15 *Paris* feet, 1 inch, 1 line as above, that is, 2173 lines $\frac{7}{9}$. The weight of the body is diminiſhed by the weight of the ambient air. Let us ſuppoſe the weight loſt thereby to be $\frac{1}{1\,1\,0\,0\,0}$ part of the whole weight; then that heavy body falling *in vacuo* will deſcribe a height of 2174 lines in one ſecond of time.

A body in every ſidereal day of 23^{h}. 56′. 4″. uniformly revolving in a circle at the diſtance of 19615800 feet from the centre, in one ſecond of time deſcribes an arc of 1433, 46 feet; the verſed ſine of which is 0,05236561 feet, or 7,54064 lines. And therefore the force with which bodies deſcend in the latitude of *Paris* is to the centrifugal force of bodies in the equator ariſing from the diurnal motion of the Earth, as 2174 to 7,54064.

The centrifugal force of bodies in the equator, is to the centrifugal force with which bodies recede directly from

from the Earth in the latitude of *Paris* 48°. 50'. 10". in the duplicate proportion of the radius to the coſine of the latitude, that is, as 7,54064 to 3,267. Add this force to the force with which bodies deſcend by their weight in the latitude of *Paris*, and a body, in the latitude of *Paris*, falling by its whole undiminiſhed force of gravity, in the time of one ſecond, will deſcribe 2177,267 lines, or 15 *Paris* feet, 1 inch, and 5,267 lines. And the total force of gravity in that latitude will be to the centrifugal force of bodies in the equator of the Earth, as 2177,267 to 7,54064, or as 289 to 1.

Wherefore if *APBQ* (*Pl.* 10. *Fig.* 1.) repreſent the figure of the Earth, now no longer ſphærical, but generated by the rotation of an ellipſis about its leſſer axe; and *ACQqca* a canal full of water, reaching from the pole *Qq* to the centre *Cc*, and thence riſing to the equator *Aa*: The weight of the water in the leg of the canal *ACca*, will be to the weight of water in the other leg *QCcq*, as 289 to 288, becauſe the centrifugal force, ariſing from the circular motion, ſuſtains and takes off one of the 289 parts of the weight (in the one leg) and the weight of 288 in the other ſuſtains the reſt. But by computation (from cor. 2. prop. 91. book 1.) I find, that if the matter of the Earth was all uniform, and without any motion, and its axe *PQ* were to the diameter *AB*, as 100 to 101; the force of gravity in the place *Q*, towards the Earth, would be to the force of gravity in the ſame place *Q* towards a ſphere deſcrib'd about the centre *C* with the radius *PC*, or *QC*, as 126 to 125. And by the ſame argument, the force of gravity in the place *A* towards the ſphæroid, generated by the rotation of the ellipſe *APBQ* about the axe *AB*, is to the force of gravity in the ſame place *A*, towards the ſphere deſcrib'd about the centre *C* with the radius *AC*, as 125 to 126. But the force of gravity in the place *A*,

towards the Earth, is a mean proportional betwixt the forces of gravity towards that ſphæroid and this ſphere; becauſe the ſphere, by having its diameter $P\underline{Q}$ diminiſhed, in the proportion of 101 to 100, is transformed into the figure of the Earth; and this figure, by having a third diameter perpendicular to the two diameters AB and $P\underline{Q}$ diminiſh'd in the ſame proportion, is converted into the ſaid ſphæroid; and the force of gravity in A, in either caſe, is diminiſh'd nearly in the ſame proportion. Therefore the force of gravity in A, towards the ſphere deſcrib'd about the centre C, with the radius AC, is to the force of gravity in A, towards the Earth, as 126 to $125\frac{1}{2}$. And the force of gravity in the place $\underline{Q}$, towards the ſphere deſcrib'd about the centre C with the radius $\underline{Q}C$, is to the force of gravity in the place A, towards the ſphere deſcrib'd about the centre C, with the radius AC, in the proportion of the diameters, (by prop. 72. book 1.) that is, as 100 to 101. If therefore we compound thoſe three proportions 126 to 125, 126 to $125\frac{1}{2}$, and 100 to 101; into one: The force of gravity in the place $\underline{Q}$ towards the Earth, will be to the force of gravity in the place A towards the Earth, as $126 \times 126 \times 100$ to $125 \times 125\frac{1}{2} \times 101$; or as 501 to 500.

Now ſince (by cor. 3. prop. 91. book 1.) the force of gravity in either leg of the canal $ACca$, or $\underline{Q}Cc\underline{q}$, is as the diſtance of the places from the centre of the Earth, if thoſe legs are conceived to be divided by tranſverſe, parallel, and equidiſtant ſurfaces, into parts proportional to the wholes, the weights of any number of parts in the one leg $ACca$, will be to the weights of the ſame number of parts in the other leg, as their magnitudes and the accelerative forces of their gravity conjunctly, that is, as 101 to 100, and 500 to 501, or as 505 to 501. And therefore if the centrifugal force of every part in the leg $ACca$, ariſing from the diurnal motion, was to the weight of the ſame part, as 4 to

4 to 505, ſo that from the weight of every part, conceived to be divided into 505 parts, the centrifugal force might take off four of thoſe parts, the weights would remain equal in each leg, and therefore the fluid would reſt in an equilibrium. But the centrifugal force of every part is to the weight of the ſame part as 1 to 289; that is, the centrifugal force which ſhould be $\frac{4}{505}$ parts of the weight, is only $\frac{1}{289}$ part thereof. And therefore, I ſay, by the rule of proportion, that if the centrifugal force $\frac{4}{505}$ make the height of the water in the leg *ACca* to exceed the height of the water in the leg *QCcq*, by one $\frac{1}{100}$ part of its whole height; the centrifugal force $\frac{1}{289}$ will make the exceſs of the height in the leg *ACca*, only $\frac{1}{7225}$ part of the height of the water in the other leg *QCcq*. And therefore the diameter of the Earth at the equator, is to its diameter from pole to pole, as 230 to 229. And ſince the mean ſemidiameter of the Earth, according to *Picart*'s menſuration, is 19615800 *Paris* feet, or 3923,16 miles (reckoning 5000 feet to a mile) the Earth will be higher at the equator, than at the poles, by 85472 feet, or 17 $\frac{1}{100}$ miles. And its height at the equator will be about 19658600 feet, and at the poles 19573000 feet.

If, the denſity and periodic time of the diurnal revolution remaining the ſame, the Planet was greater or leſs than the Earth; the proportion of the centrifugal force to that of gravity, and therefore alſo of the diameter betwixt the poles to the diameter at the equator, would likewiſe remain the ſame. But if the diurnal motion was accelerated or retarded in any proportion, the centrifugal force would be augmented or diminiſhed nearly in the ſame duplicate proportion; and therefore the difference of the diameters will be increaſed or diminiſhed in the ſame duplicate ratio very nearly. And if the denſity of the Planet was augmented or diminiſhed in any proportion, the force of

gravity tending towards it would alſo be augmented or diminiſhed in the ſame proportion; and the difference of the diameters contrarywiſe would be diminiſhed in proportion as the force of gravity is augmented, and augmented in proportion as the force of gravity is diminiſhed. Wherefore, ſince the Earth, in reſpect of the fixt Stars, revolves in $23^h.\ 56'$, but Jupiter in $9^h.\ 56'$, and the ſquares of their periodic times are as 29 to 5, and their denſities as 400 to $94\frac{1}{2}$; the difference of the diameters of Jupiter will be to its leſſer diameter, as $\frac{29}{5} \times \frac{400}{\frac{941}{2}} \times \frac{229}{1}$ to 1, or as 1 to $9\frac{1}{3}$ nearly.

Therefore the diameter of Jupiter from eaſt to weſt, is to its diameter from pole to pole nearly as $10\frac{1}{3}$ to $9\frac{1}{3}$. Therefore ſince its greateſt diameter is $37''$, its leſſer diameter lying between the poles, will be $33''\ 25'''$. Add thereto about $3''$ for the irregular refraction of light, and the apparent diameters of this Planet will become $40''$ and $36''.\ 25'''$: which are to each other as $11\frac{1}{6}$ to $10\frac{1}{6}$ very nearly. Theſe things are ſo upon the ſuppoſition, that the body of Jupiter is uniformly denſe. But now if its body be denſer towards the plane of the equator than towards the poles, its diameters may be to each other as 12 to 11, or 13 to 12, or perhaps as 14 to 13.

And *Caſſini* obſerved in the year 1691, that the diameter of Jupiter reaching from eaſt to weſt, is greater by about a fifteenth part than the other diameter. Mr. *Pound* with his 123 foot teleſcope, and an excellent micrometer, meaſured the diameters of Jupiter in the year 1719, and found them as follows.

The

The times.			*Greateſt diam.*	*Leſſer diam.*	*The diam. to each other.*		
	day.	hours.	parts.	parts.		as	
Jan.	28	6	13,40	12,28	12	to	11
Mar.	6	7	13,12	12,20	$13\frac{1}{4}$	to	$12\frac{1}{4}$
Mar.	9	7	13,12	12,08	$12\frac{2}{3}$	to	$11\frac{2}{3}$
Apr.	9	9	12,32	11,48	$14\frac{1}{2}$	to	$13\frac{1}{2}$

So that the theory agrees with the phænomena. For the Planets are more heated by the Sun's rays towards their equators, and therefore are a little more condenſed by that heat, than towards their poles.

Moreover, that there is a diminution of gravity occaſioned by the diurnal rotation of the Earth, and therefore the Earth riſes higher there than it does at the poles, (ſuppoſing that its matter is uniformly denſe) will appear by the experiments of pendulums related under the following propoſition.

Proposition XX. Problem IV.

To find and compare together the weights of bodies in the different regions of our Earth.

Becauſe the weights of the unequal legs of the canal of water *ACQqca*, are equal; and the weights of the parts proportional to the whole legs, and alike ſituated in them, are one to another as the weights of the wholes, and therefore equal betwixt themſelves; the weights of equal parts and alike ſituated in the legs, will be reciprocally as the legs, that is, reciprocally as 230 to 229. And the caſe is the ſame in all homogeneous equal bodies alike ſituated in the legs of the canal. Their weights are reciprocally as the legs, that

is, reciprocally as the diſtances of the bodies from the centre of the Earth. Therefore if the bodies are ſituated in the uppermoſt parts of the canals, or on the ſurface of the Earth, their weights will be, one to another, reciprocally as their diſtances from the centre. And by the ſame argument, the weights in all other places round the whole ſurface of the Earth, are reciprocally as the diſtances of the places from the centre; and therefore, in the hypotheſis of the Earth's being a ſphæroid, are given in proportion.

Whence ariſes this theorem, that the increaſe of weight, in paſſing from the equator to the poles, is nearly as the verſed ſine of double the latitude, or, which comes to the ſame thing, as the ſquare of the right ſine of the latitude. And the arcs of the degrees of latitude in the meridian, increaſe nearly in the ſame proportion. And therefore, ſince the latitude of *Paris* is 48°. 50′, that of places under the equator, 00°. 00′. and that of places under the poles 90°; and the verſed ſines of double thoſe arcs are 11334, 00000 and 20000, the radius being 10000; and the force of gravity at the pole is to the force of gravity at the equator, as 230 to 229, and the exceſs of the force of gravity at the pole, to the force of gravity at the equator, as 1 to 229, the exceſs of the force of gravity in the latitude of *Paris*, will be to the force of gravity at the equator as $1 \times \frac{11334}{20000}$ to 229, or as 5667 to 2290000. And therefore the whole forces of gravity in thoſe places will be, one to the other, as 2295667 to 2290000. Wherefore, ſince the lengths of pendulums vibrating in equal times, are as the forces of gravity, and in the latitude of *Paris*, the length of a pendulum vibrating ſeconds, is 3 *Paris* feet, and 8 $\frac{1}{2}$ lines, or rather, becauſe of the weight of the air 8 $\frac{5}{9}$ lines; the length of a pendulum vibrating in the ſame time under the equator, will be ſhorter by 1,087 lines. And by a like calculus the following table is made.

Latitude

Latitude of the place.	*Length of the pendulum.*	*Meaſure of one degree in the meridian.*
Deg.	Feet. Lines.	Toiſes.
0	3 . 7,468	56637
5	3 . 7,482	56642
10	3 . 7,526	56659
15	3 . 7,596	56687
20	3 . 7,692	56724
25	3 . 7,812	56769
30	3 . 7,948	56823
35	3 . 8,099	56882
40	3 . 8,261	56945
1	3 . 8,294	56958
2	3 . 8,327	56971
3	3 . 8,361	56984
4	3 . 8,394	56997
45	3 . 8,428	57010
6	3 . 8,461	57022
7	3 . 8,494	57035
8	3 . 8,528	57048
9	3 . 8,561	57061
50	3 . 8,594	57074
55	3 . 8,756	57137
60	3 . 8,907	57196
65	3 . 9,044	57250
70	3 . 9,162	57295
75	3 . 9,258	57332
80	3 . 9,329	57360
85	3 . 9,372	57377
90	3 . 9,387	57382

By this table therefore it appears, that the inequality of degrees is ſo ſmall, that the figure of the Earth, in

geographical matters, may be conſidered as ſphærical; eſpecially if the Earth be a little denſer towards the plane of the equator than towards the poles.

Now ſeveral aſtronomers ſent into remote countries to make aſtronomical obſervations, have found that pendulum clocks do accordingly move ſlower near the equator than in our climates. And firſt of all in the year 1672, M. *Richer* took notice of it in the iſland of *Cayenne*. For when, in the month of *Auguſt*, he was obſerving the tranſits of the fixt Stars over the meridian, he found his clock to go ſlower than it ought in reſpect of the mean motion of the Sun, at the rate of 2′. 28″. a day. Therefore fitting up a ſimple pendulum to vibrate in ſeconds, which were meaſured by an excellent clock, he obſerved the length of that ſimple pendulum; and this he did over and over every week for ten months together. And upon his return to *France*, comparing the length of that pendulum, with the length of the pendulum at *Paris*, (which was 3 *Paris* feet and 8 $\frac{1}{5}$ lines) he found it ſhorter by 1 $\frac{1}{4}$ line.

Afterwards our friend Dr. *Halley*, about the year 1677, arriving at the iſland of St. *Helen*, found his pendulum-clock to go ſlower there than at *London*, without marking the difference. But he ſhortned the rod of his clock, by more than the $\frac{1}{8}$ of an inch, or 1 $\frac{1}{2}$ line. And to effect this, becauſe the length of the ſcrew at the lower end of the rod was not ſufficient, he interpoſed a wooden ring betwixt the nut and the ball.

Then in the year 1682. M. *Varin* and M. *des Hayes*, found the length of a ſimple pendulum vibrating in ſeconds at the royal obſervatory of *Paris* to be 3 feet and 8$\frac{5}{9}$ lines. And by the ſame method in the iſland of *Goree*, they found the length of an iſochronal pendulum to be 3 feet and 6$\frac{5}{9}$ lines, differing from the former by two lines. And in the ſame year, going to the iſlands

iſlands of *Guadaloupe* and *Martinico*, they found that the length of an iſochronal pendulum in thoſe iſlands was 3 feet and $6\frac{1}{2}$ lines.

After this M. *Couplet*, the ſon, in the month of *July* 1697, at the royal obſervatory of *Paris*, ſo fitted his pendulum clock to the mean motion of the Sun, that for a conſiderable time together, the clock agreed with the motion of the Sun. In *November* following, upon his arrival at *Lisbon*, he found his clock to go ſlower than before, at the rate of 2′. 13″. in 24 hours. And next *March* coming to *Paraiba* he found his clock to go ſlower there than at *Paris*, and at the rate of 4′. 12″. in 24 hours. And he affirms, that the pendulum vibrating in ſeconds was ſhorter at *Lisbon* by $2\frac{1}{2}$ lines, and at *Paraiba* by $3\frac{1}{3}$ lines, than at *Paris*. He had done better to have reckon'd thoſe differences $1\frac{1}{3}$ and $2\frac{1}{9}$. For theſe differences correſpond to the differences of the times 2′. 13″. and 4′. 12″. But this gentleman's obſervations are ſo groſs, that we cannot confide in them.

In the following years 1699 and 1700. M. *des Hayes*, making another voyage to *America*, determin'd that in the iſlands of *Cayenne* and *Granada* the length of the pendulum vibrating in ſeconds was a ſmall matter leſs than 3 feet and $6\frac{1}{2}$ lines; that in the iſland of St. *Chriſtophers*, it was 3 feet and $6\frac{1}{4}$ lines; and in the iſland of St. *Domingo*, 3 feet and 7 lines.

And in the year 1704. P. *Feuillé* at *Puerto bello* in *America*, found that the length of the pendulum vibrating in ſeconds, was 3 *Paris* feet, and only $5\frac{7}{12}$ lines, that is, almoſt 3 lines ſhorter than at *Paris*; but the obſervation was faulty. For afterwards going to the iſland of *Martinico*, he found the length of the iſochronal pendulum there, 3 *Paris* feet and $5\frac{10}{12}$ lines.

Now the latitude of *Paraiba* is 6°. 38′. ſouth. That of *Puerto bello* 9°. 33′. north. And the latitudes of the ſlands *Cayenne*, *Goree*, *Guadaloupe*, *Martinico*, *Granada*,

St. *Chriſtophers* and St. *Domingo*, are reſpectively 4°. 55′, 14°. 40″, 14°. 00′, 14°. 44′, 12°. 06′, 17°. 19′. and 19°. 48′, north. And the exceſſes of the length of the pendulum at *Paris* above the lengths of the iſochronal pendulums obſerv'd in thoſe latitudes, are a little greater than by the table of the lengths of the pendulum above computed. And therefore the Earth is a little higher under the equator than by the preceding calculus, and a little denſer at the centre than in mines near the ſurface, unleſs perhaps the heats of the torrid zone have a little extended the length of the pendulums.

For M. *Picart* has obſerv'd, that a rod of iron, which in froſty weather in the winter ſeaſon was one foot long, when heated by fire, was lengthen'd into 1 foot and $\frac{1}{4}$ line. Afterwards M. *de la Hire* found that a rod of iron, which in the like winter ſeaſon was 6 feet long; when expos'd to the heat of the ſummer Sun, was extended into 6 feet and $\frac{1}{3}$ line. In the former caſe the heat was greater than in the latter. But in the latter it was greater than the heat of the external parts of an human body. For metals expos'd to the ſummer-ſun, acquire a very conſiderable degree of heat. But the rod of a pendulum-clock is never expos'd to the heat of the ſummer-ſun, nor ever acquires a heat equal to that of the external parts of an human body. And therefore though the 3 foot rod of a pendulum clock will indeed be a little longer in the ſummer than in the winter-ſeaſon; yet the difference will ſcarcely amount to $\frac{1}{4}$ line. Therefore the total difference of the lengths of iſochronal pendulums in different climates, cannot be aſcrib'd to the difference of heat. Nor indeed to the miſtakes of the *French* aſtronomers. For although there is not a perfect agreement betwixt their obſervations, yet the errors are ſo ſmall that they may be neglected; and in this they all agree, that iſochronal pendulums are ſhorter under the equator than at the royal obſervatory of *Paris*, by a difference not leſs

leſs than 1 $\frac{1}{4}$ line, nor greater than 2 $\frac{3}{5}$ lines. By the obſervations of M. *Richer* in the iſland of *Cayenne*, the difference was 1$\frac{1}{4}$ line. That difference being corrected by thoſe of M. *des Hayes* becomes 1 $\frac{1}{2}$ line or 1 $\frac{1}{4}$ line. By the leſs accurate obſervations of others the ſame was made about two lines. And this diſagreement might ariſe partly from the errors of the obſervations, partly from the diſſimilitude of the internal parts of the Earth, and the height of mountains, partly from the different heats of the air.

I take an iron rod of 3 feet long to be ſhorter by a ſixth part of one line in winter time with us here in *England*, than in the ſummer. Becauſe of the great heats under the equator, ſubduct this quantity from the difference of one line and a quarter obſerv'd by M. *Richer*, and there will remain one line $\frac{1}{12}$, which agrees very well with 1$\frac{87}{100}$ line collected by the theory a little before. M. *Richer* repeated his obſervations, made in the iſland of *Cayenne*, every week for 10 months together, and compared the lengths of the pendulum which he had there noted in the iron rods, with the lengths thereof which he obſerv'd in *France*. This diligence and care ſeems to have been wanting to the other obſervers. If this gentleman's obſervations are to be depended on, the Earth is higher under the equator than at the poles, and that by an exceſs of about 17 miles: as appeared above by the theory.

PROP.

Proposition XXI. Theorem XVII.

That the equinoctial points go backwards, and that the axe of the Earth, by a nutation in every annual revolution, twice vibrates towards the ecliptic, and as often returns to its former position.

The propoſition appears from cor. 20. prop. 66. book 1. But that motion of nutation muſt be very ſmall, and indeed ſcarce perceptible.

Proposition XXII. Theorem XVIII.

That all the motions of the Moon, and all the inequalities of thoſe motions, follow from the principles which we have laid down.

That the greater Planets, while they are carried about the Sun may, in the mean time, carry other leſſer Planets, revolving about them; and that thoſe leſſer Planets muſt move in ellipſes, which have their foci in the centres of the greater, appears from prop. 65. book 1. But then their motions will be ſeveral ways diſturb'd by the action of the Sun, and they will ſuffer ſuch inequalities as are obſerv'd in our Moon. Thus our Moon, (by cor. 2, 3, 4, and 5. prop. 66. book 1.) moves faſter, and, by a radius drawn to the Earth, deſcribes an area, greater for the time, and has its orbit leſs curv'd, and therefore approaches nearer to the Earth, in the ſyzygies than in the quadratures, excepting in ſo far as theſe effects are hinder'd by the motion of eccentricity. For (by cor. 9. prop. 66. book 1.) the

the eccentricity is greateſt, when the apogeon of the Moon is in the ſyzygies, and leaſt when the ſame is in the quadratures; and upon this account, the perigeon Moon is ſwifter, and nearer to us, but the apogeon Moon ſlower and farther from us, in the ſyzygies than in the quadratures. Moreover the apogee goes forwards, and the nodes backwards: and this is done, not with a regular, but an unequal motion. For (by cor. 7 and 8. prop. 66. book 1.) the apogee goes more ſwiftly forwards in its ſyzygies, more ſlowly backwards in its quadratures; and, by the exceſs of its progreſs above its regreſs, advances yearly *in conſequentia*. But contrarywiſe the nodes (by cor. 11. prop. 66. book 1.) are quieſcent in their ſyzygies, and go faſteſt back in their quadratures. Further, the greateſt latitude of the Moon, (by cor. 10. prop. 66 book 1.) is greater in the quadratures of the Moon, than in its ſyzygies. And (by cor. 6. prop. 66. book 1.) the mean motion of the Moon is ſlower in the perihelion of the Earth, than in its aphelion. And theſe are the principal inequalities (of the Moon) taken notice of by aſtronomers.

But there are yet other inequalities, not obſerv'd by former aſtronomers; by which the motions of the Moon are ſo diſturb'd, that to this day we have not been able to bring them under any certain rule. For the velocities or horary motions of the apogee and nodes of the Moon, and their equations as well as the difference betwixt the greateſt eccentricity in the ſyzygies, and the leaſt eccentricity in the quadratures, and that inequality, which we call the variation, are (by cor. 14. prop. 66. book 1.) in the courſe of the year, augmented and diminiſh'd, in the triplicate proportion of the Sun's apparent diameter. And beſides (by cor. 1 and 2. lem. 10. and cor. 16. prop. 66. book 1.) the variation is augmented and diminiſh'd, nearly in the duplicate proportion of the time between the

the quadratures. But in aſtronomical calculations, this inequality is commonly thrown into, and confounded with, the equation of the Moon's centre.

Proposition XXIII. Problem V.

To derive the unequal motions of the ſatellites of Jupiter and Saturn from the motions of our Moon.

From the motions of our Moon we deduce the correſponding motions of the moons or ſatellites of Jupiter, in this manner, by cor. 16. prop. 66. book 1. The mean motion of the nodes of the outmoſt ſatellite of Jupiter, is to the mean motion of the nodes of our Moon, in a proportion compounded of the duplicate proportion of the periodic time of the Earth about the Sun, to the periodic time of Jupiter about the Sun, and the ſimple proportion of the periodic time of the ſatellite about Jupiter to the periodic time of our Moon about the Earth: and therefore thoſe nodes, in the ſpace of an hundred years, are carried 8°. 24′. backwards, or *in antecedentia*. The mean motions of the nodes of the inner ſatellites, are to the mean motion of the nodes of the outmoſt, as their periodic times to the periodic time of the former, by the ſame corollary, and are thence given. And the motion of the apſis of every ſatellite *in conſequentia*, is to the motion of its nodes *in antecedentia*, as the motion of the apogee of our Moon, to the motion of its nodes (by the ſame corollary) and is thence given. But the motions of the apſides thus found, muſt be diminiſh'd in the proportion of 5 to 9, or of about 1 to 2, on account of a cauſe, which I cannot here deſcend to explain. The greateſt equations of the nodes, and of the apſis of every ſatellite, are to the greateſt equations of the nodes, and

and apogee of our Moon reſpectively, as the motions of the nodes and apſides of the ſatellites, in the time of one revolution of the former equations, to the motions of the nodes and apogee of our Moon, in the time of one revolution of the latter equations. The variation of a ſatellite, ſeen from Jupiter, is to the variation of our Moon, in the ſame proportion, as the whole motions of their nodes reſpectively, during the times, in which the ſatellite and our Moon, (after parting from) are revolv'd (again) to the Sun, by the ſame corollary; and therefore in the outmoſt ſatellite, the variation does not exceed 5″. 12‴.

PROPOSITION XXIV. THEOREM XIX.

That the flux and reflux of the Sea, ariſe from the actions of the Sun and Moon.

By cor. 19 and 20. prop. 66. book 1. It appears that the waters of the ſea ought twice to riſe and twice to fall every day, as well lunar as ſolar; and that the greateſt height of the waters in the open and deep ſeas, ought to follow the appulſe of the luminaries to the meridian of the place, by a leſs interval than 6 hours; as happens in all that eaſtern tract of the *Atlantic* and *Æthiopic* ſeas between *France* and the *Cape of Good Hope*; and on the coaſts of *Chili* and *Peru* in the *South-Sea*; in all which ſhoars the flood falls out about the ſecond, third, or fourth hour, unleſs where the motion propagated from the deep ocean is by the ſhallowneſs of the channels, through which it paſſes to ſome particular places, retarded to the fifth, ſixth, or ſeventh hour, and even later. The hours I reckon from the appulſe of each luminary to the meridian of the place, as well under, as above the horizon; and by the hours of the lunar day, I underſtand the 24th parts of that time, which the Moon, by its apparent diurnal motion, employs

ploys to come about again to the meridian of the place which it left the day before. The force of the Sun or Moon in raiſing the ſea, is greateſt in the appulſe of the luminary to the meridian of the place. But the force impreſſed upon the ſea at that time continues a little while after the impreſſion, and is afterwards encreas'd by a new, though leſs, force ſtill acting upon it. This makes the ſea riſe higher and higher, till this new force becoming too weak to raiſe it any more, the ſea riſes to its greateſt height. And this will come to paſs perhaps in one or two hours, but more frequently near the ſhores in about three hours, or even more where the ſea is ſhallow.

The two luminaries excite two motions, which will not appear diſtinctly, but between them will ariſe one mixt motion compounded out of both. In the conjunction or oppoſition of the luminaries, their forces will be conjoin'd, and bring on the greateſt flood and ebb. In the quadratures the Sun will raiſe the waters which the Moon depreſſes, and depreſs the waters which the Moon raiſes, and from the difference of their forces, the ſmalleſt of all tides will follow. And becauſe (as experience tells us) the force of the Moon is greater than that of the Sun, the greateſt height of the waters will happen about the third lunar hour. Out of the ſyzygies and quadratures, the greateſt tide, which by the ſingle force of the Moon ought to fall out at the third lunar hour, and by the ſingle force of the Sun at the third ſolar hour, by the compounded forces of both muſt fall out in an intermediate time, that approaches nearer to the third hour of the Moon, than to that of the Sun. And therefore while the Moon is paſſing from the ſyzygies to the quadratures, during which time the 3d hour of the Sun precedes the 3d hour of the Moon, the greateſt height of the waters will alſo precede the 3d hour of the Moon; and that, by the greateſt interval, a little after the octants of the Moon;

Moon; and by like intervals, the greateſt tide will follow the 3d lunar hour, while the Moon is paſſing from the quadratures to the ſyzygies. Thus it happens in the open ſea. For in the mouths of rivers, the greater tides come later to their height.

But the effects of the luminaries depend upon their diſtances from the Earth. For when they are leſs diſtant, their effects are greater, and when more diſtant, their effects are leſs, and that in the triplicate proportion of their apparent diameter. Therefore it is, that the Sun, in the winter time, being then in its perigee, has a greater effect, and makes the tides in the ſyzygies ſomething greater, and thoſe in the quadratures ſomething leſs than in the ſummer ſeaſon; and every month the Moon, while in the perigee, raiſes greater tides than at the diſtance of 15 days before or after, when it is in its apogee. Whence it comes to paſs, that two higheſt tides don't follow one the other, in two immediately ſucceeding ſyzygies.

The effect of either luminary doth likewiſe depend upon its declination or diſtance from the equator. For, if the luminary was plac'd at the pole, it would conſtantly attract all the parts of the waters, without any intenſion or remiſſion of its action, and could cauſe no reciprocation of motion. And therefore, as the luminaries decline from the equator towards either pole, they will, by degrees, loſe their force, and on this account will excite leſſer tides in the ſolſtitial than in the equinoctial ſyzygies. But in the ſolſtitial quadratures, they will raiſe greater tides than in the quadratures about the equinoxes; becauſe the force of the Moon then ſituated in the equator, moſt exceeds the force of the Sun. Therefore the greateſt tides fall out in thoſe ſyzygies, and the leaſt in thoſe quadratures, which happen about the time of both equinoxes: and the greateſt tide in the ſyzygies is always ſucceeded by the leaſt tide in the quadratures, as we find by experience.

But, becauſe the Sun is leſs diſtant from the Earth in winter than in ſummer, it comes to paſs that the greateſt and leaſt tides more frequently appear before than after the vernal equinox, and more frequently after than before the autumnal.

Moreover, the effects of the luminaries depend upon the latitudes of places. Let *ApEP Pl.*10. *Fig.* 2. repreſent the Earth cover'd with deep waters; *C* its centre; *P*, *p* its poles; *AE* the equator; *F*, any place without the equator; *Ff*, the parallel of the place; *Dd* the correſpondent parallel on the other ſide of the equator; *L*, the place of the Moon three hours before; *H*, the place of the Earth directly under it; *h*, the oppoſite place; *K*, *k* the places at 90 degrees diſtance; *CH*, *Ch*, the greateſt heights of the ſea from the centre of the Earth; and *CK*, *ck* its leaſt heights: and if with the axes *Hh*, *Kk*, an ellipſis is deſcrib'd, and by the revolution of that ellipſis about its longer axe *Hh*, a ſphæroid *HPKhpk* is form'd, this ſphæroid will nearly repreſent the figure of the ſea; and *CF*, *Cf*, *CD*, *Cd*, will repreſent the heights of the ſea in the places *Ff*, *Dd*. But further, in the ſaid revolution of the ellipſis any point *N* deſcribes the circle *NM*, cutting the parallels *Ff*, *Dd*, in any places *R*, *T*; and the equator *AE* in *S*; *CN* will repreſent the height of the ſea in all thoſe places *R*, *S*, *T*, ſituated in this circle. Wherefore in the diurnal revolution of any place *F*, the greateſt flood will be in *F*, at the 3d hour after the appulſe of the Moon to the meridian above the Horizon; and afterwards the greateſt ebb in *Q*, at the 3d hour after the ſetting of the Moon: and then the greateſt flood in *f*, at the 3d hour after the appulſe of the Moon to the meridian under the horizon, and laſtly, the greateſt ebb in *Q*, at the 3d hour after the riſing of the Moon; and the latter flood in *f*, will be leſs than the preceding flood in *F*. For the whole ſea is divided into two hemiſpherical floods, one in the hemiſphere *KHk* on the north ſide, the

the other in the oppoſite hemiſphere *Khk* which we may therefore call the northern and the ſouthern floods. Theſe floods being always oppoſite the one to the other, come by turns to the meridians of all places, after an interval of 12 lunar hours. And ſeeing the northern countries partake more of the northern flood, and the ſouthern countries more of the ſouthern flood, thence ariſe tides, alternately greater and leſs in all places without the equator, in which the luminaries riſe and ſet. But the greateſt tide will happen, when the Moon declines towards the vertex of the place, about the 3d hour after the appulſe of the Moon to the meridian above the horizon; and when the Moon changes its declination *to the other ſide of the equator*, that which was the greater tide will be chang'd into a leſſer. And the greateſt difference of the floods will fall out about the times of the ſolſtices; eſpecially if the aſcending node of the Moon is about the firſt of Aries. So it is found by experience, that the morning tides in winter exceed thoſe of the evening, and the evening tides in ſummer exceed thoſe of the morning; at *Plymouth* by the height of one foot, but at *Briſtol*, by the height of 15 inches, according to the obſervations of *Colepreſs* and *Sturmy*.

But the motions which we have been deſcribing, ſuffer ſome alteration from that force of reciprocation, which the waters, being once moved, retain a little while *by their vis inſita*. Whence it comes to paſs that the tides may continue for ſome time, tho' the actions of the luminaries ſhould ceaſe. This power of retaining the impreſs'd motion leſſens the difference of the alternate tides and makes thoſe tides which immediately ſucceed after the ſyzygies greater, and thoſe which follow next after the quadratures, leſs. And hence it is, that the alternate tides at *Plymouth* and *Briſtol*, don't differ much more one from the other than by the height of a foot or 15 inches, and that the greateſt tides of all at

thoſe ports are not the firſt but the third after the ſyzygies. And beſides all the motions are retarded in their paſſage through ſhallow channels, ſo that the greateſt tides of all in ſome ſtreights and mouths of rivers, are the fourth or even the fifth after the ſyzygies.

Farther it may happen that the tide may be propagated from the ocean through different channels towards the ſame port, and may paſs quicker through ſome channels than through others, in which caſe the ſame tide, divided into two or more ſucceeding one another, may compound new motions of different kinds. Let us ſuppoſe two equal tides flowing towards the ſame port from different places, the one preceding the other by 6 hours; and ſuppoſe the firſt tide to happen at the third hour of the appulſe of the Moon to the meridian of the port. If the Moon at the time of the appulſe to the meridian was in the equator, every 6 hours alternately there would ariſe equal floods, which meeting with as many equal ebbs would ſo ballance one the other, that for that day the water would ſtagnate and remain quiet. If the Moon then declined from the equator, the tides in the ocean would be alternately greater and leſs as was ſaid. And from thence two greater and two leſſer tides would be alternately propagated towards that port. But the two greater floods would make the greateſt height of the waters to fall out in the middle time betwixt both; and the greater and leſſer floods would make the waters to riſe to a mean height in the middle time between them, and in the middle time between the two leſſer floods the waters would riſe to their leaſt height. Thus in the ſpace of 24 hours the waters would come, not twice, as commonly, but once only to their greateſt, and once only to their leaſt height; and their greateſt height, if the Moon declined towards the elevated pole, would happen at the 6 or 30th hour after the appulſe of the Moon to the meridian; and when the Moon changed its declination this flood would

would be changed into an ebb. An example of all which Dr. *Halley* has given us, from the obſervations of ſeamen in the port of *Batſham* in the kingdom of *Tunquin* in the latitude of 20°. 50′. north. In that port, on the day which follows after the paſſage of the Moon over the equator, the waters ſtagnate: when the Moon declines to the north they begin to flow and ebb, not twice, as in other ports, but once only every day, and the flood happens at the ſetting, and the greateſt ebb at the riſing of the Moon. This tide encreaſes with the declination of the Moon till the 7th or 8th day; then for the 7 or 8 days following, it decreaſes at the ſame rate as it had increaſed before, and ceaſes when the Moon changes its declination, croſſing over the equator to the ſouth. After which the flood is immediatly chang'd into an ebb; and thenceforth the ebb happens at the ſetting, and the flood at the riſing of the Moon; till the Moon again paſſing the equator changes its declination. There are two inlets to this port, and the neighbouring channels, one from the ſeas of *China*, between the continent and the iſland of *Luconia*, the other from the *Indian* ſea, between the continent and the iſland of *Borneo*. But whether there be really two tides propagated through the ſaid channels, one from the *Indian* ſea in the ſpace of 12 hours, and one from the ſea of *China* in the ſpace of 6 hours, which therefore happening at the 3d and 9th lunar hours, by being compounded together, produce thoſe motions, or whether there be any other circumſtances in the ſtate of thoſe ſeas, I leave to be determin'd by obſervations on the neighbouring ſhoars.

Thus I have explain'd the cauſes of the motions of the Moon and of the Sea. Now it is fit to ſubjoin ſomething concerning the quantity of thoſe motions.

PRO-

Proposition XXV. Problem VI.

To find the forces with which the Sun disturbs the motions of the Moon. Pl. 10. Fig. 3.

Let S repreſent the Sun, T the Earth, P the Moon, $CADB$ the Moon's orbit. In SP take SK equal to ST; and let SL be to SK, in the duplicate proportion of SK to SP; draw LM parallell to PT; and if ST or SK is ſuppos'd to repreſent the accelerated force of gravity of the Earth towards the Sun, SL will repreſent the accelerative force of gravity of the Moon towards the Sun. But that force is compounded of the parts SM and LM, of which the force LM, and that part of SM which is repreſented by TM, diſturb the motion of the Moon, as we have ſhew'd in prop. 66. book 1. and its corollaries. Foraſmuch as the Earth and Moon are revolv'd about their common centre of gravity, the motion of the Earth about that centre will be alſo diſturb'd by the like forces, but we may conſider the ſums both of the forces and of the motions as in the Moon, and repreſent the ſum of the forces by the lines TM and ML, which are analogous to them both. The force ML (in its mean quantity) is, to the centripetal force by which the Moon may be retain'd in its orbit revolving about the Earth at reſt at the diſtance PT, in the duplicate proportion of the periodic time of the Moon about the Earth, to the periodic time of the Earth about the Sun (by cor. 17. prop. 66. book 1.) that is in the duplicate proportion of $27^{d}.\ 7^{h}.\ 43'.$ to $365^{d}.\ 6^{h}.\ 9'$; or as 1000 to 178725; or as 1 to $178\frac{29}{40}$. But in the 4th prop. of this book we found, that if both Earth and Moon were revolv'd about their common centre of gravity, the mean diſtance of the one from the other would be nearly $60\frac{1}{2}$ mean ſemidiameters of the Earth. And the

the force, by which the Moon may be kept revolving in its orbit about the Earth in reſt at the diſtance PT of $60\frac{1}{2}$ ſemidiameters of the Earth, is to the force by which it may be revolv'd in the ſame time at the diſtance of 60 ſemidiameters, as $60\frac{1}{2}$ to 60; and this force is to the force of gravity with us, very nearly as 1 to 60×60. Therefore the mean force ML is to the force of gravity on the ſurface of our Earth, as $1 \times 60\frac{1}{2}$ to $60 \times 60 \times 60 \times 178\frac{2}{60}$, or as 1 to 638092, 6. whence by the proportion of the lines TM, ML, the force TM is alſo given; and theſe are the forces with which the Sun diſturbs the motions of the Moon. *Q. E. I.*

Proposition XXVI. Problem VII.

To find the horary increment of the area, which the Moon, by a radius drawn to the Earth, deſcribes in a circular orbit.

We have above ſhew'd that the area, which the Moon deſcribes by a radius drawn to the Earth, is proportional to the time of deſcription; excepting in ſo far as the Moon's motion is diſturb'd by the action of the Sun. And here we propoſe to inveſtigate the inequality of the moment, or horary increment *of that area, or motion ſo diſturb'd.* To render the calculus more eaſy, we ſhall ſuppoſe the orbit of the Moon to be circular, and neglect all inequalities, but that only which is now under conſideration. And becauſe of the immenſe diſtance of the Sun, we ſhall further ſuppoſe, that the lines SP and ST, are parallel. By this means, the force LM *Pl.* 10. *Fig.* 4. will be always reduc'd to its mean quantity TP, as well as the force TM, to its mean quantity $3\,PK$. Theſe forces, (by cor. 2. of the laws of motion) compoſe the force TL; and this force by letting fall the

perpendicular LE upon the radius TP, is reſolv'd into the forces TE, EL; of which the force TE, acting conſtantly in the direction of the radius TP, neither accelerates or retards the deſcription of the area TPC, made by that radius TP; but EL *acting on the radius* TP in a perpendicular direction, accelerates or retards *the deſcription of the area* in proportion as it accelerates or retards the Moon. That acceleration of the Moon, in its paſſage from the quadrature C, to the conjunction A, is in every moment of time, as the *generating* accelerative force EL, that is, as $\frac{3PK \times TK}{TP}$ — Let the time be repreſented by the mean motion of the Moon, or (which comes to the ſame thing) by the angle CTP, or even by the arc CP. At right angles upon CT, erect CG equal to CT. And ſuppoſing the quadrantal arc AC to be divided into an infinite number of equal parts Pp &c. theſe *parts* may repreſent the like *infinite* number of the equal parts of time. Let fall pk perpendicular on CT; and draw TG meeting with KP, kp produc'd, in F and f; then will FK be equal to TK, and Kk be to PK as Pp to Tp, that is, in a giv'n proportion; and therefore $FK \times Kk$, or the area $FKkf$, will be as $\frac{3PK \times TK}{TP}$, that is as EL; and compounding, the whole area $GCKF$ will be as the ſum of all the forces EL impreſs'd upon the Moon in the whole time CP; and therefore alſo as the velocity generated by that ſum, that is, as the acceleration of the deſcription of the area CTP, or as the increment of the moment *thereof*. The force by which the Moon may in its periodic time $CADB$ of $27^d.\ 7^h.\ 43'$, be retain'd revolving about the Earth in reſt at the diſtance TP, would cauſe a body, falling in the time CT, to deſcribe the length $\frac{1}{2}CT$, and at the ſame time to acquire a velocity equal to that with which the Moon

is

is mov'd in its orbit. This appears from cor. 9. prop. 4. book 1. But ſince Kd, drawn perpendicular on TP, is *but* a third part of EL, and *equal to* the half of TP, or ML, in the octants, the force EL in the octants, where it is greateſt, will exceed the force ML, in the proportion of 3 to 2; and therefore will be to that force by which the Moon in its periodic time may be retain'd revolving about the Earth at reſt, as 100 to $\frac{2}{3} \times 17872\frac{1}{2}$, or 11915; and in the time CT will generate a velocity equal to $\frac{100}{11915}$ parts of the velocity of the Moon; but in the time CPA, will generate a greater velocity in the proportion of CA to CT or TP. Let the greateſt force EL in the octants be repreſented by the area $FK \times Kk$, or by the rectangle $\frac{1}{2}\ TP \times Pp$, which is equal thereto. And the velocity which that greateſt force can generate in any time CP, will be to the velocity which any other leſſer force EL can generate in the ſame time, as the rectangle $\frac{1}{2}\ TP \times CP$ to the area $KCGF$; but the velocities generated in the whole time CPA, will be one to the other as the rectangle $\frac{1}{2}\ TP \times CA$ to the triangle TCG; or as the quadrantal arc CA to the radius TP. And therefore (by prop. 9. book 5. elem.) the latter velocity generated in the whole time, will be $\frac{100}{11915}$ parts of the velocity of the Moon. To this velocity of the Moon, which is proportional to the mean moment of the area (ſuppoſing this mean moment to be repreſented by the number 11915) we add and ſubſtract the half of the other velocity; the ſum 11915 $+$ 50, or 11965 will repreſent the greateſt moment of the area in the ſyzygy A; and the difference 11915 $-$ 50, or 11865, the leaſt moment thereof in the quadratures. Therefore the areas, which in equal times, are deſcribed in the ſyzygies and quadratures, are, one to the other, as 11965 to 11865. And if to the leaſt moment 11865, we add a moment which ſhall be

to

to 100, the difference of the two former moments as the trapezium $FKCG$ to the triangle TCG, or which comes to the ſame thing, as the ſquare of the ſine PK to the ſquare of the radius TP, (that is, as Pd to TP) the ſum will repreſent the moment of the area, when the Moon is in any intermediate place P.

But theſe things take place, only in the hypotheſis that the Sun and the Earth are at reſt, and that the ſynodical revolution of the Moon is finiſhed in $27^d. 7^h. 43'$. But ſince the Moon's ſynodical period is really $29^d. 12^h. 44'$, the increments of the moments muſt be inlarged, in the ſame proportion as the time is, that is, in the proportion of 1080853 to 1000000. Upon which account, the whole increment, which was $\frac{100}{11915}$ parts of the mean moment, will now become $\frac{100}{11023}$ parts thereof. And therefore the moment of the area, in the quadrature of the Moon, will be to the moment thereof in the ſyzygy, as 11023—50 to 11023+50; or as 10973 to 11073; and to the moment thereof when the Moon is in any intermediate place P, as 10973 to $10973+Pd$; that is, ſuppoſing $TP=100$.

The area therefore, which the Moon, by a radius drawn to the Earth, deſcribes in the ſeveral little equal parts of time, is nearly as the ſum of the number 219,46, and the verſed ſine of the double diſtance of the Moon from the neareſt quadrature, conſidered in a circle which hath unity for its radius. Thus it is, when the variation in the octants is in its mean quantity. But if the variation there is greater or leſs, that verſed ſine muſt be augmented or diminiſhed in the ſame proportion.

PRO-

Proposition XXVII. Problem VIII.

From the horary motion of the Moon, to find its diſtance from the Earth.

The area which the Moon, by a radius drawn to the Earth, deſcribes in every moment of time, is as the horary motion of the Moon, and the ſquare of the diſtance of the Moon from the Earth conjunctly. And therefore the diſtance of the Moon from the Earth is in a proportion compounded of the ſubduplicate proportion of the area directly, and the ſubduplicate proportion of the horary motion inverſely. *Q. E. I.*

Cor. 1. Hence the apparent diameter of the Moon is given. For it is reciprocally as the diſtance of the Moon from the Earth. Let aſtronomers try how accurately this rule agrees with the phænomena.

Cor. 2. Hence alſo the orbit of the Moon may be more exactly defin'd from the phænomena than hitherto could be done.

Proposition XXVIII. Problem IX.

To find the Diameters of the orbit, in which, without eccentricity, the Moon would move.

The curvature of the orbit which a body deſcribes, if attracted in lines perpendicular to the orbit, is as the force of attraction directly, and the ſquare of the velocity inverſely. I eſtimate the curvatures of lines, compared one with another, according to the evaneſcent proportion of the ſines or tangents of their angles of contact to equal radij, ſuppoſing thoſe radij to be infinitely diminiſhed. But the attraction of the Moon towards the Earth in the ſyzygies, is the exceſs of its gravity

gravity towards the Earth above the force of the Sun $2PK$ (ſee *Fig.* prop. 25.) bywhich force, the accelerative gravity of the Moon towards the Sun exceeds the accelerative gravity of the Earth towards the Sun, or is exceeded by it. But in the quadratures that attraction is the ſum of the gravity of the Moon towards the Earth, and the Sun's force KT, by which the Moon is attracted towards the Earth. And theſe attractions, putting N for $\frac{AT+CT}{2}$ are nearly as $\frac{178725}{AT^2} + \frac{2000}{CT \times N}$ and $\frac{178725}{CT^2} + \frac{1000}{AT \times N}$, or as $178725\,N \times CT^2 - 2000\,AT^2 \times CT$, and $178725\,N \times AT^2 + 1000\,CT^2 \times AT$. For if the accelerative gravity of the Moon towards the Earth be repreſented by the number 178725, the mean force ML, which in the quadratures is PT or TK, and draws the Moon towards the Earth, will be 1000; and the mean force TM, in the ſyzygies will be 3000; from which, if we ſubſtract the mean force ML, there will remain 2000, the force by which the Moon in the ſyzygies is drawn from the Earth, and which we above called $2PK$. But the velocity of the Moon in the ſyzygies A and B, is to its velocity in the quadratures C and D, as CT to AT, and the moment of the area, which the Moon by a radius drawn to the Earth deſcribes in the ſyzygies, to the moment of that area *deſcribed* in the quadratures conjunctly; that is, as 11073 CT to 10973 AT. Take this ratio twice inverſely, and the former ratio once directly, and the curvature of the orb of the Moon in the ſyzygies will be to the curvature thereof in the quadratures, as $120406729 \times 178725\,AT^2 \times CT^2 \times N - 120406729 \times 2000\,AT^4 \times CT$, to $122611329 \times 178725\,AT^2 \times CT^2 \times N + 122611329 \times 1000\,CT^4 \times AT$, that is, as $2151969\,AT \times CT \times N - 24081\,AT^3$ to $2191371\,AT \times CT \times N + 12261\,CT^3$.

Becauſe

Becauſe the figure of the Moon's orbit is unknown, let us, in its ſtead, aſſume the ellipſe *DBCA*, *Pl.* 10 *Fig.* 5. in the centre of which we ſuppoſe the Earth to be ſituated, and the greater axe *de* to lie between the quadratures, as the leſſer *AB* between the ſyzygies. But ſince the plane of this ellipſe is revolved about the Earth by an angular motion, and the orbit, whoſe curvature we now examine ſhould be deſcribed in a plane void of ſuch motion; we are to conſider the figure which the Moon, while it is revolved in that ellipſe, deſcribes in this plane, that is to ſay the figure *Cpa*, the ſeveral points *p* of which are found by aſſuming any point *P* in the ellipſe, which may repreſent the place of the Moon, and drawing *Tp* equal to *TP*, in ſuch manner that the angle *PTp* may be equal to the apparent motion of the Sun from the time of the laſt quadrature in *C*; or (which comes to the ſame thing) that the angle *CTp* may be to the angle *CTP*, as the time of the ſynodic revolution of the Moon to the time of the periodic revolution thereof, or as $29^{d}.\ 12^{h}.\ 44'$, to $27^{d}.\ 7^{h}.\ 43'$. If therefore in this proportion we take the angle *CTa* to the right angle *CTA*, and make *Ta* of equal length with *TA*; we ſhall have *a* the lower, and *C* the upper apſis of this orbit. But by computation I find, that the difference betwixt the curvature of this orbit *Cpa* at the vertex *a*, and the curvature of a circle deſcribed about the centre *T*, with the interval *TA*, is to the difference betwixt the curvature of the ellipſe at the vertex *A*, and the curvature of the ſame circle, in the duplicate proportion of the angle *CTP* to the angle *CTp*; and that the curvature of the ellipſe in *A*, is to the curvature of that circle, in the duplicate proportion of *TA* to *TC*; and the curvature of that circle to the curvature of a circle deſcribed about the centre *T* with the interval *TC*, as *TC* to *TA*; but that the curvature of this *laſt arch* is to the curvature of the ellipſe in *C*, in the duplicate

plicate proportion of TA to TC; and that the difference betwixt the curvature of the ellipſe in the vertex C, and the curvature of this laſt circle, is to the difference betwixt the curvature of the figure Tpa, at the vertex C, and the curvature of this ſame *laſt* circle, in the duplicate proportion of the angle CTp to the angle CTP. All which proportions are eaſily drawn from the ſines of the angles of contact, and of the differences of thoſe angles. But by comparing thoſe proportions together, we find the curvature of the figure Cpa at a, to be to its curvature at C, as $AT^3 . \times \frac{16824}{100000} CT^2\ AT$ to $CT^3 + \frac{16824}{100000} AT^2 \times CT$. Where the number $\frac{16824}{100000}$ repreſents the difference of the ſquares of the angles CTP and CTp, applied to the ſquare of the leſſer angle CTP; or (which is all one) the difference of the ſquares of the times $27^d.\ 7^h.\ 43'$, and $29^d.\ 12^h.\ 44'$. applied to the ſquare of the time $27^d.\ 7^h.\ 43'$.

Since therefore a repreſents the ſyzygy of the Moon, and C its quadrature, the proportion now found muſt be the ſame with that proportion of the curvature of the Moon's orb in the ſyzygies, to the curvature thereof in the quadratures, which we found above. Therefore, in order to find the proportion of CT to AT, Let us multiply the extremes and the means, and the terms which come out applied to $AT \times CT$, become $2062{,}79\ CT^4 - 2151969\ N \times CT^3 + 368676\ N \times AT \times CT^2 + 36342\ AT^2 \times CT^2 - 362047\ N \times AT^2 \times CT + 2191371\ N \times AT^3 + 4051{,}4\ AT^4 = 0$. Now if for the half ſum N of the terms AT and CT, we put 1, and x for their half difference, then CT will be $= 1 + x$, and $AT = 1 - x$. And ſubſtituting thoſe values in the equation, after reſolving thereof, we ſhall find $x = 0{,}00719$; and from thence the ſemidiameter $CT = 1{,}00719$, and the ſemidiameter $AT = 0{,}99281$, which numbers are nearly as $70\frac{1}{24}$, and $69\frac{1}{24}$. Therefore the Moon's diſtance from the Earth

Earth in the ſyzygies, is to its diſtance in the quadratures (ſetting aſide the conſideration of eccentricity) as $69\frac{1}{4}$ to $70\frac{1}{4}$; or in round numbers as 69 to 70.

Proposition XXIX. Problem X.

To find the variation of the Moon.

This inequality is owing partly to the elliptic figure of the Moon's orbit, partly to the inequality of the moments of the area which the Moon by a radius drawn to the Earth deſcribes. If the Moon *P* revolved in the ellipſe *DBCA*, about the Earth quieſcent in the centre of the ellipſe, and by the radius *TP*, drawn to the Earth, deſcribed the area *CTP*, proportional to the time *of deſcription*; and the greateſt ſemidiameter *CT* of the ellipſe was to the leaſt *TA* as 70 to 69; the tangent of the angle *CTP* would be to the tangent of the angle of the mean motion computed from the quadrature *C*, as the ſemidiameter *TA* of the ellipſe, to its ſemidiameter *TC*, or as 69 to 70. But the deſcription of the area *CTP*, as the Moon advances from the quadrature to the ſyzygy, ought to be in ſuch manner accelerated, that the moment of the area in the Moon's ſyzygy, may be to the moment thereof in its quadrature, as 11073 to 10973; and that the exceſs of the moment in any intermediate place *P*, above the moment in the quadrature, may be as the ſquare of the ſine of the angle *CTP*. Which we may effect with accuracy enough, if we diminiſh the tangent of the angle *CTP*, in the ſubduplicate proportion of the number 10973 to the number 11073, that is, in proportion of the number 68,6877 to the number 69. Upon which account the tangent of the angle *CTP*, will now be to the tangent of the mean motion, as 68,6877 to 70; and the angle *CTP*, in the octants, where the mean

mean motion is 45°, will be found 44°. 27′. 28″. which ſubſtracted from 45°. the angle of the mean motion, leaves the greateſt variation 32′. 32″. Thus it would be, if the Moon in paſſing from the quadrature to the ſyzygy, deſcribed an angle *CTA* of 90 degrees only. But becauſe of the motion of the Earth, by which the Sun is apparently transferr'd *in conſequentia*, the Moon, before it overtakes the Sun, deſcribes an angle *CTa*, greater than a right angle, in the proportion of the time of the ſynodic revolution of the Moon, to the time of its periodic revolution, that is, in the proportion of $29^{d}.\ 12^{h}.\ 44'$. to $27^{d}.\ 7^{h}.\ 43'$. Whence it comes to paſs, that all the angles about the centre *T*, are dilated in the ſame proportion, and the greateſt variation, which otherwiſe would be *but* 32′. 32″, now augmented in the ſaid proportion becomes 35′. 10″.

And this is its magnitude in the mean diſtance of the Sun from the Earth, neglecting the differences, which may ariſe from the curvature of the *orbis magnus*, and the ſtronger action of the Sun upon the Moon when horn'd and new, than when gibbous and full. In other diſtances, of the Sun from the Earth, the greateſt variation is in a proportion compounded of the duplicate proportion of the time of the ſynodic revolution of the Moon (the time of the year being given) directly, and the triplicate proportion of the diſtance of the Sun from the Earth, inverſely. And therefore, in the apogee of the Sun, the greateſt variation is 33′. 14″, and in its perigee, 37′. 11″, if the eccentricity of the Sun is to the tranſverſe ſemidiameter of the *orbis magnus*, as $16\frac{1}{16}$ to 1000.

Hitherto we have inveſtigated the variation in an orb not eccentric, in which, to wit, the Moon in its octants is always in its mean diſtance from the Earth. If the Moon, on account of its eccentricity, is more or leſs removed from the Earth, than if placed in this orb, the variation may be ſomething greater, or ſomething

Plate X. Vol. II. P. 272.

Fig. 1. p. 241.

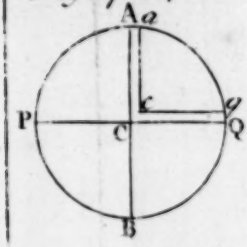


Fig. 2. p. 258.

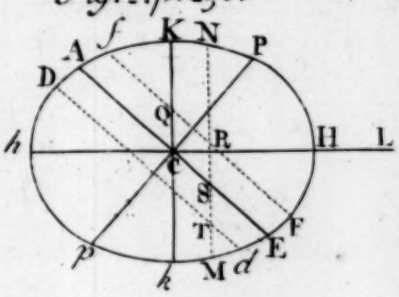


Fig. 3. p. 262.

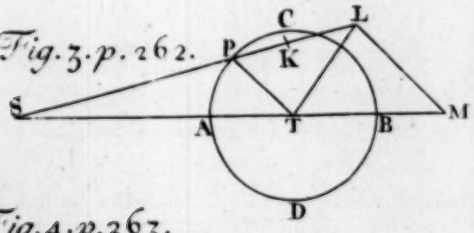


Fig. 4. p. 263.

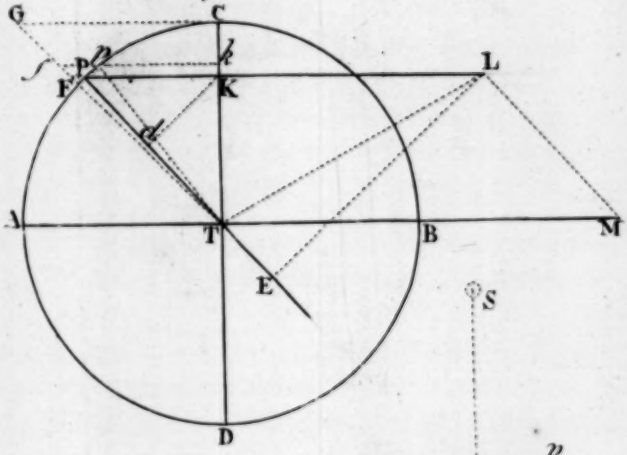
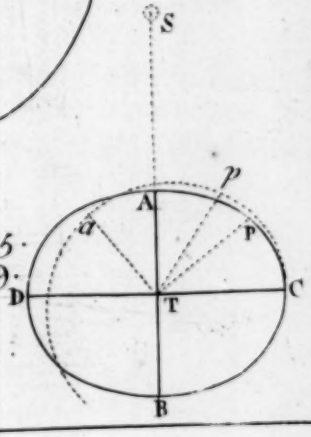


Fig. 5. p. 269. 271.



thing leſs, than according to this rule. But I leave the exceſs or defect to the determination of aſtronomers from the phænomena.

Proposition XXX. Problem XI.

To find the horary motion of the nodes of the Moon in a circular orbit, Pl. 11. Fig. 1.

Let *S* repreſent the Sun, *T* the Earth, *P* the Moon, *NPn* the orbit of the Moon, *Npn* the orthographic projection of the orbit upon the plane of the ecliptic; *N*, *n* the nodes; *nTNm*, the line of the nodes produced indefinitely; *PI*, *PK* perpendiculars upon the lines *ST*, *Qq*; *Pp* a perpendicular upon the plane of the ecliptic; *A*, *B* the Moon's ſyzygies in the plane of the ecliptic; *AZ* a perpendicular let fall upon *Nn*, the line of the nodes; *Q*, *q* the quadratures of the Moon in the plane of the ecliptic, and *pK*, a perpendicular on the line *Qq* lying between the quadratures. The force of the Sun to diſturb the motion of the Moon (by prop. 25.) is twofold, one proportional to the line *LM*, the other to the line *MT*, in the ſcheme of that propoſition. And the Moon by the former force is drawn towards the Earth, by the latter towards the Sun, in a direction parallel to the right line *ST* joining the Earth and the Sun. The former force *LM* acts in the direction of the plane of the Moon's orbit, and therefore makes no change upon the ſituation thereof, and is upon that account to be neglected. The latter force *MT*, by which the plane of the Moon's orbit is diſturbed, is the ſame with the force 3 *PK* or 3 *IT*. And this force (by prop. 25.) is to the force, by which the Moon may, in its periodic time, be uniformly revolved in a circle about the Earth at reſt, as 3 *IT* to the radius of the circle multiplied by the

number 178,725, or as IT to the radius thereof multiplied by 59,575. But in this calculus, and all that follows I conſider all the lines drawn from the Moon to the Sun, as parallel to the line which joins the Earth and the Sun, becauſe what inclination there is, almoſt as much diminiſhes all effects in ſome caſes, as it augments them in others, and we are now enquiring after the mean motions of the nodes, neglecting ſuch niceties as are of no moment, and would only ſerve to render the calculus more perplext.

Now ſuppoſe PM to repreſent an arc which the Moon deſcribes in the leaſt moment of time, and ML a little line, the half of which the Moon, by the impulſe of the ſaid force 3 IT, would deſcribe in the ſame time. And joining PL, MP, let them be produced to m and l, where they cut the plane of the ecliptic, and upon Tm let fall the perpendicular PH. Now ſince the right line ML is parallel to the plane of the ecliptic, and therefore can never meet with the right line ml which lies in that plane, and yet both thoſe right lines lye in one common plane LM Pml, they will be parallel, and upon that account the triangles LMP, lmP will be ſimilar. And ſeeing MPm lies in the plane of the orbit, in which the Moon did move while in the place P; the point m will fall upon the line Nn, which paſſes through the nodes N, n, of that orbit. And becauſe the force by which the half of the little line LM is generated, if the whole had been together, and at once impreſſed in the point P, would have generated that whole line, and cauſed the Moon to move in the arc whoſe chord is LP; that is to ſay, would have transferred the Moon from the plane $MPmT$ into the plane $LPlT$; therefore the angular motion of the nodes generated by that force, will be equal to the angle mTl. But ml is to mP, as ML to MP; and ſince MP, becauſe of the time given, is alſo given, ml will be as the rectangle

tangle $ML \times mP$, that is, as the rectangle $IT \times mP$. And, if Tml is a right angle, the angle mTl will be as $\frac{ml}{Tm}$ and therefore as $\frac{IT \times Pm}{Tm}$, that is, (becauſe Tm and mP, TP and PH are proportional) as $\frac{IT \times PH}{TP}$; and therefore, becauſe TP is given, as $IT \times PH$. But if the angle Tml or STN is oblique, the angle mTl will be yet leſs, in proportion of the ſine of the angle STN to the radius, or AZ to AT. And therefore the velocity of the nodes, is as $IT \times PH \times AZ$, or as the ſolid content of the ſines of the three angles, TPI, PTN, and STN.

If theſe are right angles, as happens when the nodes are in the quadratures, and the Moon in the ſyzygy, the little line ml will be removed to an infinite diſtance, and the angle mTl will become equal to the angle mPl. But in this caſe the angle mPl is to the angle PTM, which the Moon in the ſame time by its apparent motion deſcribes about the Earth, as 1 to 59,575. For the angle mPl is equal to the angle LPM, that is, to the angle of the Moon's deflexion from a rectilinear path, which angle, if the gravity of the Moon ſhould have then ceaſed, the ſaid force of the Sun 3 IT would by it ſelf have generated in that given time; and the angle PTM is equal to the angle of the Moon's deflexion from a rectilinear path, which angle, if the force of the Sun 3 IT ſhould have then ceaſed, the force alone by which the Moon is retained in its orbit would have generated in the ſame time. And theſe forces (as we have above ſhew'd) are, the one to the other, as 1 to 59,575. Since therefore, the mean horary motion of the Moon (in reſpect of the fixt Stars) is $32'.\ 56''.\ 27'''.\ 12\frac{1}{2}^{iv}$, the horary motion of the node in this caſe will be $35''.\ 10'''.\ 33^{iv}.\ 12^{v}$. But in other caſes, the horary motion will be to $33''.$

$10'''.\ 33^{iv}.\ 12^{v}.$ as the ſolid content of the ſines of the three angles TPI, PTN and STN (or of the diſtances of the Moon from the quadrature, of the Moon from the node, and of the node from the Sun) to the cube of the radius. And as often as the ſine of any angle is changed from poſitive to negative, and from negative to poſitive, ſo often muſt the regreſſive be changed into a progreſſive, and the progreſſive into a regreſſive motion. Whence it comes to paſs, that the nodes are progreſſive, as often as the Moon happens to be placed between either quadrature, and the node neareſt to that quadrature. In other caſes, they are regreſſive, and by the exceſs of the regreſs above the progreſs, they are monthly transferred *in antecedentia*.

Cor. 1. Hence if from P and M, the extreme points of a leaſt arc PM, *Pl.* 11. *Fig.* 2. on the line Qq joining the quadratures we let fall the perpendiculars PK, Mk, and produce the ſame till they cut the line of the nodes Nn, in D and d; the horary motion of the nodes will be as the area $MPDd$, and the ſquare of the tine AZ conjunctly. For let PK, PH and AZ be the three ſaid ſines, *viz.* PK the ſine of the diſtance of the Moon from the quadrature, PH the ſine of the diſtance of the Moon from the node, and AZ the ſine of the diſtance of the node from the Sun: and the velocity of the node will be as the ſolid content of $PK \times PH \times AZ$. But PT is to PK, as PM to Kk; and therefore, becauſe PT and PM are given, Kk will be as PK. Likewiſe AT is to PD, as AZ to PH, and therefore PH is as the rectangle $PD \times AZ$, and by compounding thoſe proportions, $PK \times PH$ is as the ſolid content $Kk \times PD \times AZ$, and $PK \times PH \times AZ$, as $Kk \times PD \times AZ^2$. that is, as the area $PDdM$ and AZ^2 conjunctly. *Q. E. D.*

Cor.

Cor. 2. In any given poſition of the nodes, their mean horary motion is half their horary motion in the Moon's ſyzygies; and therefore is to $16''.\ 35'''.\ 16^{iv}.\ 36^{v}$, as the ſquare of the ſine of the diſtance of the nodes from the ſyzygies to the ſquare of the radius, or as AZ^2, to AT^2. For if the Moon, by an uniform motion deſcribes the ſemicircle QAq, the ſum of all the areas $PDdM$ during the time of the Moon's paſſage from Q to M, will make up the area $QMdE$, terminating at the tangent QE of the circle. And by the time that the Moon has arrived at the point n, that ſum will make up the whole area $EQAn$ deſcribed by the line PD; but when the Moon proceeds from n to q, the line PD will fall without the circle, and will deſcribe the area nqe, terminating at the tangent qe of the circle; which area, becauſe the nodes were before regreſſive, but are now progreſſive, muſt be ſubducted from the former area, and being it ſelf equal to the area QEN, will leave the ſemicircle $NQAn$. While therefore the Moon deſcribes a ſemicircle, the ſum of all the areas $PDdM$ will be the area of that ſemicircle; and while the Moon deſcribes a complete circle, the ſum of thoſe areas will be the area of the whole circle. But the area $PDdM$, when the Moon is in the ſyzygies is the rectangle of the arc PM into the radius PT; and the ſum of all the areas, *every one* equal to this area, in the time that the Moon deſcribes a complete circle is the rectangle of the whole circumference into the radius of the circle; and this rectangle, being double the area of the circle, will be double the quantity of the former ſum. If therefore the nodes went on with that velocity uniformly continued, which they acquire in the Moon's ſyzygies, they would deſcribe a ſpace double of that which they deſcribe in fact; and therefore the mean motion, by which, if uniformly continued, they would deſcribe the ſame ſpace with that which they do in fact deſcribe

by an unequal motion, is *but* one half of that motion which they are poſſeſſed of in the Moon's ſyzygies. Wherefore ſince their greateſt horary motion, if the nodes are in the quadratures, is $33''.\ 10'''.\ 33^{iv}.\ 12^{v}$, their mean horary motion in this caſe will be $16''.\ 35'''.\ 16^{iv}.\ 36^{v}$. And ſeeing the horary motion of the nodes is every where as AZ^2 and the area $PDdM$ conjunctly, and therefore in the Moon's ſyzygies, the horary motion of the nodes is as AZ^2 and the area $PDdM$ conjunctly, that is, (becauſe the area $PDdM$ deſcribed in the ſyzygies is given) as AZ^2; therefore the mean motion alſo will be as AZ^2, and therefore when the nodes are without the quadratures, this motion will be to $16''.\ 35'''.\ 16^{iv}.\ 36^{v}$. as AZ^2 to AT^2. *Q. E. D.*

Proposition XXXI. Problem XII.

To find the horary motion of the nodes of the Moon in an elliptic orbit, Pl. 12. Fig. 1.

Let $Qpmaq$ repreſent an ellipſe, deſcribed with the greater axe Qq, and the leſſer axe ab; $QAqB$ a circle circumſcribed; T the Earth in the common centre of both; S the Sun; p the Moon moving in this ellipſe; and pm an arc which it deſcribes in the leaſt moment of time; N and n the nodes joined by the line Nn; pK and mk perpendiculars upon the axe Qq, produced both ways till they meet the circle in P and M, and the line of the nodes in D and d. And if the Moon, by a radius drawn to the Earth, deſcribes an area proportional to the time *of deſcription*, the horary motion of the node in the ellipſe will be as the area $pDdm$, and AZ^2 conjunctly.

For let PF touch the circle in P, and produced meet TN in F; and pf touch the ellipſe in p, and produced meet the ſame TN in f, and both tangents concur

Plate XI. Vol. II. P. 278.

Fig. 1. p. 273.

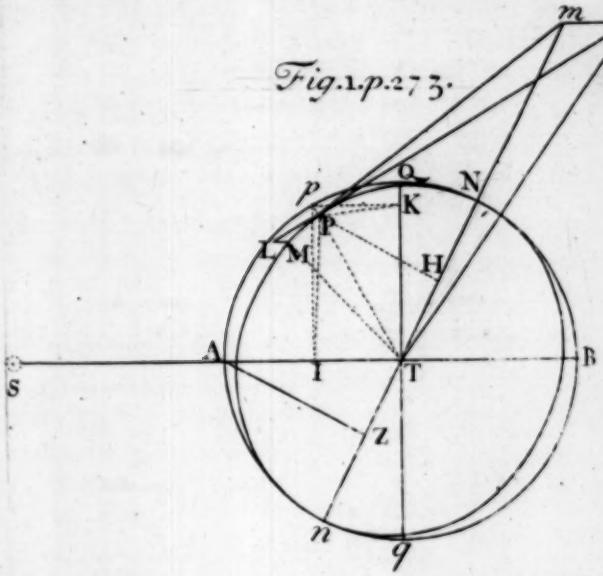
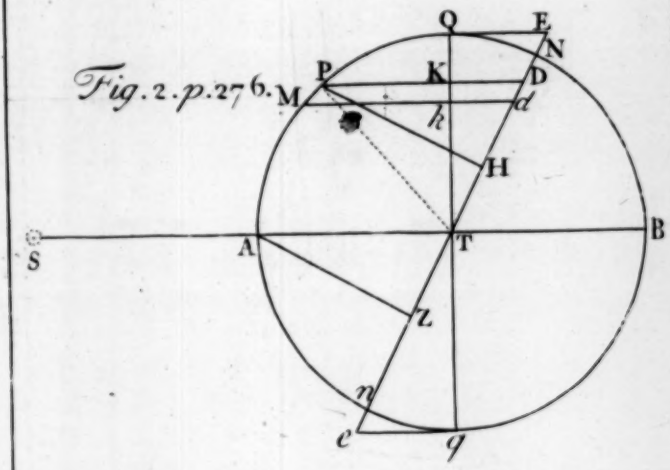


Fig. 2. p. 276.



in the axe TQ at Υ. And let ML repreſent the ſpace which the Moon, by the impulſe of the abovementioned force $3IT$ or $3PK$, would deſcribe with a tranſverſe motion, in the mean time while, revolving in the circle it deſcribes the arc PM; and ml denote the ſpace, which the Moon revolving in the ellipſe would deſcribe in the ſame time by the impulſe of the ſame force $3IT$ or $3PK$; and let LP and lp be produced till they meet the plane of the ecliptic in G and g, and FG and fg be joined, of which FG produced may cut pf, pg, and TQ in c, e and R reſpectively; and fg produced may cut TQ in r. Becauſe the force $3IT$ or $3PK$ in the circle, is to the force $3IT$ or $3pK$ in the ellipſe, as PK to pK, or as AT to aT; the ſpace ML, generated by the former force, will be to the ſpace ml generated by the latter, as PK to pK, that is, becauſe of the ſimilar figures $P\Upsilon Kp$, and $F\Upsilon Rc$, as FR to cR. But (becauſe of the ſimilar triangles PLM, PGF) ML is to FG, as PL to PG, that is (on account of the parallels Lk, PK, GR) as pl to pe, that is, (becauſe of the ſimilar triangles plm, cpe) as lm to ce; and inverſely as LM is to lm, or as FR is to cR, ſo is FG to ce. And therefore if fg was to ce, as fy to $c\Upsilon$, that is as fr to cR, (that is as fr to FR and FR to cR conjunctly, that is, as fT to FT, and FG to ce conjunctly) becauſe the ratio of FG to ce, expung'd on both ſides, leaves the ratios fg to FG and fT to FT, fg would be to FG, as fT to FT; and therefore the angles which FG and fg would ſubtend at the Earth T would be equal each to other. But theſe angles, (by what we have ſhew'd in the preceding propoſition) are the motions of the nodes, while the Moon deſcribes, in the circle the arc PM, in the ellipſe the arc pm: And therefore the motions of the nodes in the circle, and in the ellipſe, would be equal to each other. Thus I ſay it would be if fg was to ce, as fT to $c\Upsilon$, that

is, if fg was equal to $\frac{ce \times f\Upsilon}{c\Upsilon}$. But becauſe of the ſimilar triangles fgp, cep, fg is to ce as fp to cp; and therefore fg is equal to $\frac{ce \times fp}{cp}$; and therefore the angle which fg ſubtends in fact, is to the former angle which FG ſubtends, that is to ſay, the motion of the nodes in the ellipſe is to the motion of the ſame in the circle, as this fg or $\frac{ce \times fp}{cp}$, to the former fg or $\frac{ce \times f\Upsilon}{c\Upsilon}$, that is as $fp \times c\Upsilon$ to $f\Upsilon \times cp$, or as fp to $f\Upsilon$, and $c\Upsilon$ to cp, that is, if ph parallel to TN meet FP in h, as Fh to $F\Upsilon$ and $F\Upsilon$ to FP; that is, as Fh to FP or Dp to DP, and therefore as the area $Dpmd$ to the area $DPMd$. And therefore ſeeing (by corol. 1. prop. 30.) the latter area and AZ^2 conjunctly are proportional to the horary motion of the nodes in the circle, the former area and AZ^2 conjunctly will be proportional to the horary motion of the nodes in the ellipſe. $\underset{\frown}{Q}.E.D.$

Cor. Since therefore in any given poſition of the nodes, the ſum of all the areas $pDdm$, in the time while the Moon is carried from the quadrature to any place m, is the area $mp\underset{\frown}{QE}d$ terminated at the tangent of the ellipſe $\underset{\frown}{QE}$; and the ſum of all thoſe areas, in one entire revolution, is the area of the whole ellipſe: the mean motion of the nodes in the ellipſe will be to the mean motion of the nodes in the circle, as the ellipſe to the circle; that is, as $T\alpha$ to TA or 69 to 70. And therefore ſince (by corol. 2. prop. 30.) the mean horary motion of the nodes in the circle is to $16''.35'''.16^{iv}.36^{v}$. as AZ^2 to AT^2, if we take the angle $16'.21'''.3^{iv}.30^{v}$. to the angle $16'.35'''.16^{iv}.36^{v}$. as 69 to 70, the mean horary motion of the nodes in the ellipſe will be to $16''.21'''.33^{iv}.30^{v}$. as AZ^2 to AT^2; that is, as the ſquare of the ſine of the diſtance of the node from the Sun to the ſquare of the radius.

But

But the Moon, by a radius drawn to the Earth, deſcribes the area in the ſyzygies with a greater velocity than it does that in the quadratures, and upon that account the time is contracted in the ſyzygies, and prolong'd in the quadratures; and together with the time the motion of the nodes is likewiſe augmented or diminiſh'd. But the moment of the area in the quadrature of the Moon, was to the moment thereof in the ſyzygies as 10973 to 11073; and therefore the mean moment in the octants is to the exceſs in the ſyzygies, and to the defect in the quadratures, as 11023, the half ſum of thoſe numbers, to their half difference 50. Wherefore ſince the time of the Moon's mora in the ſeveral little equal parts of its orbit, is reciprocally as its velocity; the mean time in the octants will be to the exceſs of the time in the quadratures, and to the defect *of the time* in the ſyzygies, ariſing from this cauſe, nearly as 11023 to 50. But reckoning from the quadratures to the ſyzygies, I find that the exceſs of the moments of the area, in the ſeveral places, above the leaſt moment in the quadratures, is nearly as the ſquare of the ſine of the Moon's diſtance from the quadratures; and therefore the difference betwixt the moment in any place, and the mean moment in the octants, is as the difference betwixt the ſquare of the ſine of the Moon's diſtance from the quadratures, and the ſquare of the ſine of 45 degrees, or half the ſquare of the radius; and the increment of the time in the ſeveral places between the octants and quadratures, and the decrement thereof between the octants and ſyzygies is in the ſame proportion. But the motion of the nodes while the Moon deſcribes the ſeveral little equal parts of its orbit, is accelerated or retarded in the duplicate proportion of the time. For that motion while the Moon deſcribes PM, is (*cæteris paribus*) as ML, and ML is in the duplicate proportion of the time. Wherefore the motion of the nodes in the ſyzygies,

in

in the time while the Moon deſcribes giv'n little parts of its orbit, is diminiſh'd in the duplicate proportion of the number 11073 to the number 11023; and the decrement is to the remaining motion as 100 to 10973; but to the whole motion as 100 to 11073 nearly. But the decrement in the places between the octants and ſyzygies, and the increment in the places between the octants and quadratures, is to this decrement, nearly as the whole motion in theſe places to the whole motion in the ſyzygies, and the difference betwixt the ſquare of the ſine of the Moon's diſtance from the quadrature, and the half ſquare of the radius, to the half ſquare of the radius conjunctly. Wherefore, if the nodes are in the quadratures, and we take two places, one on one ſide, one on the other, equally diſtant from the octant and other two diſtant by the ſame interval, one from the ſyzygy, the other from the quadrature, and from the decrements of the motions in the two places between the ſyzygy and octant, we ſubtract the increments of the motions in the two other places between the octant and the quadrature; the remaining decrement will be equal to the decrement in the ſyzygy: as will eaſily appear by computation. And therefore the mean decrement, which ought to be ſubducted from the mean motion of the nodes, is the fourth part of the decrement in the ſyzygy. The whole horary motion of the nodes in the ſyzygies (when the Moon by a radius drawn to the Earth, was ſuppos'd to deſcribe an area proportional to the time) was $32''.\ 42'''.\ 7^{iv}.$ And we have ſhew'd, that the decrement of the motion of the nodes, in the time while the Moon, now moving with greater velocity, deſcribes the ſame ſpace, was to this motion as 100 to 11073; and therefore this decrement is $17''.\ 43^{iv}.\ 11^{v}.$ The fourth part of which $4'''.\ 25^{iv}.\ 48^{v}.$ ſubtracted from the mean horary motion above found $16''.\ 21'''.\ 3^{iv}.\ 30^{v}.$ leaves $16''.\ 16'''.\ 37^{iv}.\ 42^{v}.$ their correct mean horary motion.

If

If the nodes are without the quadratures, and two places are conſider'd, one on one ſide, one on the other equally diſtant from the ſyzygies; the ſum of the motions of the nodes when the Moon is in thoſe places, will be to the ſum of their motions, when the Moon is in the ſame places and the nodes in the quadratures, as AZ^2, to AT^2. And the decrements of the motions, ariſing from the cauſes but now explained, will be mutually as the motions themſelves, and therefore the remaining motions will be mutually betwixt themſelves as AZ^2. to AT^2. And the mean motions will be as the remaining motions. And therefore in any giv'n poſition of the nodes, their correct mean horary motion is to $16''. 16'''. 37^{iv}. 42^{v}.$ as AZ^2, to AT^2. that is, as the ſquare of the ſine of the diſtance of the nodes from the ſyzygies to the ſquare of the radius.

PROPOSITION XXXIII. PROBLEM XIII.

To find the mean motion of the nodes of the Moon. Pl. 12. Fig. 2.

The yearly mean motion is the ſum of all the mean horary motions, throughout the courſe of the year. Suppoſe that the node is in *N*, and that after ev'ry hour is elaps'd, it is drawn back again to its former place; ſo that, notwithſtanding its proper motion, it may conſtantly remain in the ſame ſituation, with reſpect to the fixt Stars; while in the mean time the Sun *S*, by the motion of the Earth, is ſeen to leave the node and to proceed till it compleats its apparent annual courſe by an uniform motion. Let Aa repreſent a given leaſt arc, which the right line TS always drawn to the Sun, by its interſection with the circle NAn, deſcribes in the leaſt given moment of time; and the mean

mean horary motion (from what we have above ſhew'd) will be as AZ^2, that is (becauſe AZ and ZY are proportional) as the rectangle of AZ into ZY, that is, as the area $AZYa$. And the ſum of all the mean horary motions from the beginning will be as the ſum of all the areas $aYZA$, that is as the area NAZ. But the greateſt $AZYa$ is equal to the rectangle of the arc Aa into the radius of the circle; and therefore the ſum of all theſe rectangles in the whole circle, will be to the like ſum of all the greateſt rectangles, as the area of the whole circle to the rectangle of the whole circumference into the radius, that is, as 1 to 2. But the horary motion correſponding to that greateſt rectangle, was $16''.16'''.37^{iv}.42^{v}$. and this motion in the complete courſe of the ſidereal year $365^d.6^h.9'$. amounts to $39°.38'.7''.50'''$. and therefore the half thereof $19°.49'.3''.55'''$. is the mean motion of the nodes correſponding to the whole circle. And the motion of the nodes, in the time while the Sun is carry'd from N to A is to $19°.49'.3''.55'''$. as the area NAZ to the whole circle.

Thus it would be if the node was after every hour drawn back again to its former place, that ſo, after a compleat revolution, the Sun at the year's end would be found again in the ſame node which it had left when the year begun. But becauſe of the motion of the node in the mean time, the Sun muſt needs meet the node ſooner, and now it remains that we compute the abreviation of the time. Since then the Sun, in the courſe of the year, travels 360 degrees, and the node in the ſame time by its greateſt motion would be carried $39°.38'.7''.50'''$, or 39,6355 degrees; and the mean motion of the node in any place N, is to its mean motion in its quadratures, as AZ^2 to AT^2: the motion of the Sun will be to the motion of the node in N, as $360\,AT^2$, to $39{,}6355\,AZ^2$; that is, as 9,0827646 AT^2 to AZ^2. Wherefore if we ſuppoſe the circumference

ference NAn of the whole circle to be divided into little equal parts, ſuch as Aa, the time in which the Sun would deſcribe the little arc Aa, if the circle was quieſcent, will be to the time of which it would deſcribe the ſame arc, ſuppoſing the circle together with the nodes to be revolv'd about the centre T, reciprocally as $9,0827646\ AT^2$ to $9,0827646\ AT^2 + AZ^2$. For the time is reciprocally as the velocity with which the little arc is deſcrib'd, and this velocity is the ſum of the velocities of both Sun and node. If therefore the ſector NTA repreſent the time in which the Sun by it ſelf, without the motion of the node, would deſcribe the arc NA, and the indefinitely ſmall part ATa of the ſector repreſent the little moment of the time, in which it would deſcribe the leaſt arc Aa; and (letting fall $a\Upsilon$ perpendicular upon Nn) if in AZ we take dZ, of ſuch length, that the rectangle of dZ into $Z\Upsilon$, may be to the leaſt part ATa of the ſector, as AZ^2 to $9,0827646\ AT^2 + AZ^2$, that is to ſay, that dZ may be to $\frac{1}{2}AZ$, as AT^2 to $9,0827646\ AT^2 + AZ^2$; the rectangle of dZ into $Z\Upsilon$ will repreſent the decrement of the time ariſing from the motion of the node, while the arc Aa is deſcrib'd. And if the curve $NdGn$ is the locus where the point d is always found, the curvilinear area NdZ will be as the whole decrement *of time* while the whole arc NA is deſcrib'd. And therefore, the exceſs of the ſector NAT above the area NdZ will be as the whole time. But becauſe the motion of the node in a leſs time, is leſs in proportion of the time, the area $AaYZ$ muſt alſo be diminiſh'd in the ſame proportion. Which may be done by taking in AZ the line eZ of ſuch length, that it may be to the length of AZ, as AZ^2 to $9,0827646\ AT^2 + AZ^2$. For ſo the rectangle of eZ into $Z\Upsilon$, will be to the area $AZYa$, as the decrement of the time in which the arc Aa is deſcrib'd, to the whole time in which it would

would have been deſcrib'd, if the node had been quieſcent. And therefore that rectangle will be as the decrement of the motion of the node. And if the curve *NeFn* is the locus of the point *e*, the whole area *NeZ*, which is the ſum of all the decrements *of that motion*, will be as the whole decrement *thereof* during the time in which the arc *AN* is deſcrib'd; and the remaining area *NAe* will be as the remaining motion, which is the true motion of the node, during the time in which the whole arc *NA* is deſcrib'd by the joint motions of both Sun and node. Now the area of the ſemicircle is to the area of the figure *NeFn* found by the method of infinite ſeries, nearly as 793 to 60. But the motion correſponding *or proportional* to the whole circle was 19° 49′. 3″. 55‴. and therefore the motion correſponding to double the figure *NeFn* is 1°. 29′. 58″. 2‴. which taken from the former motion leaves 18°. 19′. 5″. 53‴. the whole motion of the node with reſpect to the fixed Stars in the interval between two of its conjunctions with the Sun; and this motion ſubducted from the annual motion of the Sun 360°. leaves 341°. 40′. 54″. 7‴. the motion of the Sun in the interval between the ſame conjunctions. But as this motion is to the annual motion 360°. ſo is the motion of the node but juſt now found 18° 19′. 5″. 53‴. to its annual motion which will therefore be 19°. 18′. 1″. 23‴. And this is the mean motion of the nodes in the ſidereal year. By aſtronomical tables it is 19°. 21′. 21″. 50‴. The difference is leſs than $\frac{1}{300}$ part of the whole motion, and ſeems to ariſe from the eccentricity of the Moon's orbit, and its inclination to the plane of the ecliptic. By the eccentricity of this orbit, the motion of the nodes is too much accelerated, and on the other hand, by the inclination of the orbit, the motion of the nodes is ſomething retarded, and reduc'd to its juſt velocity.

PRO-

PROPOSITION XXXIII. PROBLEM XIV.

To find the true motion of the nodes of the Moon. Pl. 12. Fig. 3.

In the time which is as the area *NTA—NdZ* (in the preceding *Fig.*) that motion is as the area *NAe*, and is thence giv'n. But becauſe the calculus is too difficult it will be better to uſe the following conſtruction of the problem. About the centre *C*, with any interval *CD*, deſcribe the circle *BEFD*, produce *DC* to *A*, ſo as *AB* may be to *AC*, as the mean motion to half the mean true motion when the nodes are in their quadratures (that is, as 19°. 18′. 1″. 23‴. to 19°. 49′. 3″. 55‴. and therefore *BC* to *AC*, as the difference of thoſe motions 0°. 31′. 2″. 32‴. to the latter motion 19°. 49′. 3″. 55‴. that is, as 1 to $38\frac{3}{10}$). Then through the point *D*, draw the indefinite line *Gg*, touching the circle in *D*; and if we take the angle *BCE*, or *BCF*, equal to the double diſtance of the Sun from the place of the node, as found by the mean motion; and drawing *AE* or *AF*, cutting the perpendicular *DG* in *G*, we take another angle which ſhall be to the whole motion of the node, in the interval between its ſyzygies (that is to 9°. 11′. 3″.) as the tangent *DG* to the whole circumference of the circle *BED*; and add this *laſt* angle (for which the angle *DAG* may be us'd) to the mean motion of the nodes, while they are paſſing from the quadratures to the ſyzygies, and ſubtract it from their mean motion, while they are paſſing from the ſyzygies to the quadratures; we ſhall have their true motion. For the true motion ſo found will nearly agree with the true motion which comes out from aſſuming the time as the area *NTA—NdZ*, and the motion of the node as the area *NAe*,

as

as whoever will pleaſe to examine and make the computations will find. And this is the ſemi-menſtrual equation of the motion of the nodes. But there is alſo a menſtrual equation, but which is by no means neceſſary for finding of the Moon's latitude. For ſince the variation of the inclination of the Moon's orbit to the plane of the ecliptic is liable to a twofold inequality: the one ſemi-menſtrual, the other menſtrual: the menſtrual inequality *of this variation*, and the menſtrual equation of the nodes, ſo moderate and correct each other, that in computing the latitude of the Moon both may be neglected.

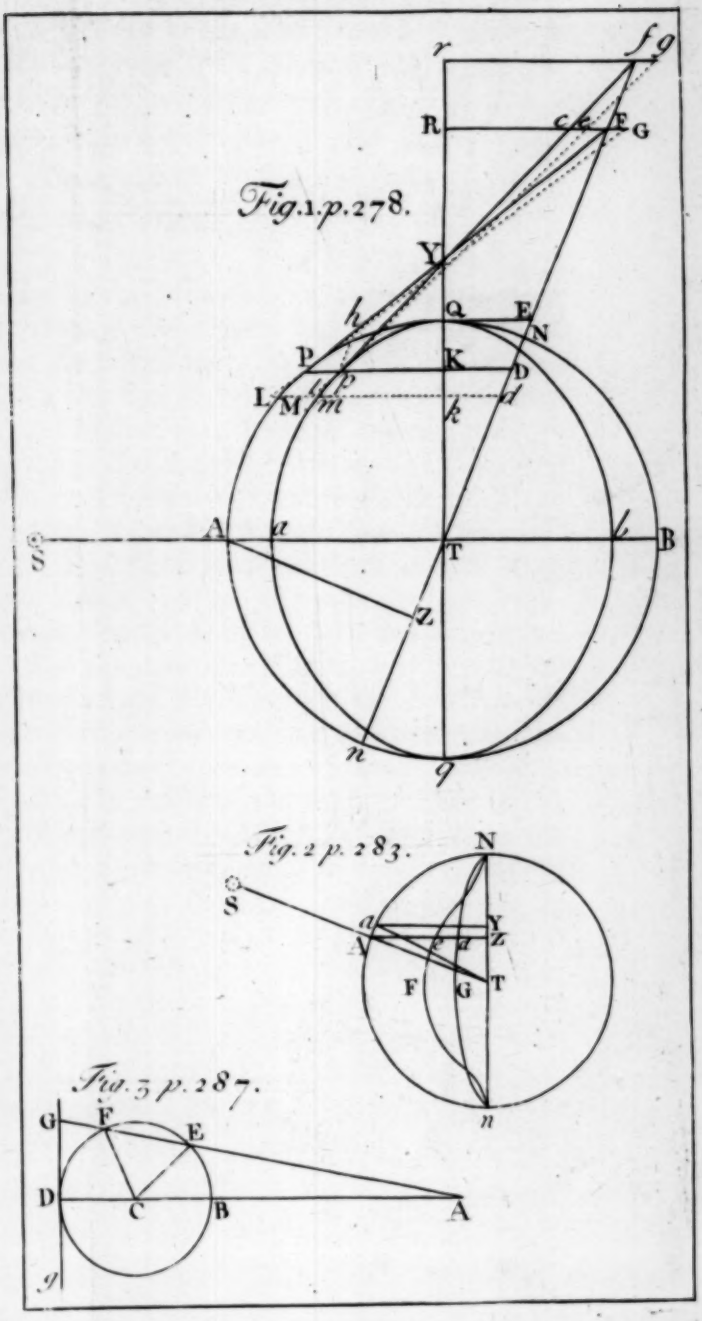
Cor. From this and the preceding prop. it appears that the nodes are quieſcent in their ſyzygies, but regreſſive in their quadratures, by an hourly motion of $16'.\ 19''.\ 26^{iv}$. And that the equation of the motion of the nodes in the octants is $1°.\ 30'$. all which exactly agree with the phænomena of the heavens.

Scholium.

Mr. *Machin* Aſtron. Prof. Greſh. and Dr. *Henry Pemberton* ſeparately found out the motion of the nodes by a different method. Mention has been made of this method in another place. Their ſeveral papers, both of which I have ſeen, contained two propoſitions, and exactly agreed with each other in both of them. Mr. *Machin*'s paper coming firſt to my hands, I ſhall here inſert it.

OF

Plate XII. Vol. II. P. 288.



Of the motion of the Moon's nodes.

Proposition I.

" *The mean motion of the Sun from the node, is* " *defined by a geometric mean proportional, be-* " *tween the mean motion of the Sun, and that* " *mean motion with which the Sun recedes* " *with the greateſt ſwiftneſs from the node* " *in the quadratures.*

" Let T (*Pl.* 13. *Fig.* 1.) be the Earth's place, " Nn the line of the Moon's nodes at any given time, " KTM a perpendicular thereto, TA a right line revol- " ving about the centre with the ſame angular velocity " with which the Sun and the node recede from one " another, in ſuch ſort that the angle between the qui- " eſcent right line Nn, and the revolving line TA, may " be always equal to the diſtance of the places of the " Sun and node. Now if any right line TK be divi- " ded into parts, TS and SK, and thoſe parts be taken " as the mean horary motion of the Sun to the mean " horary motion of the node in the quadratures, and " there be taken the right line TH, a mean proportio- " nal between the part TS and the whole TK, this " right line will be proportional to the Sun's mean mo- " tion from the node.

" For let there be deſcribed the circle $NKnM$ from " the centre T and with the radius TK, and about the " ſame centre, with the ſemi-axes TH and TN, let there " be deſcribed an ellipſis $NHnL$. And in the time in " which the Sun recedes from the node through the arc " Na, if there be drawn the right line Tba, the area of " the ſector NTa will be the exponent of the ſum of " the motions of the Sun and node in the ſame time. " Let therefore the extremely ſmall arc aA be that " which the right line Tba, revolving according to the " aboveſaid law, will uniformly deſcribe in a given par-

" ticle of time, and the extremely ſmall ſector TAa will " be as the ſum of the velocities with which the Sun " and node are carried two different ways in that time. " Now the Sun's velocity is almoſt uniform, its inequa- " lity being ſo ſmall as ſcarcely to produce the leaſt in- " equality in the mean motion of the nodes. The other " part of this ſum, namely the mean quantity of the ve- " locity of the node, is increaſed in the receſs from the " ſyzygies in a duplicate ratio of the ſine of its diſtance " from the Sun (by corol. prop. 31. of this book) and " being greateſt in its quadratures with the Sun in K, " is in the ſame ratio to the Sun's velocity as SK to TS, " that is, as (the difference of the ſquares of TK and " TH, or) the rectangle KHM to TH^2. But the " ellipſis NBH divides the ſector ATa, the exponent " of the ſums of theſe two velocities, into two parts " $ABba$ and BTb, proportional to the velocities. For " produce BT to the circle in β, and from the point " B let fall upon the greater axis the perpendicular BG, " which being produced both ways may meet the circle " in the points F and f; and becauſe the ſpace $ABba$ " is to the ſector TBb as the rectangle $AB\beta$ to BT^2, " (that rectangle being equal to the difference of the " ſquares of TA and TB, becauſe the right line $A\beta$ " is equally cut in T, and unequally in B;) therefore " when the ſpace $ABba$ is the greateſt of all in K, " this ratio will be the ſame as the ratio of the rectangle " KHM to HT^2. But the greateſt mean velocity of " the node was ſhewn above to be in that very ratio to " the velocity of the Sun; and therefore in the quadra- " tures the ſector ATa is divided into parts proportio- " nal to the velocities. And becauſe the rectangle KHM " is to HT^2, as FBf to BG^2, and the rectangle $AB\beta$ " is equal to the rectangle FBf; therefore the little a- " rea $ABba$, where it is greateſt, is to the remaining " ſector TBb, as the rectangle $AB\beta$ to BG^2. But the " ratio of theſe little areas always was as the rectangle

" $AB\beta$

" $AB\beta$ to BT^2, and therefore the little area $ABba$ in " the place A is leſs than its correſpondent little area in " the quadratures, in the duplicate ratio of BG to BT, " that is, in the duplicate ratio of the ſine of the Sun's " diſtance from the node. And therefore the ſum of all " the little areas $ABba$, to wit, the ſpace ABN will " be as the motion of the node in the time in which " the Sun hath been going over the arc NA ſince he " left the node. And the remaining ſpace, namely the " elliptic ſector NTB, will be as the Sun's mean moti- " on in the ſame time. And becauſe the mean annual " motion of the node is that motion which it performs " in the time that the Sun completes one period of its " courſe, the mean motion of the node from the Sun " will be to the mean motion of the Sun it ſelf, as the " area of the circle to the area of the ellipſis; that is as " the right line TK to the right line TH, which is a " comean proportional between TK and TS; or which " mes to the ſame, as the mean proportional TH to the " right line TS.

PROPOSITION II.

The mean motion of the Moon's nodes being given, to find their true motion.

" Let the angle A be the diſtance of the Sun from " the mean place of the node, or the Sun's mean motion " from the node. Then if we take the angle B, whoſe " tangent is to the tangent of the angle A, as TH to " TK, that is, in the ſubduplicate ratio of the mean ho- " rary motion of the Sun to the mean horary motion " of the Sun from the node, when the node is in the " quadrature, that angle B will be the diſtance of the " Sun from the node's true place. For join FT, and by " the demonſtration of the laſt proportion, the angle " FTN will be the diſtance of the Sun from the mean

" place of the node, and the angle *ATN* the diſtance " from the true place, and the tangents of theſe angles " are between themſelves as *TK* to *TH*.

" Cor. Hence the angle *FTA* is the equation of " the Moon's nodes, and the ſine of this angle where " it is greateſt in the octants, is to the radius as *KH* " to *TK* + *TH*. But the ſine of this equation in a- " ny other place *A* is to the greateſt ſine, as the ſine " of the ſums of the angles *FTN* + *ATN* to the radius; " that is, nearly as the ſine of double the diſtance of " the Sun from the mean place of the node (namely " 2 *FTN*) to the radius.

Scholium.

" If the mean horary motion of the nodes in the qua- " dratures be 16″. 16‴. 37ᵢᵛ. 42ᵛ. that is in a whole " ſidereal year 39°. 38′. 7″. 50‴. *TH* will be to " *TK* in the ſub-duplicate ratio of the number " 9,0827646 to the number 10, 827646, that is, as " 18, 6524761 to 19, 6524761. And therefore *TH* " is to *HK* as 18,6524761 to 1, that is, as the moti- " on of the Sun in a ſidereal year to the mean motion " of the node 19°. 18′. 1″. 23$\frac{2}{3}$‴.

" But if the mean motion of the Moon's nodes in " 20 Julian years is 386°. 50′. 15″. as is collected from " the obſervations made uſe of in the theory of the " Moon, the mean motion of the nodes in one ſidereal " year will be 19°. 20′. 31″. 58‴. And *TH* will be " to *HK* as 360°. to 19°. 20′. 31″. 58‴. that is, " as 18,61214 to 1, and from hence the mean horary " motion of the nodes in the quadratures will come out " 16″. 18‴. 48ᵢᵛ. And the greateſt equation of the " nodes in the octants will be 1° 29′. 57″.

Pro-

PROPOSITION XXXIV. PROBLEM XV.

To find the horary variation of the inclination of the Moon's orbit to the plane of the ecliptic.

Let *A* and *a*, (*Pl.* 13. *Fig.* 2.) repreſent the ſyzygies; *Q* and *q* the quadratures; *N* and *n* the nodes; *P* the place of the Moon in its orbit; *p* the orthographic projection of that place upon the plane of the ecliptic; and *mTl* the momentaneous motion of the nodes as above. If upon *Tm* we let fall the perpendicular *PG*, and joining *pG* we produce it till it meet *Tl* in *g*, and join alſo *Pg*; the angle *PGp* will be the inclination of the Moon's orbit to the plane of the ecliptic when the Moon is in *P*; and the angle *Pgp* will be the inclination of the ſame after a ſmall moment of time is elaps'd; and therefore the angle *GPg* will be the momentaneous variation of the inclination. But this angle *GPg* is to the angle *GTg*, as *TG* to *PG* and *Pp* to *PG* conjunctly. And therefore if for the moment of time we aſſume an hour; ſince the angle *GTg* (by prop. 30.) is to the angle 33″. 10‴. 33^{iv}. as $IT \times PG \times AZ$, to AT^3, the angle *GPg* (or the horary variation of the inclination) will be to the angle 33″. 10‴. 33^{iv}. as $IT \times AZ \times TG \times \frac{Pp}{PG}$ to AT^3. *Q. E. I.*

And thus it would be if the Moon was uniformly revolv'd in a circular orbit. But if the orbit is elliptical, the mean motion of the nodes will be diminiſh'd in proportion of the leſſer axis to the greater, as we have ſhewn above. And the variation of the inclination will be alſo diminiſh'd in the ſame proportion.

COR. 1. Upon *Nn* erect the perpendicular *TF*, and let *pM* be the horary motion of the Moon in the plane

of the ecliptic; upon QT let fall the perpendiculars pK, Mk, and produce them till they meet TF in H and h; then IT will be to AT, as Kk to Mp; and TG to Hp as TZ to AT; and therefore $IT \times TG$ will be equal to $\frac{Kk \times Hp \times TZ}{Mp}$, that is, equal to the area $HpMh$ multiplied into the ratio $\frac{TZ}{Mp}$: and therefore the horary variation of the inclination will be to $33''.\ 10'''.\ 33^{iv}.$ as the area $HpMh$ multiply'd into $AZ \times \frac{TZ}{Mp} \times \frac{Pp}{PG}$ to AT^3.

Cor. 2. And therefore, if the Earth and nodes were after every hour drawn back from their new, and inſtantly reſtor'd to their old places, ſo as their ſituation might continue given for a whole periodic month together; the whole variation of the inclination during that month would be to $33''.\ 10'''.\ 33^{iv}$, as the aggregate of all the areas $HpMh$, generated in the time of one revolution of the point p, (with due regard in ſumming to their proper ſigns $+$ and $-$) multiply'd into $AZ \times TZ \times \frac{Pp}{PG}$ to $Mp \times AT^3$, that is, as the whole circle $QAqa$ multiply'd into $AZ \times TZ \times \frac{Pp}{PG}$ to $Mp \times AT^3$, that is, as the circumference $QAqa$ multiply'd into $AZ \times TZ \times \frac{Pp}{PG}$ to $2\,Mp \times AT^2$.

Cor. 3. And therefore, in a giv'n poſition of the nodes, the mean horary variation, from which, if uniformly continu'd through the whole month, that menſtrual variation might be generated, is to $33''.\ 10'''.\ 33^{iv}.$ as $AZ \times TZ \times \frac{Pp}{PG}$ to $2\,AT^2$, or as $Pp \times \frac{AZ \times TZ}{\frac{1}{2}AT}$ to $PG \times 4\,AT$, that is (becauſe Pp is to PG, as the ſine

ſine of the aforeſaid inclination to the radius; and $\frac{AZ \times TZ}{\frac{1}{2}AT}$ to 4 AT, as the ſine of double the angle ATn to four times the radius) as the ſine of the ſame inclination multiply'd into the ſine of double the diſtance of the nodes from the Sun, to four times the ſquare of the radius.

Cor. 4. Seeing the horary variation of the inclination, when the nodes are in the quadratures, is (by this prop.) to the angle 33″. 10‴. 33^{iv}, as $IT \times AZ \times TG \times \frac{Pp}{PG}$ to AT^3, that is, as $\frac{IT \times TG}{\frac{1}{2}AT} \times \frac{Pp}{PG}$, to 2 AT, that is, as the ſine of double the diſtance of the Moon from the quadratures multiply'd into $\frac{Pp}{PG}$ to twice the radius: the ſum of all the horary variations during the time that the Moon, in this ſituation of the nodes, paſſes from the quadrature to the ſyzygy (that is in the ſpace of 177$\frac{1}{4}$ hours) will be to the ſum of as many angles 33″. 10‴. 33^{iv}. or 5878″, as the ſum of all the ſines of double the diſtance of the Moon from the quadratures multiply'd into $\frac{Pp}{PG}$, to the ſum of as many diameters; that is, as the diameter multiplied into $\frac{Pp}{PG}$ to the circumference; that is, if the inclination be 5°. 1′, as $7 \times \frac{874}{10000}$ to 22, or as 278 to 10000. And therefore the whole variation, compos'd out of the ſum of all the horary variations in the foreſaid time, is 163″. or 2′. 43″.

PROPOSITION XXXV. PROBLEM XVI.

To a given time to find the inclination of the Moon's orbit to the plane of the ecliptic.

Let AD (*Pl.* 14. *Fig.* 1.) be the ſine of the greateſt inclination, and AB the ſine of the leaſt. Biſect BD in C; and round the centre C, with the interval BC, deſcribe the circle BGD. In AC take CE in the ſame proportion to EB as EB to twice BA. And if to the time giv'n we ſet off the angle AEG equal to double the diſtance of the nodes from the quadratures, and upon AD let fall the perpendicular GH; AH will be the ſine of the inclination requir'd.

For GE^2 is equal to $GH^2 + HE^2 = BHD + HE^2 = HBD + HE^2 - BH^2 = HBD + BE^2 - 2BH \times BE = BE^2 + 2EC \times BH = 2EC \times AB + 2EC \times BH = 2EC \times AH$. Wherefore ſince $2EC$ is giv'n, GE^2 will be as AH. Now let AEg repreſent double the diſtance of the nodes from the quadratures, in a given moment of time after, and the arc Gg, on account of the giv'n angle GEg, will be as the diſtance GE. But Hh is to Gg, as GH to GC, and therefore Hh is as the rectangle $GH \times Gg$, or $GH \times GE$, that is, as $\frac{GH}{GE} \times GE^2$ or $\frac{GH}{GE} \times AH$; that is, as AH and the ſine of the angle AEG conjunctly. If therefore in any one caſe, AH be the ſine of inclination, it will increaſe by the ſame increments as the ſine of inclination doth, by cor. 3. of the preceding prop. and therefore will always continue equal to that ſine. But when the point G falls upon either point B or D, AH is equal to this ſine, and therefore remains always equal thereto. *Q. E. D.*

In this demonſtration I have ſuppos'd, that the angle

Fig. 1. *p.* 289.

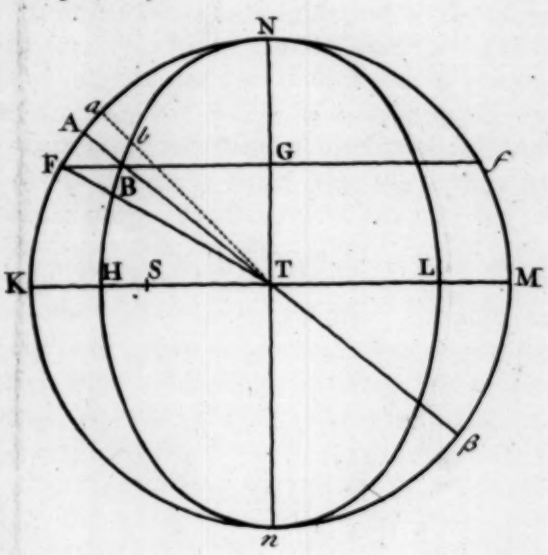
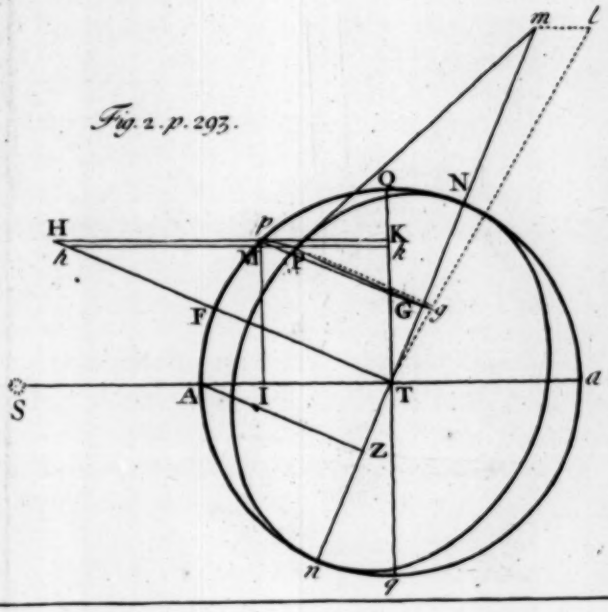


Fig. 2. *p.* 293.



gle *BEG* repreſenting double the diſtance of the nodes from the quadratures, increaſeth uniformly. For I cannot deſcend to ev'ry minute circumſtance of inequality. Now ſuppoſe that *BEG* is a right angle, and that *Gg* is in this caſe the horary increment of double the diſtance of the nodes from the Sun; then by cor. 3. of the laſt prop. the horary variation of the inclination in the ſame caſe, will be to $33''.\ 10'''.\ 33^{iv}.$ as the rectangle of *AH* the ſine of the inclination into the ſine of the right angle *BEG*, double the diſtance of the nodes from the Sun, to four times the ſquare of the radius; that is, as *AH* the ſine of the mean inclination to four times the radius, that is, ſeeing the mean inclination is about $5°.\ 8\frac{1}{2}'$, as its ſine 896 to 40000, the quadruple of the radius, or as 224 to 10000. But the whole variation, correſponding to *BD* the difference of the ſines, is to this horary variation, as the diameter *BD* to the arc *Gg*, that is, conjunctly as the diameter *BD* to the ſemi-circumference *BGD*, and as the time of $2079\frac{7}{10}$ hours, in which the node proceeds from the quadratures to the ſyzygies, to one hour, that is, as 7 to 11 and $2079\frac{7}{10}$ to 1. Wherefore compounding all theſe proportions, we ſhall have the whole variation *BD* to $33''.\ 10'''.\ 33^{iv}.$ as $224 \times 7 \times 2079\frac{7}{10}$ to 110000, that is, as 29645 to 1000; and from thence that variation *BD* will come out $16'.\ 23\frac{1}{2}''$.

And this is the greateſt variation of the inclination, abſtracting from the ſituation of the Moon in its orbit. For if the nodes are in the ſyzygies, the inclination ſuffers no change from the various poſitions of the Moon. But if the nodes are in the quadratures, the inclination is leſs when the Moon is in the ſyzygies than when it is in the quadratures, by a difference of $2'.\ 43''.$ as we ſhew'd in cor. 4. of the preceding prop. and the whole mean variation *BD*, diminiſh'd by $1'.\ 21\frac{1}{2}''.$ the half of this exceſs, becomes $15'.\ 2''.$ when

the Moon is in the quadratures; and increas'd by the ſame, becomes 17′. 45″. when the Moon is in the ſyzygies. If therefore the Moon be in the ſyzygies, the whole variation in the paſſage of the nodes from the quadratures to the ſyzygies will be 17′. 45″. And therefore if the inclination be 5°. 17′. 20″. when the nodes are in the ſyzygies, it will be 4°. 59′. 35″. when the nodes are in the quadratures and the Moon in the ſyzygies. The truth of all which is confirm'd by obſervations.

Now if the inclination of the orbit ſhould be requir'd, when the Moon is in the ſyzygies, and the nodes any where between them and the quadratures; let *AB* be to *AD*, as the ſine of 4°. 59′. 35″. to the ſine of 5°. 17′. 20″. and take the angle *AEG*, equal to double the diſtance of the nodes from the quadratures; and *AH* will be the ſine of the inclination deſir'd. To this inclination of the orbit the inclination of the ſame is equal, when the Moon is 90°. diſtant from the nodes. In other ſituations of the Moon, this menſtrual inequality to which the variation of the inclination is obnoxious in the calculus of the Moon's latitude, is balanc'd and in a manner took off, by the menſtrual inequality of the motion of the nodes (as we ſaid before) and therefore may be neglected in the computation of the ſaid latitude.

Scholium.

By theſe computations of the lunar motions, I was willing to ſhew that by the theory of gravity the motions of the Moon could be calculated from their phyſical cauſes. By the ſame theory I moreover found, that the annual equation of the mean motion of the Moon ariſes from the various dilatation which the orbit of the Moon ſuffers from the action of the Sun, accord-

according to cor. 6. prop. 66. book 1. The force of this action is greater in the perigeon Sun, and dilates the Moon's orbit; in the apogeon Sun it is leſs, and permits the orbit to be again contracted. The Moon moves ſlower in the dilated, and faſter in the contracted orbit; and the annual equation, by which this inequality is regulated, vaniſhes in the apogee and perigee of the Sun. In the mean diſtance of the Sun from the Earth it ariſes to about 11′. 50″. In other diſtances of the Sun, it is proportional to the equation of the Sun's centre, and is added to the mean motion of the Moon, while the Earth is paſſing from its aphelion to its perihelion, and ſubducted while the Earth is in the oppoſite ſemicircle. Taking for the radius of the *orbis magnus*, 1000, and $16\frac{1}{8}$ for the Earth's eccentricity, this equation when of the greateſt magnitude, by the theory of gravity comes out 11′. 49″. But the eccentricity of the Earth ſeems to be ſomething greater, and with the eccentricity this equation will be augmented in the ſame proportion. Suppoſe the eccentricity $16\frac{11}{12}$, and the greateſt equation will be 11′. 51″.

Further, I found that the apogee and nodes of the Moon move faſter in the perihelion of the Earth, where the force of the Sun's action is greater, than in the aphelion thereof, and that in the reciprocal triplicate proportion of the Earth's diſtance from the Sun. And hence ariſe annual equations of thoſe motions proportional to the equation of the Sun's centre. Now the motion of the Sun is in the reciprocal duplicate proportion of the Earth's diſtance from the Sun, and the greateſt equation of the centre, which this inequality generates, is 1°. 56′. 20″. correſponding to the abovemention'd eccentricity of the the Sun $16\frac{11}{12}$. But if the motion of the Sun had been in the reciprocal triplicate proportion of the diſtance, this inequality would have generated the greateſt equation 2°. 54′. 30″.

30″. And therefore the greateſt equations which the inequalities of the motions of the Moon's apogee and nodes do generate, are to 2°. 54′. 30″. as the mean diurnal motion of the Moon's apogee and the mean diurnal motion of its nodes are to the mean diurnal motion of the Sun. Whence the greateſt equation of the mean motion of the apogee comes out 19′. 43″. and the greateſt equation of the mean motion of the nodes 9′. 24″. The former equation is added, and the latter ſubducted, while the Earth is paſſing from its perihelion to its aphelion, and contrariwiſe when the Earth is in the oppoſite ſemicircle.

By the theory of gravity I likewiſe found, that the action of the Sun upon the Moon is ſomething greater when the tranſverſe diameter of the Moon's orbit paſſeth through the Sun, than when the ſame is perpendicular upon the line which joins the Earth and the Sun: And therefore the Moon's orbit is ſomething larger in the former than in the latter caſe. And hence ariſes another equation of the Moon's mean motion, depending upon the ſituation of the Moon's apogee in reſpect of the Sun; which is in its greateſt quantity, when the Moon's apogee is in the octants of the Sun, and vaniſhes when the apogee arrives at the quadratures or ſyzygies. And it is added to the mean motion, while the Moon's apogee is paſſing from the quadrature of the Sun to the ſyzygy, and ſubducted while the apogee is paſſing from the ſyzygy to the quadrature. This equation, which I ſhall call the ſemi-annual, when greateſt in the octants of the apogee, ariſes to about 3′. 45″. ſo far as I could collect from the phænomena. And this is its quantity in the mean diſtance of the Sun from the Earth. But it is increaſed and diminiſhed in the reciprocal triplicate proportion of the Sun's diſtance, and therefore is nearly 3′. 34″. when that diſtance is greateſt, and 3′. 56″. when leaſt. But when the Moon's apogee is without the octants, it becomes

leſs,

leſs, and is to its greateſt quantity, as the ſine of double the diſtance of the Moon's apogee from the neareſt ſyzygy, or quadrature to the radius.

By the ſame theory of gravity, the action of the Sun upon the Moon is ſomething greater, when the line of the Moon's nodes paſſes through the Sun, than when it is at right angles with the line which joins the Sun and the Earth. And hence ariſes another equation of the Moon's mean motion, which I ſhall call the ſecond ſemi-annual, and this is greateſt when the nodes are in the octants of the Sun, and vaniſhes when they are in the ſyzygies or quadratures; and in other poſitions of the nodes is proportional to the ſine of double the diſtance of either node from the neareſt ſyzygy or quadrature. And it is added to the mean motion of the Moon, if the Sun is *in antecedentiâ* to the node which is neareſt to him, and ſubducted if *in conſequentiâ*; and in the octants, where it is of the greateſt magnitude, it ariſes to 47″. in the mean diſtance of the Sun from the Earth, as I find from the theory of gravity. In other diſtances of the Sun this equation, greateſt in the octants of the nodes, is reciprocally as the cube of the Sun's diſtance from the Earth, and therefore in the Sun's perigee it comes to about 49″, and in its apogee to about 45″.

By the ſame theory of gravity, the Moon's apogee goes forward at the greateſt rate, when it is either in conjunction with or in oppoſition to the Sun, but in its quadratures with the Sun it goes backward. And the eccentricity comes, in the former caſe, to its greateſt quantity, in the latter to its leaſt, by cor. 7. 8. and 9. prop. 66. book 1. And thoſe inequalities by the corollaries we have nam'd, are very great, and generate the principal, which I call the ſemi-annual, equation of the apogee. And this ſemi-annual equation in its greateſt quantity comes to about 12°. 18″. as near-

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ly as I could collect from the phænomena. Our countryman *Horrox* was the firſt who advanced the theory of the Moon's moving in an ellipſe about the Earth placed in its lower focus. Dr. *Halley* improved the notion, by putting the centre of the ellipſe in an epicycle whoſe centre is uniformly revolved about the Earth. And from the motion in this epicycle the mentioned inequalities in the progreſs and regreſs of the apogee, and in the quantity of eccentricity do ariſe. Suppoſe the mean diſtance of the Moon from the Earth to be divided into 100000 parts, and let *T* (*Pl.* 14. *Fig.* 2.) repreſent the Earth, and *TC* the Moon's mean eccentricity of 5505 ſuch parts. Produce *TC* to *B*, ſo as *CB* may be the ſine of the greateſt ſemi-annual equation 12°. 18′, to the radius *TC*; and the circle *BDA* deſcribed about the centre *C*, with the interval *CB*, will be the epicycle ſpoke of, in which the centre of the Moon's orbit is placed, and revolved according to the order of the letters *BDA*. Set off the angle *BCD* equal to twice the annual argument, or twice the diſtance of the Sun's true place from the place of the Moon's apogee once equated, and *CTD* will be the ſemi-annual equation of the Moon's apogee, and *TD* the eccentricity of its orbit, tending to the place of the apogee now twice equated. But having the Moon's mean motion, the place of its apogee, and its eccentricity, as well as the longer axe of its orbit 200000; from theſe *data* the true place of the Moon in its orbit, together with its diſtance from the Earth, may be determined by the methods commonly known.

In the perihelion of the Earth where the force of the Sun is greateſt, the centre of the Moon's orbit moves faſter about the centre *C*, than in the aphelion, and that in the reciprocal triplicate proportion of the Sun's diſtance from the Earth. But becauſe the equation of the Sun's centre is included in the annual argument, the centre of the Moon's orbit moves faſter in

its

its epicycle *BDA*, in the reciprocal duplicate proportion of the Sun's diſtance from the Earth. Therefore that it may move yet faſter in the reciprocal ſimple proportion of the diſtance; ſuppoſe that from *D* the centre of the orbit a right line *DE* is drawn, tending towards the Moon's apogee once equated, that is, parallel to *TC*; and ſet off the angle *EDF* equal to the exceſs of the foreſaid annual argument above the diſtance of the Moon's apogee from the Sun's perigee *in conſequentia*; or, which comes to the ſame thing, take the angle *CDF* equal to the complement of the Sun's true anomaly to 360°. And let *DF* be to *DC*, as twice the eccentricity of the *orbis magnus* to the Sun's mean diſtance from the Earth and the Sun's mean diurnal motion from the Moon's apogee to the Sun's mean diurnal motion from its own apogee conjunctly, that is, as $33\frac{1}{8}$ to 1000, and 52′. 27″. 16‴. to 59′. 8″. 10‴. conjunctly; or as 3 to 100. And imagine the centre of the Moon's orbit, placed in the point *F*, to be revolved in an epicycle whoſe centre is *D*, and radius *DF*, while the point *D* moves in the circumference of the circle *DABD*. For by this means the centre of the Moon's orbit comes to deſcribe a certain curve line, about the centre *C*, with a velocity which will be almoſt reciprocally as the cube of the Sun's diſtance from the Earth, as it ought to be.

The calculus of this motion is difficult, but may be render'd more eaſy by the following approximation. Aſſuming as above the Moon's mean diſtance from the Earth of 100000 parts, and the eccentricity *TC* of 5505 ſuch parts, the line *CB* or *CD* will be found $1172\frac{1}{4}$, and *DF* $35\frac{1}{3}$ of thoſe parts. And this line *DF* at the diſtance *TC* ſubtends the angle at the Earth, which the removal of the centre of the orbit from the place *D* to the place *F* generates in the motion of this centre; and double this line *DF* in a parallel poſition, at the diſtance of the upper focus of the Moon's

orbit from the Earth, ſubtends at the Earth the ſame angle as *DF* did before, which that removal generates in the motion of this upper focus; but at the diſtance of the Moon from the Earth this double line 2*DF* at the upper focus, in a parallel poſition to the firſt line *DF*, ſubtends an angle at the Moon which the ſaid removal generates in the motion of the Moon, which angle may be therefore called the ſecond equation of the Moon's centre. And this equation, in the mean diſtance of the Moon from the Earth, is nearly as the ſine of the angle which that line *DF* contains with the line drawn from the point *F* to the Moon, and when in its greateſt quantity amounts to 2′. 25″. But the angle which the line *DF* contains with the line drawn from the point *F* to the Moon, is found either by ſubtracting the angle *EDF* from the mean anomaly of the Moon, or by adding the diſtance of the Moon from the Sun, to the diſtance of the Moon's apogee from the apogee of the Sun. And as the radius to the ſine of the angle thus found, ſo is 2′. 25″. to the ſecond equation of the centre; to be added, if the forementioned ſum be leſs than a ſemicircle, to be ſubducted if greater. And from the Moon's place in its orbit thus corrected, its longitude may be found in the ſyzygies of the luminaries.

The atmoſphere of the Earth to the height of 35 or 40 miles refracts the Sun's light. This refraction ſcatters and ſpreads the light over the Earth's ſhadow; and the diſſipated light near the limits of the ſhadow dilates the ſhadow. Upon which accounts, to the diameter of the ſhadow, as it comes out by the parallax, I add 1 or $1\frac{1}{3}$ minute in lunar eclipſes.

But the theory of the Moon ought to be examined and proved from the phænomena, firſt in the ſyzygies; then in the quadratures; and laſt of all in the octants; and whoſo pleaſes to undertake the work, will find it not amiſs to aſſume the following mean motions of the Sun

Sun and Moon, at the royal obſervatory of *Greenwich* to the laſt day of *December* at noon, *anno* 1700, O. S. *viz.* The mean motion of the Sun ♑ 20°. 43′. 40″. and of its apogee ♋ 7°. 44′. 30″. the mean motion of the Moon ♒ 15°. 21′. 00″; of its apogee, ♓ 8°. 20′. 00″. and of its aſcending node, ☊ 27°. 24′. 20″; and the difference of meridians betwixt the obſervatory at *Greenwich* and the royal obſervatory at *Paris*, 0ʰ. 9′. 20″. but the mean motion of the Moon and of its apogee, are not yet obtained with ſufficient accuracy.

PROPOSITION XXXVI. PROBLEM XVII.

To find the force of the Sun to move the Sea.

The Sun's force ML or PT to diſturb the motions of the Moon, was, (by prop. 25.) in the Moon's quadratures, to the force of gravity with us, as 1 to 638092,6. And the force $TM—LM$, or $2PK$ in the Moon's ſyzygies, is double that quantity. But deſcending to the ſurface of the Earth, theſe forces are diminiſhed in proportion of the diſtances from the centre of the Earth, that is, in the proportion of $60\frac{1}{2}$ to 1; and therefore the former force on the Earth's ſurface is to the force of gravity, as 1 to 38604600. And by this force the Sea is depreſſed in ſuch places as are 90 degrees diſtant from the Sun. But by the other force which is twice as great, the Sea is rais'd; not only in the places directly under the Sun, but in thoſe alſo which are directly oppoſed to it. And the ſum of theſe forces is to the force of gravity, as 1 to 12868200. And becauſe the ſame force excites the ſame motion, whether it depreſſes the waters in thoſe places which are 90 degrees diſtant from the Sun; or raiſes them in the places which are directly under, and directly oppoſed to the Sun; the foreſaid ſum will be

the total force of the Sun to diſturb the Sea, and will have the ſame effect as if the whole was employed in raiſing the Sea in the places directly under and directly oppos'd to the Sun, and did not act at all in the places which are 90 degrees removed from the Sun.

And this is the force of the Sun to diſturb the Sea in any given place, where the Sun is at the ſame time both vertical, and in its mean diſtance from the Earth. In other poſitions of the Sun, its force to raiſe the Sea is as the verſed ſine of double its altitude above the horizon of the place directly, and the cube of the diſtance from the Earth reciprocally.

Cor. Since the centrifugal force of the parts of the Earth, ariſing from the Earth's diurnal motion, which is to the force of gravity as 1 to 289, raiſes the waters under the equator to a height exceeding that under the poles by 85472 *Paris* feet, as above in prop. 19. the force of the Sun which we have now ſhewed to be to the force of gravity, as 1 to 12868200, and therefore is to that centrifugal force as 289 to 12868200, or as 1 to 44527, will be able to raiſe the waters in the places directly under and directly oppos'd to the Sun, to a height exceeding that in the places which are 90 degrees removed from the Sun, only by one *Paris* foot and $113\frac{1}{10}$ inches. For this meaſure is to the meaſure of 85472 feet, as 1 to 44527.

Proposition XXXVII. Problem XVIII.

To find the force of the Moon to move the Sea.

The force of the Moon to move the Sea is to be deduced from its proportion to the force of the Sun, and this proportion is to be collected from the proportion of the motions of the Sea, which are the effects of thoſe forces. Before the mouth of the river *Avon*, three miles below *Briſtol*, the height of the aſcent of the

the water, in the vernal and autumnal ſyzygies of the luminaries, (by the obſervations of *Samuel Sturmy*) amounts to about 45 feet, but in the quadratures to 25 only. The former of thoſe heights ariſes from the ſum of the foreſaid forces, the latter from their difference. If therefore S and L are ſuppoſed to repreſent reſpectively the forces of the Sun and Moon, while they are in the equator, as well as in their mean diſtances from the Earth, we ſhall have L-|-S to L—S as 45 to 25, or as 9 to 5.

At *Plymouth* (by the obſervations of *Samuel Colepreſs*) the tide in its mean height riſes to about 16 feet, and in the ſpring and autumn the height thereof in the ſyzygies may exceed that in the quadratures by more than 7 or 8 feet. Suppoſe the greateſt difference of thoſe heights to be 9 feet, and L-|-S will be to L—S, as $20\frac{1}{2}$ to $11\frac{1}{2}$, or as 41 to 23; a proportion that agrees well enough with the former. But becauſe of the great tide at *Briſtol*, we are rather to depend upon the obſervations of *Sturmy*, and therefore till we procure ſomething that is more certain, we ſhall uſe the proportion of 9 to 5.

But becauſe of the reciprocal motions of the waters, the greateſt tides do not happen at the times of the ſyzygies of the luminaries, but as we have ſaid before, are the third in order after the ſyzygies; (or reckoning from the ſyzygies) follow next after the third appulſe of the Moon to the meridian of the place after the ſyzygies; or rather (as *Sturmy* obſerves) are the third after the day of the new or full Moon, or rather nearly after the twelfth hour from the new or full Moon, and therefore fall nearly upon the forty third hour after the new or full of the Moon. But in this port they fall out about the ſeventh hour after the appulſe of the Moon to the meridian of the place; and therefore follow next after the appulſe of the Moon to the meridian, when the Moon is diſtant from the Sun, or from oppoſition

with the Sun by about 18 or 19 degrees *in conſequentia*. So the ſummer and winter ſeaſons come not to their height in the ſolſtices themſelves, but when the Sun is advanced beyond the ſolſtices by about a tenth part of its whole courſe, that is, by about 36 or 37 degrees. In like manner the greateſt tide is raiſed after the appulſe of the Moon to the meridian of the place, when the Moon has paſſed by the Sun, *or the oppoſition thereof*, by about a tenth part of the whole motion from *one greateſt* tide to *the next following greateſt* tide. Suppoſe that diſtance about $18\frac{1}{2}$ degrees. And the Sun's force in this diſtance of the Moon from the ſyzygies and quadratures, will be of leſs moment to augment and diminiſh that part of the motion of the Sea which proceeds from the motion of the Moon, than in the ſyzygies and quadratures themſelves, in the proportion of the radius to the co-ſine of double this diſtance, or of an angle of 37 degrees, that is, in proportion of 10000000 to 7986355. And therefore in the preceding analogy, in place of S we muſt put 0,7986355 S.

But further, the force of the Moon in the quadratures muſt be diminiſhed, on account of its declination from the equator. For the Moon in thoſe quadratures, or rather in $18\frac{1}{2}$ degrees paſt the quadratures, declines from the equator by about 22°. 13′. And the force of either luminary to move the Sea is diminiſhed as it declines from the equator, nearly in the duplicate proportion of the co-ſine of the declination. And therefore the force of the Moon in thoſe quadratures is only 0,8570327 L; whence we have L+0,7986355 S, to 0,8570327 L—0,7986355 S, as 9 to 5.

Further yet, the diameters of the orbit, in which the Moon ſhould move, ſetting aſide the conſideration of eccentricity, are one to the other, as 69 to 70. And therefore the Moon's diſtance from the Earth in the ſyzygies, is to its diſtance in the quadratures, *cæteris paribus*,

paribus, as 69 to 70. And its diſtances, when $18\frac{1}{2}$ degrees advanced beyond the ſyzygies, where the greateſt tide was excited, and when $18\frac{1}{2}$ degrees paſſed by the quadratures, where the leaſt tide was produced, are to its mean diſtance as 69,098747 and 69,897345 to $69\frac{1}{2}$. But the force of the Moon to move the Sea is in the reciprocal triplicate proportion of its diſtance. And therefore its forces, in the greateſt and leaſt of thoſe diſtances, are to its force in its mean diſtance, as 0,9830427 and 1,017522 to 1. From whence we have 1,017522 L × 0,7986355 S to 0,9830427 × 0,8570327 L—0,7986355 S as 9 to 5. And S to L, as 1 to 4,4815. Wherefore ſince the force of the Sun is to the force of gravity as 1 to 12868200, the Moon's force will be to the force of gravity, as 1 to 2871400.

Cor. 1. Since the waters excited by the Sun's force riſe to the height of a foot and $11\frac{1}{30}$ inches, the Moon's force will raiſe the ſame to the height of 8 feet and $7\frac{5}{22}$ inches; and the joint forces of both will raiſe the ſame to the height of $10\frac{1}{2}$ feet; and when the Moon is in its perigee, to the height of $12\frac{1}{2}$ feet, and more, eſpecially when the wind ſets the ſame way as the tide. And a force of that quantity is abundantly ſufficient to excite all the motions of the Sea, and agrees well with the proportion of thoſe motions. For in ſuch Seas as lye free and open from eaſt to weſt, as in the *Pacific* Sea, and in thoſe tracts of the *Atlantic* and *Ethiopic* Seas which lye without the tropics, the waters commonly riſe to 6, 9, 12, or 15 feet. But in the *Pacific* Sea, which is of a greater depth as well as of a larger extent, the tides are ſaid to be greater than in the *Atlantic* and *Ethiopic* Seas. For to have a full tide raiſed, an extent of Sea from eaſt to weſt is required of no leſs than 90 degrees. In the *Ethiopic* Sea, the waters riſe to a leſs height within the tropics than in the temperate zones,

becauſe of the narrowneſs of the Sea between *Africa* and the ſouthern parts of *America*. In the middle of the open Sea the waters cannot riſe without falling, together and at the ſame time, upon both the eaſtern and weſtern ſhoars; when notwithſtanding in our narrow Seas, they ought to fall on thoſe ſhores by alternate turns. Upon which account, there is commonly but a ſmall flood and ebb in ſuch iſlands, as lie far diſtant from the continent. On the contrary in ſome ports, where to fill and empty the bays alternately, the waters are with great violence forced in and out through ſhallow chanels, the flood and ebb muſt be greater than ordinary, as at *Plymouth* and *Chepſtow-Bridge* in *England*, at the mountains of St. *Michael*, and the town of *Auranches* in *Normandy*, and at *Cambaia* and *Pegu* in the *Eaſt-Indies*. In theſe places the Sea is hurryed in and out with ſuch violence, as ſometimes to lay the ſhoars under water, ſometimes to leave them dry, for many miles. Nor is this force of the influx and efflux to be broke, till it has raiſed and depreſſed the waters to 30, 40, or 50 feet and above. And a like account is to be given of long and ſhallow chanels or ſtreights, ſuch as the *Magellanic* ſtreights and thoſe chanels which environ *England*. The tide in ſuch ports and ſtreights, by the violence of the influx and efflux, is augmented above meaſure. But on ſuch ſhoars as ly towards the deep and open Sea, with a ſteep deſcent, where the waters may freely riſe and fall without that precipitation of influx and efflux, the proportion of the tides agrees with the forces of the Sun and Moon.

Cor. 2. Since the Moon's force to move the Sea is to the force of gravity, as 1 to 2871400, it is evident that this force is far leſs than to appear ſenſibly in ſtatical or hydroſtatical experiments, or even in thoſe of pendulums. It is in the tides only that this force ſhews it ſelf by any ſenſible effect.

Cor

Cor. 3. Becauſe the force of the Moon to move the Sea is to the like force of the Sun as 4,4815 to 1; and thoſe forces (by cor. 14. prop. 66. book 1.) are as the denſities of the bodies of the Sun and Moon and the cubes of their apparent diameters conjunctly; the denſity of the Moon will be to the denſity of the Sun as 4,4815 to 1 directly, and the cube of the Moon's diameter to the cube of the Sun's diameter inverſely; that is, (ſeeing the mean apparent diameters of the Moon and Sun are 31′. 16½″. and 32′. 12″.) as 4891 to 1000. But the denſity of the Sun was to the denſity of the Earth, as 1000 to 4000; and therefore the denſity of the Moon is to the denſity of the Earth as 4891 to 4000, or as 11 to 9. Therefore the body of the Moon is more denſe and more earthly, than the Earth it ſelf.

Cor. 4. And ſince the true diameter of the Moon, (from the obſervations of aſtronomers) is to the true diameter of the Earth, as 100 to 365, the maſs of matter in the Moon will be to the maſs of matter in the Earth as 1 to 39,788.

Cor. 5. And the accelerative gravity on the ſurface of the Moon will be about three times leſs than the accelerative gravity on the ſurface of the Earth.

Cor. 6. And the diſtance of the Moon's centre from the centre of the Earth will be to the diſtance of the Moon's centre from the common centre of gravity of the Earth and Moon, as 40,788 to 39,788.

Cor. 7. And the mean diſtance of the centre of the Moon from the centre of the Earth will be (in the Moon's octants) nearly 60$\frac{3}{7}$ of the greateſt ſemidiameters of the Earth. For the greateſt ſemidiameter of the Earth was 19658600 *Paris* feet, and the mean diſtance of the centres of the Earth and Moon, conſiſting of 60$\frac{3}{7}$ ſuch ſemidiameters, is equal to 1187379440 feet. And this diſtance (by the preceeding cor.) is to the diſtance of the Moon's centre

from the common centre of gravity of the Earth and Moon, as 40,788 to 39,788; which latter diſtance therefore is 1158268534 feet. And ſince the Moon, in reſpect of the fixt Stars, performs its revolution in $27^{d}.\ 7^{h}.\ 43\frac{4'}{5}$. the verſed-ſine of that angle which the Moon in a minute of time deſcribes is 12752341, to the radius 1000,000000,000000. And as the radius is to this verſed-ſine, ſo are 1158268534 feet to 14,7706353 feet. The Moon therefore falling towards the Earth, by that force which retains it in its orbit, would in one minute of time deſcribe 14,7706353 feet. And if we augment this force in the proportion of $178\frac{29}{40}$ to $177\frac{29}{40}$, we ſhall have the total force of gravity at the orbit of the Moon, by cor. prop. 3. And the Moon falling by this force, in one minute of time would deſcribe 14,8538067 feet. And at the 60th part of the diſtance of the Moon from the Earth's centre. That is, at the diſtance of 197896573 feet from the centre of the Earth, a body falling by its weight, would, in one ſecond of time, likewiſe deſcribe 14,8538067 feet. And therefore at the diſtance of 19615800, which compoſe one mean ſemidiameter of the Earth, a heavy body would deſcribe in falling 15,11175, or 15 feet, 1 inch and $4\frac{1}{11}$ lines in the ſame time. This will be the deſcent of bodies in the latitude of 45 degrees. And by the foregoing table to be found under prop. 20. the deſcent in the latitude of *Paris* will be a little greater by an exceſs of about $\frac{2}{3}$ parts of a line. Therefore by this computation heavy bodies in the latitude of *Paris* falling *in vacuo* will deſcribe 15 *Paris* feet, 1 inch, $4\frac{2}{3}\frac{1}{3}$ lines very nearly in one ſecond of time. And if the gravity be diminiſhed by taking away a quantity equal to the centrifugal force ariſing in that latitude from the Earth's diurnal motion; heavy bodies falling there will deſcribe in one ſecond of time 15 feet, 1 inch, and $1\frac{1}{2}$ line. And with this velocity heavy bodies do

do really fall in the latitude of *Paris*, as we have ſhewn above in prop. 4. and 19.

Cor. 8. The mean diſtance of the centres of the Earth and Moon in the ſyzygies of the Moon is equal to 60 of the greateſt ſemidiameters of the Earth, ſubducting only about one 30th part of a ſemidiameter. And in the Moon's quadratures the mean diſtance of the ſame centres is $60\frac{1}{6}$ ſuch ſemidiameters of the Earth. For theſe two diſtances are to the mean diſtance of the Moon in the octants, as 69 and 70 to $69\frac{1}{2}$, by prop. 28.

Cor. 9. The mean diſtance of the centres of the Earth and Moon in the ſyzygies of the Moon is 60 mean ſemidiameters of the Earth, and a 10th part of one ſemidiameter; and in the Moon's quadratures the mean diſtance of the ſame centres is 61 mean ſemidiameters of the Earth, ſubducting one 30th part of one ſemidiameter.

Cor. 10. In the Moon's ſyzygies its mean horizontal parallax in the latitudes of 0,30,38,45,52,60,90 degrees, is 57′. 20″. 57′. 16″. 57′. 14″. 57′. 12″. 57′ 10″. 57′. 8″. 57′. 4″. reſpectively.

In theſe computations I don't conſider the magnetic attraction of the Earth whoſe quantity is very ſmall and unknown. If this quantity ſhould ever be found out, and the meaſures of degrees upon the meridian, the lengths of iſochronous pendulums in different parallels, the laws of the motions of the Sea, and the Moon's parallax, with the apparent diameters of the Sun and Moon, ſhould be more exactly determined from phænomena; we ſhould then be inabled to bring this calculation to a greater accuracy.

PROPOSITION XXXVII. PROBLEM XIX.

To find the figure of the Moon's body.

If the Moon's body were fluid like our Sea, the force of the Earth to raiſe that fluid, in the neareſt and remoteſt parts, would be to the force of the Moon, by which our Sea is raiſed in the places under and oppoſite to the Moon, as the accelerative gravity of the Moon towards the Earth, to the accelerative gravity of the Earth towards the Moon, and the diameter of the Moon to the diameter of the Earth conjunctly, that is, as 39,788 to 1, and 100 to 365 conjunctly, or as 1081 to 100. Wherefore, ſince our Sea, by the force of the Moon, is raiſed to $8\frac{4}{5}$ feet; the lunar fluid would be raiſed by the force of the Earth to 93 feet. And upon this account, the figure of the Moon would be a ſpheroid, whoſe greateſt diameter produced would paſs through the centre of the Earth, and exceed the diameters perpendicular thereto, by 186 feet. Such a figure therefore the Moon affects, and muſt have put on from the beginning. *Q. E. I.*

COR. Hence it is, that the ſame face of the Moon always reſpects the Earth; nor can the body of the Moon poſſibly reſt in any other poſition, but would return always by a libratory motion to this ſituation. But thoſe librations however muſt be exceeding ſlow, becauſe of the weakneſs of the forces which excite them; ſo that the face of the Moon which ſhould be always obverted to the Earth, may for the reaſon aſſigned in prop. 17. be turned towards the other focus of the Moon's orbit, without being immediately drawn back, and converted again towards the Earth.

LEM-

LEMMA I.

If APEp (Pl. 14. Fig. 3.) *repreſent the Earth uniformly denſe, mark'd with the centre* C, *the poles* P, p, *and the equator* AE; *and if about the centre* C, *with the radius* CP, *we ſuppoſe the ſphere* Pape *to be deſcribed, and* QR *to denote the plane on which a right line, drawn from the centre of the Sun to the centre of the Earth, inſiſts at right angles, and farther ſuppoſe, that the ſeveral particles of the whole exterior Earth* Pap APepE, *without the height of the ſaid ſphere, endeavour to recede towards this ſide and that ſide from the plane* QR, *every particle by a force proportional to its diſtance from that plane; I ſay in the firſt place, that the whole force and efficacy of all the particles, that are ſituate in* AE *the circle of the equator, and diſpoſed uniformly without the globe, encompaſſing the ſame after the manner of a ring, to wheel the Earth about its centre, is to the whole force and efficacy of as many particles, in that point* A *of the equator which is at the greateſt diſtance from the plane* QR, *to wheel the Earth about its centre with a like circular motion, as* 1 *to* 2. *And that circular motion will be performed about an axis lying in the common ſection of the equator and the plane* QR.

For let there be deſcribed from the centre *K*, with the diameter *IL*, the ſemicircle *INLK*. Suppoſe the ſemicircumference *INL* to be divided into innumerable

rable equal parts, and from the ſeveral parts N to the diameter IL let fall the ſines NM. Then the ſums of the ſquares of all the ſines NM will be equal to the ſums of the ſquares of the ſines KM, and both ſums together will be equal to the ſums of the ſquares of as many ſemidiameters KN; and therefore the ſum of the ſquares of all the ſines NM will be but half ſo great as the ſum of the ſquares of as many ſemidiameters KN.

Suppoſe now the circumference of the circle AE to be divided into the like number of little equal parts, and from every ſuch part F a perpendicular FG to be let fall upon the plane QR, as well as the perpendicular AH from the point A. Then the force by which the particle F recedes from the plane QR, will (by ſuppoſition) be as that perpendicular FG, and this force multiplied by the diſtance CG will repreſent the power of the particle F to turn the Earth round its centre. And therefore the power of a particle in the place F, will be to the power of a particle in the place A, as $FG \times GC$ to $AH \times HC$; that is, as FC^2 to AC^2: and therefore the whole power of all the particles F, in their proper places F, will be to the power of the like number of particles in the place A, as the ſum of all the FC^2 to the ſum of all the AC^2, that is, (by what we have demonſtrated before) as 1 to 2. *Q. E. D.*

And becauſe the action of thoſe particles is exerted in the direction of lines perpendicularly receding from the plane QR, and that equally from each ſide of this plane, they will wheel about the circumference of the circle of the equator, together with the adherent body of the Earth, round an axe, which lies as well in the plane QR, as in that of the equator.

L E M.

LEMMA II.

The ſame things ſtill ſuppoſed, I ſay in the ſecond place, that the total force or power of all the particles ſituated every where about the ſphere to turn the Earth about the ſaid axe, is to the whole force of the like number of particles, uniformly diſpos'd round the whole circumference of the equator AE *in the faſhion of a ring, to turn the whole Earth about with the like circular motion, as* 2 *to* 5. Pl. 14. Fig. 4.

For, let IK be any leſſer circle parallel to the equator AE, and let L, l be any two equal particles in this circle, ſituated without the ſphere $Pape$. And if upon the plane QR, which is at right angles with a radius drawn to the Sun, we let fall the perpendiculars LM, lm; the total forces by which theſe particles recede from the plane QR, will be proportional to the perpendiculars LM, lm. Let the right line Ll be drawn parallel to the plane $Pape$, and biſect the ſame in X; and thro' the point X draw Nn, parallel to the plane QR, and meeting the perpendiculars LM, lm in N and n; and upon the plane QR let fall the perpendicular XY. And the contrary forces of the particles L and l, to wheel about the Earth contrarywiſe, are as $LM \times MC$, and $lm \times mC$, that is, as $LN \times MC + NM \times MC$, and $ln \times mC - nm \times mC$; or $LN \times MC + NM \times MC$, and $LN \times mC - NM \times mC$, and $LN \times Mm - NM \times \overline{MC + mC}$, the difference of the two, is the force of both taken together to turn the Earth round. The affirmative part of this difference $LN \times Mm$, or $2LN \times NX$, is to $2AH \times HC$, the force of two particles of the ſame ſize ſituated

ated in A, as LX^2 to AC^2. And the negative part $\overline{NM \times MC + mC}$, or $2XY \times CY$, is to $2AH \times HC$, the force of the same two particles situated in A, as CX^2 to AC^2. And therefore the difference of the parts, that is, the force of the two particles L and l, taken together, to wheel the Earth about, is to the force of two particles, equal to the former and situated in the place A, to turn in like manner the Earth round, as $LX^2 - CX^2$ to AC^2. But if the circumference IK of the circle IK is supposed to be divided into an infinite number of little equal parts L, all the LX^2 will be to the like number of IX^2, as 1 to 2 (by lem. 1.) and to the same number of AC^2, as IX^2 to $2AC^2$; and the same number of CX^2, to as many AC^2, as $2CX^2$ to $2AC^2$. Wherefore the united forces of all the particles in the circumference of the circle IK, are to the joint forces of as many particles in the place A, as $IX^2 - 2CX^2$ to $2AC^2$; and therefore (by lem. 1.) to the united forces of as many particles in the circumference of the circle AE, as $IX^2 - 2CX^2$ to AC^2.

Now if Pp the diameter of the sphere is conceiv'd to be divided into an infinite number of equal parts, upon which a like number of circles IK are supposed to insist, the matter in the circumference of every circle IK will be as IX^2. And therefore the force of that matter to turn the Earth about will be as IX^2 into $IX^2 - 2CX^2$. And the force of the same matter, if it was situated in the circumference of the circle AE, would be as IX^2 into AC^2. And therefore the force of all the particles of the whole matter, situated without the sphere in the circumferences of all the circles, is to the force of the like number of particles situated in the circumference of the greatest circle AE, as all the IX^2 into $IX^2 - 2CX^2$ to as many IX^2 into AC^2, that is, as all the $AC^2 - CX^2$ into $AC^2 - 3CX^2$ to as many $AC^2 - CX^2$ into

AC

AC^2, that is, as all the $AC^4 - 4AC^2 \times CX^2 + 3CX^4$ to as many $AC^4 - AC^2 \times CX^2$, that is, as the whole fluent quantity whoſe fluxion is $AC^4 - 4AC^2 \times CX^2 + 3CX^4$, to the whole fluent quantity whoſe fluxion is $AC^4 - AC^2 \times CX^2$; and therefore by the method of fluxions, as $AC^4 \times CX - \frac{4}{3} AC^2 \times CX^3 + \frac{1}{5} CX^5$ to $AC^4 \times CX - \frac{1}{3} AC^2 \times CX^3$, that is, if for CX we write the whole Cp, or AC, as $\frac{4}{15} AC^5$ to $\frac{2}{3} AC^5$, that is, as 2 to 5. *Q. E. D.*

LEMMA III.

The ſame things ſtill ſuppoſed, I ſay in the third place, that the motion of the whole Earth about the axe abovenamed, ariſing from the motions of all the particles, will be to the motion of the foreſaid ring about the ſame axe, in a proportion compounded of the proportion of the matter in the Earth to the matter in the ring; and the proportion of three ſquares of the quadrantal arc of any circle, to two ſquares of its diameter, that is, in the proportion of the matter to the matter, and of the number 925275, *to the number* 1000000.

For the motion of a cylinder, revolv'd about its quieſcent axe, is to the motion of the inſcrib'd ſphere revolv'd together with it, as any four equal ſquares to three circles inſcrib'd in three of thoſe ſquares: And the motion of this cylinder is to the motion of an exceeding thin ring, ſurrounding both ſphere and cylinder in their common contact, as double the matter in the cylinder to triple the matter in the ring: And this motion of the ring, uniformly continued about

bout the axe of the cylinder, is to the uniform motion of the ſame about its own diameter perform'd in the ſame periodic time, as the circumference of a circle to double its diameter.

HYPOTHESIS II.

If the other parts of the Earth were took away, and the remaining ring was carried alone about the Sun in the orbit of the Earth by the annual motion, while by the diurnal motion it was in the mean time revolved about its own axe, inclined to the plane of the ecliptic by an angle of $23\frac{1}{2}$ *degrees; the motion of the equinoctial points would be the ſame, whether the ring were fluid, or whether it conſiſted of a hard and rigid matter.*

PROPOSITION XXXIX. PROBLEM XX.

To find the preceſſion of the equinoxes.

The middle horary motion of the Moon's nodes, in a circular orbit when the nodes are in the quadratures, was $16''.\ 35'''.\ 16^{iv}.\ 36^{v}.$ the half of which $8''.\ 17'''.\ 38^{iv}.\ 18^{v}.$ (for the reaſons above explain'd) is the mean horary motion of the nodes in ſuch an orbit, which motion in a whole ſidereal year becomes $20^{\circ}.\ 11'.\ 46''.$ Becauſe therefore the nodes of the Moon in ſuch an orbit would be yearly transfer'd $20^{\circ}.\ 11'.\ 46''.$ in *antecedentia*; and if there were more Moons, the motion of the nodes of every one, (by cor. 16. prop. 66. book 1.) would be as its periodic time; if upon the ſurface of the Earth, a Moon was revolv'd in the time of a ſidereal day, the annual motion of the nodes of this Moon would be to $20^{\circ}.\ 11'.\ 46''.$

as

Fig. 1. *p.* 296.

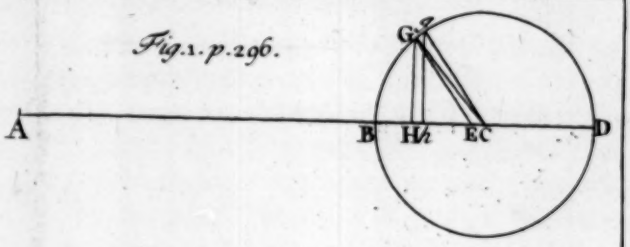


Fig. 2. *p.* 302.

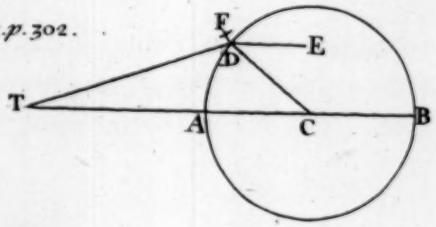
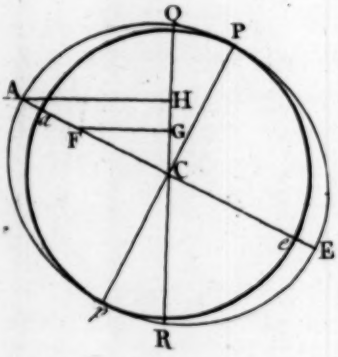


Fig. 3. *p.* 315.



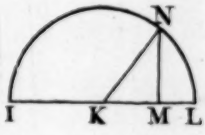
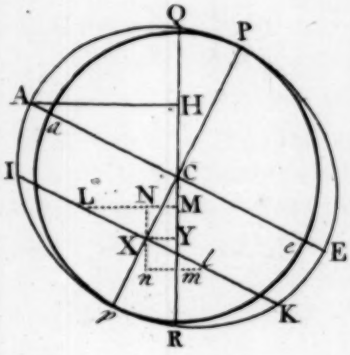


Fig. 4. *p.* 317.



as $23^h.\ 56'.$ the ſidereal day, to $27^d.\ 7^h.\ 43'.$ the periodic time of our Moon, that is, as 1436 to 39343. And the ſame thing would happen to the nodes of a ring of Moons encompaſſing the Earth, whether theſe Moons did not mutually touch each the other, or whether they were molten and form'd into a continued ring, or whether that ring ſhould become rigid and inflexible.

Let us then ſuppoſe that this ring is in quantity of matter equal to the whole exterior Earth *PapAPepE*, which lies without the ſphere *Papep* (ſee *Fig. Lem.* 2.) and becauſe this ſphere is to that exterior Earth, as aC^2 to $AC^2 - aC^2$, that is, (ſeeing *PC* or *aC* the leaſt ſemidiameter of the Earth is to *AC* the greateſt ſemidiameter of the ſame as 229 to 230) as 52441 to 459; if this ring encompaſs'd the Earth round the equator, and both together were revolv'd about the diameter of the ring, the motion of the ring (by lem. 3.) would be to the motion of the inner ſphere, as 459 to 52441 and 1000000 to 925275 conjunctly, that is, as 4590 to 485223; and therefore the motion of the ring would be to the ſum of the motions of both ring and ſphere, as 4590 to 489813. Wherefore if the ring adheres to the ſphere, and communicates its motion to the ſphere, by which its nodes or equinoctial points recede: the motion remaining in the ring will be to its former motion, as 4590 to 489813, upon which account the motion of the equinoctial points will be diminiſh'd in the ſame proportion. Wherefore the annual motion of the equinoctial points of the body, compoſed of both ring and ſphere, will be to the motion, $20^\circ.\ 11'.\ 46''.$ as 1436 to 39343 and 4590 to 489813 conjunctly, that is as 100 to 292369. But the forces by which the nodes of a number of Moons (as we explained above) and therefore by which the equinoctial points of the ring recede (that is the forces $3IT$ in *Fig. prop.* 30) are in the

ſeveral particles as the diſtances of thoſe particles from the plane QR; and by theſe forces the particles recede from that plane: and therefore (by lem. 2.) if the matter of the ring was ſpread all over the ſurface of the ſphere, after the faſhion of the figure $PapAPepE$, in order to make up that exterior part of the Earth, the total force or power of all the particles to wheel about the Earth round any diameter of the equator, and therefore to move the equinoctial points, would become leſs than before, in the proportion of 2 to 5. Wherefore the annual regreſs of the equinoxes now would be to 20°. 11'. 46". as 10 to 73092: that is, would be 9". 56"'. 50^{iv}.

But becauſe the plane of the equator is inclin'd to that of the ecliptic, this motion is to be diminiſh'd in the proportion of the ſine 91706, (which is the co-ſine of 23 $\frac{1}{2}$ deg.) to the radius 100000. And the remaining motion will now be 9". 7"'. 20^{iv}. which is the annual preceſſion of the equinoxes, ariſing from the force of the Sun.

But the force of the Moon to move the ſea was to the force of the Sun nearly as 4,4815 to 1. And the force of the Moon to move the equinoxes is to that of the Sun in the ſame proportion. Whence the annual preceſſion of the equinoxes, proceeding from the force of the Moon, comes out 40". 52"'. 52^{iv}. and the total annual preceſſion, ariſing from the united forces of both, will be 50". 00"'. 12^{iv}. the quantity of which motion agrees with the phænomena. For the preceſſion of the equinoxes, by aſtronomical obſervations, is about 50". yearly.

If the height of the Earth at the equator exceeds its height at the poles by more than 17$\frac{1}{6}$ miles, the matter thereof will be more rare near the ſurface, than at the center; and the preceſſion of the equinoxes will be augmented by the exceſs of height, and diminiſhed by the greater rarity.

And now we have deſcribed the ſyſtem of the Sun, the Earth, Moon and Planets, it remains that we add ſomething about the Comets.

LEMMA IV.

That the Comets are higher than the Moon, and in the regions of the Planets.

As the Comets were placed by aſtronomers above the Moon becauſe they were found to have no diurnal parallax; ſo their annual parallax is a convincing proof of their deſcending into the regions of the Planets. For all the Comets which move in a direct courſe according to the order of the ſigns, about the end of their appearance become more than ordinarily ſlow or retrograde, if the Earth is between them and the Sun: and more than ordinarily ſwift, if the Earth is approaching to a heliocentric oppoſition with them. Whereas, on the other hand, thoſe which move againſt the order of the ſigns, towards the end of their appearance, appear ſwifter than they ought to be, if the Earth is between them and the Sun; and ſlower, and perhaps retrograde, if the Earth is in the other ſide of its orbit. And theſe appearances proceed chiefly from the diverſe ſituations which the Earth acquires in the courſe of its motion, after the ſame manner as it happens to the Planets, which appear ſometimes retrograde, ſometimes more ſlowly, and ſometimes more ſwiftly, progreſſive, according as the motion of the Earth falls in with that of the Planet, or is directed the contrary way. If the Earth move the ſame way with the Comet, but, by an angular motion about the Sun, ſo much ſwifter that right lines drawn from the Earth to the Comet converge towards the parts beyond the Comet; the Comet ſeen from the Earth becauſe of its ſlower motion will ap-

pear retrograde; and even if the Earth is ſlower than the Comet, the motion of the Earth being ſubducted, the motion of the Comet will at leaſt appear retarded. But if the Earth tends the contrary way to that of the Comet, the motion of the Comet will from thence appear accelerated. And from this apparent acceleration, or retardation, or regreſſive motion, the diſtance of the Comet may be inferr'd in this manner. Let $\Upsilon Q A$, $\Upsilon Q B$, $\Upsilon Q C$ (*Pl.* 15. *Fig.* 1.) be three obſerved longitudes of the Comet about the time of its firſt appearing, and $\Upsilon Q F$ its laſt obſerved longitude before its diſappearing. Draw the right line ABC, whoſe parts AB, BC, intercepted between the right lines QA and QB, QB and QC, may be one to the other, as the two times between the three firſt obſervations. Produce AC to G, ſo as AG may be to AB as the time between the firſt and laſt obſervation to the time between the firſt and ſecond; and join QG. Now if the Comet did move uniformly in a right line, and the Earth either ſtood ſtill, or was likewiſe carried forwards in a right line by an uniform motion: the angle ΥQG would be the longitude of the Comet at the time of the laſt obſervation. The angle therefore FQG, which is the difference of the longitude, proceeds from the inequality of the motions of the Comet and the Earth. And this angle, if the Earth and Comet move contraryways, is added to the angle ΥQG, and accelerates the apparent motion of the Comet. But if the Comet move the ſame way with the Earth, it is ſubtracted, and either retards the motion of the Comet, or perhaps renders it retrograde, as we have but now explained. This angle therefore, proceeding chiefly from the motion of the Earth, is juſtly to be eſteem'd the parallax of the Comet; neglecting, to wit, ſome little increment or decrement that may ariſe from the unequal motion of the Comet in its orbit. And from this parallax we thus deduce the diſtance of the Comet.

Let

Let *S*, (Pl. 15. Fig. 2.) repreſent the Sun, *acT* the *orbis magnus*, *a* the Earth's place in the firſt obſervation, *c* the place of the Earth in the third obſervation, *T* the place of the Earth in the laſt obſervation, and *T* ♈ a right line drawn to the beginning of Aries. Set off the angle ♈ *TV*, equal to the angle ♈ *QF*, that is, equal to the longitude of the Comet at the time when the Earth is in *T*; join *ac*, and produce it to *g*, ſo as *ag* may be to *ac*, as *AG* to *AC*; and *g* will be the place at which the Earth would have arrived in the time of the laſt obſervation, if it had continued to move uniformly in the right line *ac*. Wherefore if we draw *g* ♈, parallel to *T* ♈, and make the angle ♈ *gV*, equal to the angle ♈ *QG*, this angle ♈ *gV* will be equal to the longitude of the Comet ſeen from the place *g*, and the angle *TVg* will be the parallax which ariſes from the Earth's being transferr'd from the place *g* into the place *T*; and therefore *V* will be the place of the Comet in the plane of the ecliptic. And this place *V* is commonly lower than the orb of Jupiter.

The ſame thing may be deduced from the incurvation of the way of the Comets. For theſe bodies move almoſt in great circles, while their velocity is great, but about the end of their courſe, when that part of their apparent motion which ariſes from the parallax bears a greater proportion to their whole apparent motion, they commonly deviate from thoſe circles, and when the Earth goes to one ſide, they deviate to the other. And this deflexion, becauſe of its correſponding with the motion of the earth, muſt ariſe chiefly from the parallax. And the quantity thereof is ſo conſiderable, as, by my computation, to place the diſappearing Comets a good deal lower than Jupiter. Whence it follows that when they approach nearer to us in their perigees and perihelions, they often deſcend below the orbs of Mars and the inferior Planets.

The near approach of the Comets is further confirmed from the light of their heads. For the light of a celeſtial body, illuminated by the Sun and receding to remote parts, is diminiſhed in the quadruplicate proportion of the diſtance; to wit, in one duplicate proportion, on account of the increaſe of the diſtance from the Sun, and in another duplicate proportion, on account of the decreaſe of the apparent diameter. Wherefore if both the quantity of light and the apparent diameter of a Comet are given, its diſtance will be alſo given, by taking the diſtance of the Comet to the diſtance of a Planet, in the direct proportion of their diameters and the reciprocal ſubduplicate proportion of their lights. Thus in the Comet of the year 1682, Mr. *Flamſtead* obſerved with a teleſcope of 16 feet, and meaſured with a micrometer, the leaſt diameter of its head, 2′. 00. But the nucleus, or ſtar in the middle of the head, ſcarcely amounted to the tenth part of this meaſure; and therefore its diameter was only 11″ or 12″. But in the light and ſplendor of its head, it ſurpaſs'd that of the Comet in the year 1680. and might be compared with the Stars of the firſt or ſecond magnitude. Let us ſuppoſe that Saturn with its ring was about four times more lucid; and becauſe the light of the ring was almoſt equal to the light of the globe within, and the apparent diameter of the globe is about 21″. and therefore the united light of both globe and ring would be equal to the light of a globe whoſe diameter is 30″. it follows that the diſtance of the Comet was to the diſtance of Saturn, as 1 to $\sqrt{4}$ inverſly and 12″ to 30 directly; that is, as 24 to 30, or 4 to 5. Again the Comet in the month of *April* 1665, as *Hevelius* informs us, excelled almoſt all the fixt Stars in ſplendor, and even Saturn it ſelf, as being of a much more vivid colour. For this Comet was more lucid than that other which had appeared about the end of the preceding year and had been compared to the Stars of the firſt magnitude.

The

The diameter of its head was about 6′. but the nucleus, compared with the Planets by means of a teleſcope, was plainly leſs than Jupiter; and ſometimes judged leſs, ſometimes judged equal to the globe of Saturn within the ring. Since then the diameters of the heads of the Comets ſeldom exceed 8′ or 12′. and the diameter of the nucleus or central ſtar is but about a tenth or perhaps fifteenth part of the diameter of the head; it appears that theſe ſtars are generally of about the ſame apparent magnitude with the Planets. But in regard their light may be often compared with the light of Saturn, yea and ſometimes exceeds it; it is evident, that all Comets in their perihelions, muſt either be placed below, or not far above Saturn. And they are much miſtaken, who remove them almoſt as far as the fixt Stars. For if it was ſo, the Comets could receive no more light from our Sun, than our Planets do from the fixt Stars.

So far we have gone, without conſidering the obſcuration which Comets ſuffer from that plenty of thick ſmoak, which encompaſſeth their heads, and through which the heads always ſhew dull, as through a cloud. For by how much the more a body is obſcured by this ſmoak, by ſo much the more near it muſt be allowed to come to the Sun, that it may vye with the Planets in the quantity of light which it reflects. Whence it is probable that the Comets deſcend far below the orb of Saturn, as we proved before from their parallax. But above all the thing is evinced from their tails, which muſt be owing either to the Sun's light reflected by a ſmoke ariſing from them, and diſperſing it ſelf through the æther, or to the light of their own heads. In the former caſe, we muſt ſhorten the diſtance of the Comets, leſt we be obliged to allow that the ſmoak ariſing from their heads, is propagated through ſuch a vaſt extent of ſpace and with ſuch a velocity and expanſion, as will ſeem altogether incre-

dible. In the latter caſe, the whole light of both head and tail is to be aſcribed to the central nucleus. But then if we ſuppoſe all this light to be united and condens'd within the diſc of the nucleus, certainly the nucleus will by far exceed Jupiter it ſelf in ſplendor, eſpecially when it emits a very large and lucid tail. If therefore, under a leſs apparent diameter, it reflects more light, it muſt be much more illuminated by the Sun, and therefore much nearer to it. And the ſame argument will bring down the heads of Comets ſometimes within the orb of Venus, *viz.* when being hid under the Sun's rays, they emit ſuch huge and ſplendid tails, like beams of fire, as ſometimes they do. For if all that light was ſuppoſed to be gathered together into one Star, it would ſometimes exceed not one Venus only, but a great many ſuch united into one.

Laſtly, the ſame thing is infer'd from the light of the heads, which increaſes in the receſs of the Comets from the Earth towards the Sun; and decreaſes in their return from the Sun towards the Earth. For ſo the Comet of the year 1665 (by the obſervations of *Hevelius*) from the time that it was firſt ſeen, was always loſing of its apparent motion, and therefore had already paſſed its perigee; but yet the ſplendor of its head was daily increaſing, till being hid under the Sun's rays, the Comet ceas'd to appear. The Comet of the year 1683 (by the obſervations of the ſame *Hevelius*) about the end of *July*, when it firſt appeared, moved at a very ſlow rate, advancing only about 40 or 45 minutes in its orb in a day's time. But from that time its diurnal motion was continually upon the increaſe, till *September* 4, when it aroſe to about 5 degrees. And therefore in all this interval of time, the Comet was approaching to the Earth. Which is likewiſe proved from the diameter of its head, meaſured with a micrometer. For *Auguſt* 6. *Hevelius* found it only 6'. 05". including the *coma*, which *Sept.* 2. he obſerved to be 9'.07".

9'.07". and therefore its head appeared far leſs about the beginning, than towards the end of the motion: tho' about the beginning, becauſe nearer to the Sun, it appeared far more lucid than towards the end, as the ſame *Hevelius* declares. Wherefore in all this interval of time, on account of its receſs from the Sun, it decreas'd in ſplendor, notwithſtanding its acceſs towards the Earth. The Comet of the year 1618 about the middle of *December*, and that of the year 1680, about the end of the ſame month, did both move with their greateſt velocity, and were therefore then in their perigees. But the greateſt ſplendor of their heads was ſeen two weeks before, when they had juſt got clear of the Sun's rays; and the greateſt ſplendor of their tails, a little more early, when yet nearer to the Sun. The head of the former Comet (according to the obſervations of *Cyſatus*) *December* 1. appeared greater than the Stars of the firſt magnitude, and *December* 16. (then in the perigee) it was but little diminiſhed in magnitude, but in the ſplendor and brightneſs of its light, a great deal. *January* 7, *Kepler* being uncertain about the head left off obſerving. *December* 12. the head of the latter Comet was ſeen and obſerv'd by Mr. *Flamſtead*, when but 9 degrees diſtant from the Sun; which is ſcarcely to be done in a Star of the third magnitude. *December* 15 and 17. it appeared as a Star of the third magnitude, its luſtre being diminiſhed by the brightneſs of the clouds near the ſetting Sun. *December* 26. when it mov'd with the greateſt velocity, being almoſt in its perigee, it was leſs than the mouth of *Pegaſus*, a Star of the third magnitude. *Jan.* 3. it appeared as a Star of the fourth. *Jan.* 9. as one of the fifth. *Jan.* 13. it was hid by the ſplendor of the Moon then in her increaſe. *January* 25. it was ſcarcely equal to the Stars of the ſeventh magnitude. If we compare equal intervals of time, on one ſide and on the other, from the perigee, we ſhall find that the head of the Comet, which at both

both intervals of time, was far, but yet equally, remov'd from the Earth, and ſhould have therefore ſhone with equal ſplendor, appear'd brighteſt on the ſide of the perigee towards the Sun; and diſappeared on the other. Therefore from the great difference of light in the one ſituation and in the other, we conclude the great vicinity of the Sun and Comet in the former. For the light of Comets uſes to be regular, and to appear greateſt when the heads move faſteſt, and are therefore in their perigees; excepting in ſo far as it is increaſed by their nearneſs to the Sun.

Cor. 1. Therefore the Comets ſhine by the Sun's light, which they reflect.

Cor. 2. From what has been ſaid, we may likewiſe underſtand, why Comets are ſo frequently ſeen in that hemiſphere in which the Sun is, and ſo ſeldom in the other. If they were viſible in the regions far above Saturn, they would appear more frequently in the parts oppoſite to the Sun. For ſuch as were in thoſe parts would be nearer to the Earth, whereas the preſence of the Sun muſt obſcure and hide thoſe that appear in the hemiſphere in which he is. Yet looking over the hiſtory of Comets, I find that four or five times more have been ſeen in the hemiſphere towards the Sun, than in the oppoſite hemiſphere; beſides, without doubt, not a few, which have been hid by the light of the Sun. For Comets deſcending into our parts neither emit tails nor are ſo well illuminated by the Sun as to diſcover themſelves to our naked eyes, until they are come nearer to us than Jupiter. But the far greater part of that ſpherical ſpace, which is deſcrib'd about the Sun with ſo ſmall an interval, lies on that ſide of the Earth which regards the Sun; and the Comets in that greater part are commonly more ſtrongly illuminated, as being for the moſt part nearer to the Sun.

Cor.

Cor. 3. Hence alſo it is evident, that the celeſtial ſpaces are void of reſiſtance. For though the Comets are carried in oblique paths, and ſometimes contrary to the courſe of the Planets, yet they move every way with the greateſt freedom, and preſerve their motions for an exceeding long time, even where contrary to the courſe of the Planets. I am out in my judgment, if they are not a ſort of Planets, revolving in orbits returning into themſelves with a perpetual motion. For as to what ſome writers contend, that they are no other than meteors, led into this opinion by the perpetual changes that happen to their heads, it ſeems to have no foundation. For the heads of Comets are encompaſſed with huge atmoſpheres, and the lowermoſt parts of theſe atmoſpheres muſt be the denſeſt. And therefore it is in the clouds only, not in the bodies of the Comets themſelves, that theſe changes are ſeen. Thus the Earth, if it was view'd from the Planets, would, without all doubt, ſhine by the light of its clouds, and the ſolid body would ſcarcely appear through the ſurrounding clouds. Thus alſo the belts of Jupiter are form'd in the clouds of that Planet, for they change their poſition one to another, and the ſolid body of Jupiter is hardly to be ſeen through them. And much more muſt the bodies of Comets be hid under their atmoſpheres, which are both deeper and thicker.

Proposition XL. Theorem XX.

That the Comets move in some of the conic sections, having their foci in the center of the Sun; and by radij drawn to the Sun describe areas proportional to the times.

This propoſition appears from cor. 1. prop. 13. book 1. compared with prop. 8. 12. and 13. book 3.

Cor. 1. Hence if Comets are revolv'd in orbits returning into themſelves, thoſe orbits will be ellipſes; and their periodic times be to the periodic times of the Planets in the ſeſquiplicate proportion of their principal axes. And therefore the Comets, which for the moſt part of their courſe are higher than the Planets, and upon that account deſcribe orbits with greater axes, will require a longer time to finiſh their revolutions. Thus if the axe of a Comet's orbit was four times greater than the axe of the orbit of Saturn, the time of the revolution of the Comet would be to the time of the revolution of Saturn, that is, to 30 years, as $4\sqrt{4}$ (or 8) to 1, and would therefore be 240 years.

Cor. 2. But their orbits will be ſo near to parabolas, that parabolas may be us'd for them without ſenſible error.

Cor. 3. And therefore by cor. 7. prop. 16. book 1. the velocity of every Comet will always be to the velocity of any Planet, ſuppos'd to be revolv'd at the ſame diſtance in a circle about the Sun, nearly in the ſubduplicate proportion of double the diſtance of the Planet from the centre of the Sun, to the diſtance of the Comet from the Sun's centre very nearly. Let us ſuppoſe the radius of the *orbis magnus*, or the greateſt ſemidiameter of the ellipſe which the Earth deſcribes,

to

to conſiſt of 100000000 parts; and then the Earth by its mean diurnal motion will deſcribe 1720212 of thoſe parts, and 71675½ by its horary motion. And therefore the Comet, at the ſame mean diſtance of the Earth from the Sun, with a velocity which is to the velocity of the Earth as √ 2 to 1, would by its diurnal motion deſcribe 2432747 parts, and 101364½ parts by its horary motion. But at greater or leſs diſtances both the diurnal and horary motion will be to this diurnal and horary motion in the reciprocal ſubduplicate proportion of the diſtances, and is therefore given.

Cor. 4. Wherefore, if the *latus rectum* of the parabola is quadruple of the radius of the *orbis magnus*, and the ſquare of that radius is ſuppos'd to conſiſt of 100000000 parts: the area which the Comet will daily deſcribe by a radius drawn to the Sun will be 1216373½ parts; and the horary area will be 50682¼ parts. But if the *latus rectum* is greater or leſs in any proportion, the diurnal and horary area will be leſs or greater, in the ſubduplicate of the ſame proportion reciprocally.

Lemma V.

To find a curve line of the parabolic kind, which ſhall paſs through any given number of points. Pl. 15. Fig. 3.

Let thoſe points be *A, B, C, D, E, F,* &c. and from the ſame to any right line *H N,* given in poſition, let fall as many perpendiculars *A H, B I, C K, D L, E M, F N,* &c.

Caſe 1. If *H I, I K, K L,* &c. the intervals of the points *H, I, K, L, M, N,* &c. are equal, take *b,* 2 *b,* 3 *b,* 4 *b,*

$4b$, $5b$, &c. the firſt differences of the perpendiculars AH, BI, CK, &c. their ſecond differences c, $2c$, $3c$, $4c$, &c. their third, d, $2d$, $3d$, &c. that is to ſay, ſo as $AH - BI$ may be $= b$, $BI - CK = 2b$, $CK - DL = 3b$, $DL + EM = 4b$, $-EM + FN = 5b$, &c. then $b - 2b = c$, &c. and ſo on to the laſt difference, which is here f. Then erecting any perpendicular RS, which may be conſidered as an ordinate of the curve required; in order to find the length of this ordinate, ſuppoſe the intervals HI, IK, KL, LM, &c. to be units, and let $AH = a$, $-HS = p$, $\frac{1}{2}p$ into $-IS = q$, $\frac{1}{3}q$ into $+SK = r$, $\frac{1}{4}r$ into $+SL = s$, $\frac{1}{5}s$ into $+SM = t$; proceeding, to wit, to ME, the laſt perpendicular but one, and prefixing negative ſigns before the terms HS, IS, &c. which lye from S towards A; and affirmative ſigns before the terms SK, SL, &c. which lie on the other ſide of the point S. And obſerving well the ſigns, RS will be $= a + bp + cq + dr + es + ft$, + &c.

Caſe 2. But if HI, IK, *&c.* the intervals of the points H, I, K, L, &c. are unequal, take b, $2b$, $3b$, $4b$, $5b$, &c. the firſt differences of the perpendiculars AH, BI, CK, &c. divided by the intervals between thoſe perpendiculars; c, $2c$, $3c$, $4c$, &c. their ſecond differences divided by the intervals between every two; d, $2d$, $3d$, &c. their third differences, divided by the interval between every three; e, $2e$, &c. their fourth differences, divided by the intervals between every four; and ſo forth; that is, in ſuch manner, that b may be $= \frac{AH - BI}{HI}$, $2b = \frac{BI - CK}{IK}$, $3b = \frac{CK - DL}{KL}$, &c.

then $c = \frac{b - 2b}{HK}$, $2c = \frac{2b - 3b}{IL}$, $3c = \frac{3b - 4b}{KM}$, &c.

then $d = \frac{c - 2c}{HL}$, $2d = \frac{2c - 3c}{IM}$, &c. And thoſe differences being found, let AH be $= a$, $-HS = p$,

p into

p into $-IS=q$, q into $+SK=r$, r into $+SL=s$, s into $+SM=t$; proceeding, to wit, to ME, the laſt perpendicular but one; and the ordinate RS will be $=a+bp+cq+dr+es+ft$, &c.

Cor. Hence the areas of all curves may be nearly found. For if ſome number of points of the curve to be ſquar'd are found, and a parabola be ſuppos'd to be drawn through thoſe points; the area of this parabola will be nearly the ſame with the area of the curvilinear figure propos'd to be ſquar'd. But the parabola can be always ſquar'd geometrically by methods vulgarly known.

Lemma VI.

Certain obſerved places of a Comet being given, to find the place of the ſame to any intermediate given time.

Let HI, IK, KL, LM (in the preceding Fig.) repreſent the times between the obſervations; HA, IB, KC, LD, ME, five obſerv'd longitudes of the Comet, and HS the given time between the firſt obſervation and the longitude required. Then if a regular curve $ABCDE$ is ſuppos'd to be drawn through the points A, B, C, D, E, and the ordinate RS is found out by the preceding lemma, RS will be the longitude required.

After the ſame method, from five obſerv'd latitudes we may find the latitude to a given time.

If the differences of the obſerved longitudes are ſmall, ſuppoſe of 4 or 5 degrees, three or four obſervations will be ſufficient to find a new longitude and latitude. But if the differences are greater, as of 10 or 20 degrees, five obſervations ought to be uſed.

LEMMA

LEMMA VII.

Through a given point P, (Pl. 15. Fig. 4.) *to draw a right line* B C, *whoſe parts* P B, P C, *cut off by two right lines* A B, A C, *given in poſition, may be, one to the other, in a given proportion.*

From the given point *P*, ſuppoſe any right line *PD* to be drawn to either of the right lines given as *AB*, and produce the ſame towards *AC* the other given right line, as far as *E*, ſo as *PE* may be to *PD* in the given proportion. Let *EC* be parallel to *AD*. Draw *CPB*, and *PC* will be to *PB*, as *PE* to *PD*. *Q. E. F.*

LEMMA VIII.

Let A B C (Pl. 16. Fig. 1.) *be a parabola, having its focus in* S. *By the chord* A C *biſected in* I *cut off the ſegment* A B C I, *whoſe diameter is* I μ, *and vertex* μ. *In* I μ *produced take* μ O *equal to one half of* I μ. *Join* O S, *and produce it to* ξ, *ſo as* S ξ *may be equal to* 2 S O. *Now, ſuppoſing a Comet to revolve in the arc* C B A, *draw* ξ B, *cutting* A C *in* E; *I ſay, the point* E *will cut off from the chord* A C *the ſegment* A E, *nearly proportional to the time.*

For, if we join *EO*, cutting the parabolic arc *ABC* in *Y*, and draw μ *X* touching the ſame arc in the vertex μ, and meeting *EO* in *X*, the curvilinear area *AEXμA*

Plate XV. Vol. II. P. 336.

Fig. 1. p. 324.

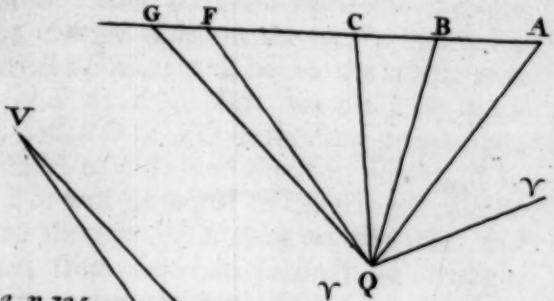


Fig. 2. p. 325.

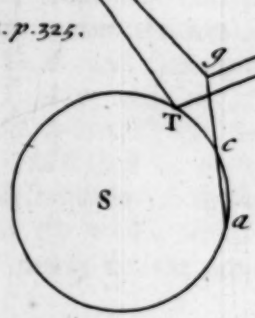


Fig. 3. p. 333.

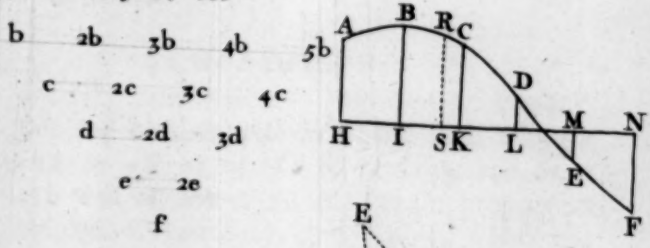
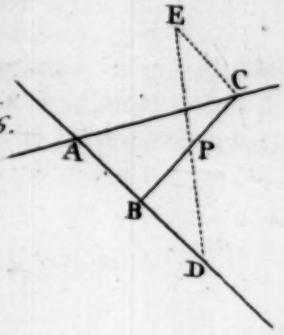


Fig. 4. p. 336.



$AEX\mu A$ will be to the curvilinear area $AC\Upsilon\mu A$, as AE to AC. And therefore ſince the triangle ASE is to the triangle ASC in the ſame proportion, the whole area $ASEX\mu A$ will be to the whole area $ASC\Upsilon\mu A$, as AE to AC. But becauſe ξO is to SO as 3 to 1, and EO to XO in the ſame proportion, SX will be parallel to EB: and therefore joining BX, the triangle SEB will be equal to the triangle XEB. Wherefore if to the area $ASEX\mu A$ we add the triangle EXB, and from the ſum ſubduct the triangle SEB, there will remain the area $ASBX\mu A$ equal to the area $ASEX\mu A$, and therefore in proportion to the area $ASC\Upsilon\mu A$ as AE to AC. But the area $ASB\Upsilon\mu A$ is nearly equal to the area $ASBX\mu A$, and this area $ASB\Upsilon\mu A$ is to the area $ASC\Upsilon\mu A$, as the time of deſcription of the arc AB to the time of deſcription of the whole arc AC. And therefore AE is to AC nearly in the proportion of the times. *Q. E. D.*

Cor. When the point B falls upon the vertex μ of the parabola, AE is to AC accurately in the proportion of the times.

Scholium.

If we join $\mu\xi$ cutting AC in δ, and in it take ξn in proportion to μB, as $27MI$ to $16M\mu$, and draw Bn: this Bn will cut the chord AC in the proportion of the times, more accurately than before. But the point n is to be taken beyond, or on this ſide the point ξ, according as the point B is more or leſs diſtant from the principal vertex of the parabola than the point μ.

LEMMA IX.

The right lines $I\mu$ *and* μM *and the length* $\frac{AIC}{4S\mu}$ *are equal among themſelves.*

For $4S\mu$ is the latus rectum of the parabola belonging to the vertex μ.

LEMMA X.

Produce Sμ *to* N *and* P, (Pl. 16. Fig. 1.) *ſo as* μN *may be one third of* μI, *and* SP *may be to* SN *as* SN *to* Sμ: *and in the time that a Comet would deſcribe the arc* AμC, *if it was ſuppos'd to move always forwards with the velocity which it hath in a height equal to* SP, *it would deſcribe a length equal to the chord* AC.

For if the Comet with the velocity, which it hath in μ, was in the ſaid time ſuppos'd to move uniformly forwards in the right line which touches the parabola in μ; the area which it would deſcribe by a radius drawn to the point *S*, would be equal to the parabolic area $ACS\mu A$. And therefore the ſpace contain'd under the length deſcrib'd in the tangent and the length $S\mu$, would be to the ſpace contain'd under the lengths AC and SM, as the area $ASC\mu A$ to the triangle ASC, that is, as SN to SM. Wherefore AC is to the length deſcrib'd in the tangent, as $S\mu$ to SN. But ſince the velocity of the Comet in the height SP (by cor. 6, prop. 16. book 1.) is to the velocity of the ſame

ſame in the height $S\mu$, in the reciprocal ſubduplicate proportion of SP to $S\mu$, that is, in the proportion of $S\mu$ to SN; the length deſcrib'd with this velocity will be to the length in the ſame time deſcrib'd in the tangent, as $S\mu$ to SN. Wherefore ſince AC, and the length deſcrib'd with this new velocity, are in the ſame proportion to the length deſcrib'd in the tangent, they muſt be equal betwixt themſelves. *Q. E. D.*

Cor. Therefore a Comet, with that velocity which it hath in the height $S\mu + \frac{1}{3} I\mu$, would, in the ſame time, deſcribe the chord AC nearly.

Lemma XI.

If a Comet void of all motion was let fall from the height SN, *or* $S\mu + \frac{1}{3}$ Iμ, *towards the Sun; and was ſtill impell'd to the Sun by the ſame force, uniformly continued, by which it was impell'd at firſt; the ſame in one half of that time in which it might deſcribe the arc* AC *in its own orbit, would in deſcending, deſcribe a ſpace equal to the length* Iμ.

For in the ſame time that the Comet would require to deſcribe the parabolic arc AC, it would (by the laſt lemma) with that velocity which it hath in the height SP, deſcribe the chord AC; and therefore (by cor. 7. prop. 16. book 1.) if it was in the ſame time ſuppos'd to revolve by the force of its own gravity, in a circle whoſe ſemidiameter was SP, it would deſcribe an arc of that circle, the length of which would be to the chord of the parabolic arc AC, in the ſubduplicate proportion of 1 to 2. Wherefore if with that weight, which in the height SP it hath towards the Sun, it ſhould fall from that height towards the Sun, it would

(by cor. 9. prop. 4. book 1.) in half the ſaid time deſcribe a ſpace equal to the ſquare of half the ſaid chord apply'd to quadruple the height SP, that is, it would deſcribe the ſpace $\frac{AI^2}{4SP}$. But ſince the weight of the Comet towards the Sun in the height SN, is to the weight of the ſame towards the Sun in the height SP, as SP to $S\mu$: the Comet, by the weight which it hath in the height SN, in falling from that height towards the Sun, would in the ſame time deſcribe the ſpace $\frac{AI^2}{4S\mu}$, that is, a ſpace equal to the length $I\mu$ or μM. *Q. E. D.*

Proposition XLI. Problem XXI.

From three obſervations given to determine the orbit of a Comet moving in a parabola.

This being a problem of very great difficulty, I try'd many methods of reſolving it; and ſeveral of thoſe problems, the compoſition whereof I have giv'n in the firſt book, tended to this purpoſe. But afterwards I contrived the following ſolution, which is ſomething more ſimple.

Select three obſervations diſtant one from another by intervals of time nearly equal. But let that interval of time in which the Comet moves more ſlowly, be ſomewhat greater than the other; ſo, to wit, that the difference of the times may be to the ſum of the times, as the ſum of the times to about 600 days; or that the point E (*Pl.* 16 *Fig.* 1.) may fall upon M nearly, and may err therefrom, rather towards I than towards A. If ſuch direct obſervations are not at hand, a new place of the Comet muſt be found by lem. 6.

Let

Let *S* (*Pl.* 16. *Fig.* 2.) repreſent the Sun; *T*, *t*, τ, three places of the Earth in the *orbis magnus*; *TA*, *tB*, τ*C*, three obſerv'd longitudes of the Comet; *V* the time between the firſt obſervation and the ſecond; *W* the time between the ſecond and the third; *X* the length, which, in the whole time, *V* + *W*, the Comet might deſcribe with that velocity which it hath in the mean diſtance of the Earth from the Sun: which length is to be found by cor. 3. prop. 40. book 3. and *tV* a perpendicular upon the chord *T*τ. In the mean obſerved longitude *tB*, take at pleaſure the point *B*, for the place of the Comet in the plane of the ecliptic; and from thence towards the Sun *S*, draw the line *BE*, which may be to the perpendicular *tV*, as the content under *SB* and St^2 to the cube of the hypotenuſe of the right angl'd triangle, whoſe ſides are *SB* and the tangent of the latitude of the Comet in the ſecond obſervation to the radius *tB*. And through the point *E*, (by lemma 7.) draw the right line *AEC*, whoſe parts *AE* and *EC*, terminating in the right lines *TA* and τ*C*, may be, one to the other, as the times *V* and *W*: then *A* and *C* will be nearly the places of the Comet in the plane of the ecliptic in the firſt and third obſervations, if *B* was its place rightly aſſum'd in the ſecond.

Upon *AC*, biſected in *I*, erect the perpendicular *Ii*. Through *B* draw the obſcure line *Bi* parallel to *AC*. Join the obſcure line *Si*, cutting *AC* in λ, and compleat the parallelogram *iI*λμ. Take *I*σ equal to 3*I*λ, and through the Sun *S*, draw the obſcure line σξ equal to 3*S*σ + 3*i*λ. Then, cancelling the letters *A*, *E*, *C*, *I*, from the point *B* towards the point ξ, draw the new obſcure line *BE*, which may be to the former *BE* in the duplicate proportion of the diſtance *BS* to the quantity *S*μ + $\frac{1}{3}$*i*λ. And through the point *E*, draw again the right line *AEC* by the ſame rule as before, that is, ſo as its parts *AE* and *EC* may be one to the other as the times *V* and *W*, between

the obſervations. Thus A and C will be the places of the Comet more accurately.

Upon AC, biſected in I, erect the perpendiculars AM, CN, IO, of which AM and CN may be the tangents of the latitudes in the firſt and third obſervations, to the radij TA and τC. Join MN, cutting IO in O. Draw the rectangular parallelogram $iI\lambda\mu$, as before. In IA produc'd, take ID equal to $S\mu + \frac{2}{3}i\lambda$. Then in MN, towards N, take MP, which may be to the above found length X, in the ſubduplicate proportion of the mean diſtance of the Earth from the Sun (or of the ſemidiameter of the *orbis magnus*) to the diſtance OD. If the point P fall upon the point N; A, B, and C will be three places of the Comet, through which its orbit is to be deſcrib'd in the plane of the ecliptic. But if the point P falls not upon the point N; in the right line AC take CG equal to NP, ſo as the points G and P may lie on the ſame ſide of the line NC.

By the ſame method, as the points E, A, C, G, were found from the aſſum'd point B, from other points b and β aſſum'd at pleaſure, find out the new points e, a, c, g; and ε, α, $\varkappa$, γ. Then through G, g, and γ, draw the circumference of a circle $G\,g\,\gamma$, cutting the right line τC in Z: and Z will be one place of the Comet in the plane of the ecliptic. And in AC, ac, $\alpha\varkappa$, taking AF, af, $\alpha\phi$ equal reſpectively to CG, cg, $\varkappa\gamma$; through the points F, f, and ϕ, draw the circumference of a circle $F\,f\,\phi$, cutting the right line AT in X; and the point X will be another place of the Comet in the plane of the ecliptic. And at the points X and Z, erecting the tangents of the latitudes of the Comet to the radij TX, and τZ, two places of the Comet in its own orbit will be determin'd. Laſtly, if (by prop. 19, book 1.) to the focus S, a parabola is deſcrib'd paſſing through thoſe two places, this parabola will be the orbit of the Comet. *Q. E. I.*

The

The demonſtration of this conſtruction follows from the preceding lemmas: becauſe the right line AC is cut in E in the proportion of the times by lem. 7, as it ought to be by lem. 8: and BE, by lem. 11, is a portion of the right line BS or $B\xi$ in the plane of the ecliptic, intercepted between the arc ABC and the chord AEC; and MP, (by cor. lem. 10.) is the length of the chord of that arc, which the Comet ſhould deſcribe in its proper orbit between the firſt and third obſervation, and therefore is equal to MN, providing B is a true place of the Comet in the plane of the ecliptic.

But it will be convenient to aſſume the points B, b, β, not at random, but nearly true. If the angle AQt, at which the projection of the orbit in the plane of the ecliptic cuts the right line tB, is rudely known; at that angle with Bt draw the obſcure line AC, which may be to $\frac{1}{2}T\tau$ in the ſubduplicate proportion of SQ to St. And drawing the right line SEB, ſo as its part EB may be equal to the length Vt, the point B will be determin'd which we are to uſe for the firſt time. Then cancelling the right line AC, and drawing a new AC according to the preceding conſtruction, and moreover, finding the length MP; in tB take the point b, by this rule, that if TA, and τC interſect each other in Υ, the diſtance Υb may be to the diſtance ΥB in a proportion compounded of the proportion of MP to MN and the ſubduplicate proportion of SB to Sb. And by the ſame method you may find the third point β, if you pleaſe to repeat the operation the third time. But if this method is follow'd, two operations generally will be ſufficient. For if the diſtance Bb happens to be very ſmall; after the points F, f, and G, g, are found, draw the right lines Ff and Gg, and they will cut TA and τC in the points requir'd X and Z.

EXAMPLE.

Let the Comet of the year 1680 be propos'd. The following table ſhews the motion thereof, as obſerv'd by *Flamſtead*, and calculated afterwards by him from his obſervations, and corrected by Dr. *Halley* from the ſame obſervations.

	Time		Sun's	Comet's	
	Appar.	True.	Longitude.	Longitude.	Lat. N.
	h. ′ ″	h. ′ ″	° ′ ″	° ′ ″	° ′
1680 *Dec.* 12	4.46	4.46. 0	♑ 1.51.23	♑ 6.32.30	8.28. 0
21	6.32½	6.36.59	11.06.44	♒ 5.08.12	21.42.13
24	6.12	6.17.52	14.09.26	18.49.23	25.23. 5
26	5.14	5.20.44	16.09.22	28.24.13	27.00.52
29	7.55	8.03.02	19.19.43	♓ 13.10.41	28.09.58
30	8.02	8.10.26	20.21.09	17.38.20	28.11.53
1681 *Jan.* 5	5.51	6.01.38	26.22.18	♈ 8.48.53	26.15. 7
9	6.49	7.00.53	♒ 0.29.02	18.44.04	24.11.56
10	5.54	6.06.10	1.27.43	20.40.50	23.43.52
13	6.56	7.08.55	4.33.20	25.59.48	22.17.28
25	7.44	7.58.42	16.45.36	♉ 9.35. 0	17.56.30
30	8.07	8.21.53	21.49.58	13.19.51	16.42.18
Feb. 2	6.20	6.34.51	24.46.59	15.13.53	16.04. 1
5	6.50	7.04.41	27.49.51	16.59.06	15.27. 3

To theſe you may add ſome obſervations of mine.

	Ap.	Comet's	
	Time.	Longitude.	Lat. North.
	h. ′	° ′ ″	° ′ ″
1681 *Feb.* 25	8.30	♉ 26.18.35	12.46.46
27	8.15	27.04.20	22.36.12
Mar. 1	11. 0	27.52.42	12.23.40
2	8. 0	28.12.48	12.19.38
5	11.30	29.18. 0	12.03.16
7	9.30	♊ 0. 4. 0	11.57. 0
9	8.30	0.43. 4	11.45.52

Theſe obſervations were made by a teleſcope of 7 feet, with a micrometer and threads plac'd in the focus of

Fig. I. p. 336.

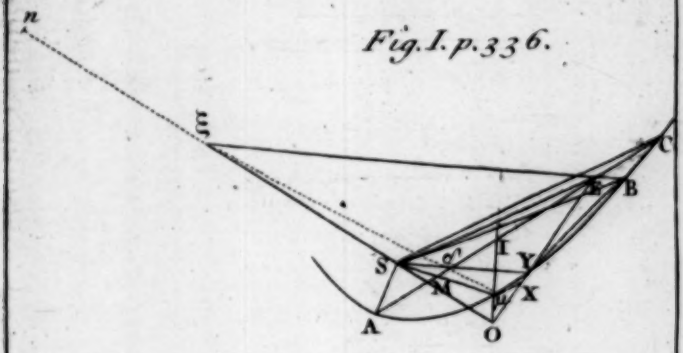


Fig. 2. p. 338.

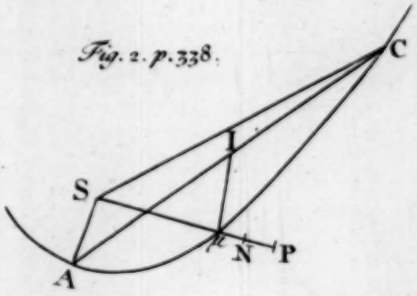


Fig. 3. p. 341.

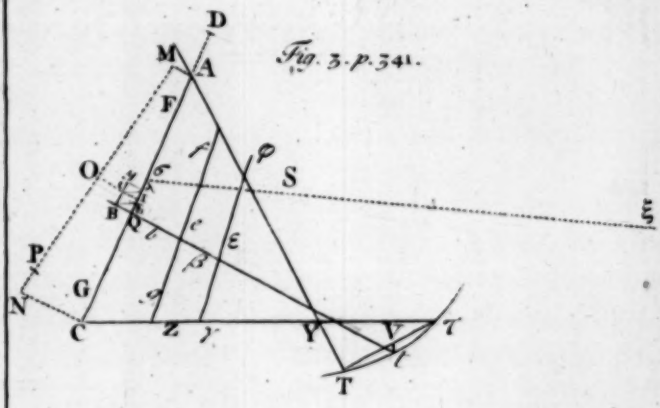
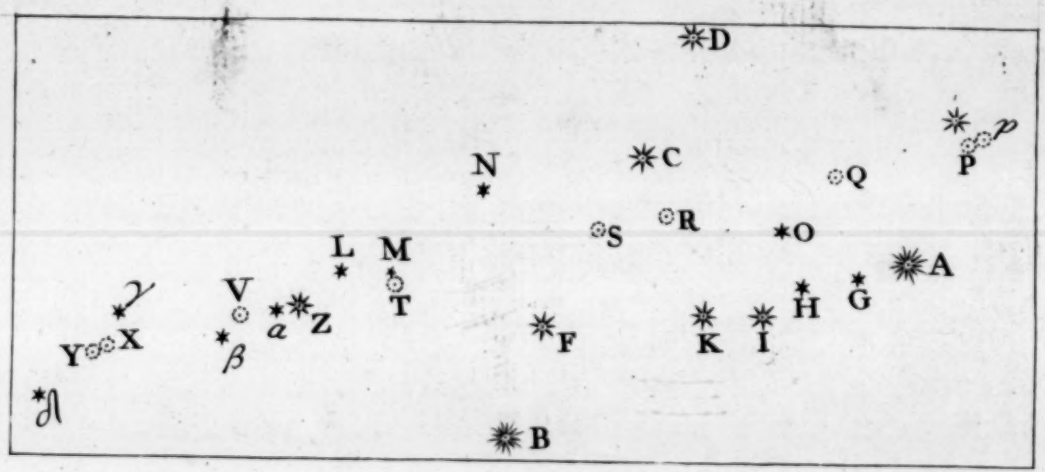


Plate XVII. *Vol.* II. *Pag.* 345.



of the teleſcope; by which inſtruments we determin'd the poſitions both of the fixt Stars among themſelves and of the Comet in reſpect of the fixt Stars. Let *A* (*Pl.* 17.) repreſent the Star of the fourth magnitude in the left heel of *Perſeus*, (*Bayer's o*) *B* the following Star of the third magnitude in the left foot (*Bayers ζ*) *C* a Star of the ſixth magnitude (*Bayer's n*) in the heel of the ſame foot, and *D*, *E*, *F*, *G*, *H*, *I*, *K*, *L*, *M*, *N*, *O*, *Z*, *α*, *β*, *γ*, *δ*, other ſmaller Stars in the ſame foot. And let *p*, *P*, *Q*, *R*, *S*, *T*, *V*, X, repreſent the places of the Comet in the obſervations above ſet down; and reckoning the diſtance *AB* of $80\frac{7}{12}$ parts, *AC* was $52\frac{1}{4}$ of thoſe parts, *BC*, $58\frac{1}{6}$; *AD*, $57\frac{1}{12}$; *BD*, $82\frac{6}{11}$; *CD*, $23\frac{2}{3}$; *AE*, $29\frac{4}{7}$; *CE*, $57\frac{1}{2}$; *DE*, $49\frac{11}{12}$; *AI*, $27\frac{7}{12}$; *BI*, $52\frac{1}{6}$; *CI*, $36\frac{7}{12}$; *DI*, $53\frac{5}{11}$; *AK*, $38\frac{2}{3}$; *BK*, 43; *CK*, $31\frac{1}{9}$; *FK*, 29; *FB*, 23, *FC*, $36\frac{1}{4}$; *AH*, $18\frac{4}{7}$; *DH*, $50\frac{7}{8}$; *BN*, $46\frac{5}{12}$; *CN*, $31\frac{1}{3}$; *BL*, $45\frac{5}{12}$; *NL*, $31\frac{5}{7}$. *HO* was to *HI* as 7 to 6, and produc'd did paſs between the Stars *D* and *E*, ſo as the diſtance of the Star *D* from this right line was $\frac{1}{6}$ *CD*. *LM* was to *LN* as 2 to 9, and produc'd did paſs through the Star *H*. Thus were the poſitions of the fixt Stars determin'd in reſpect of one another.

Mr. *Pound* has ſince obſerved a ſecond time the poſitions of theſe fixed Stars amongſt themſelves, and collected their longitudes and latitudes according to the following table.

The

The fixed Stars.	Their Longitudes.	Latitude North.	The fixed Stars.	Their Longitudes.	Latitude North.
	° ′ ″	° ′ ″		° ′ ″	° ′ ″
A	♉ 26.41.50	12. 8.36	L	♉ 29.33.34	12. 7.48
B	28.40.23	11.17.54	M	29.18.54	12. 7.20
C	27.58.30	12.40.25	N	28.48.29	12.31. 9
E	26.27.17	12.52. 7	Z	29.44.48	11.57.13
F	28.28.37	11.52.22	α	29.52. 3	11.55.48
G	26.56. 8	12. 4.58	β	♊ 0. 8.23	11.48.56
H	27.11.45	12. 2. 1	γ	0.40.10	11.55.18
I	27.25. 2	11.53.11	δ	1. 3.20	11.30.42
K	27.42. 7	11.53.26			

The positions of the Comet to these fix'd Stars were observ'd to be as follows.

Friday, *Feb.* 25. O. S. at $8\frac{1}{2}^{h}$, P. M. the distance of the Comet in p from the Star E, was less than $\frac{1}{13}AE$, and greater than $\frac{1}{5}AE$, and therefore nearly equal to $\frac{1}{14}AE$; and the angle ApE was a little obtuse, but almost right. For from A, letting fall a perpendicular on pE, the distance of the Comet from that perpendicular was $\frac{1}{5}pE$.

The same night at $9\frac{1}{2}^{h}$, the distance of the Comet in P from the Star E, was greater than $\frac{1}{4\frac{1}{2}}AE$, and less than $\frac{1}{5\frac{1}{4}}AE$, and therefore nearly equal to $\frac{10}{4\frac{2}{8}}$ of AE, or $\frac{8}{39}AE$. But the distance of the Comet from the perpendicular let fall from the Star A upon the right line PE, was $\frac{4}{5}PE$.

Sunday, *Feb.* 27. $8\frac{1}{4}^{h}$ P. M. the distance of the Comet in Q, from the Star O, was equal to the distance of the Stars O and H; and the right line QO produc'd pass'd between the Stars K and B. I could not, by reason of intervening clouds, determine the position of the Star to greater accuracy.

Tuesday, *March* 1. 11^{h}. P. M. the Comet in R, lay exactly in a line between the Stars K and C, so as the

the part CR of the right line CRK, was a little greater than $\frac{1}{3}CK$ and a little leſs than $\frac{1}{3}CK + \frac{1}{8}CR$, and therefore $= \frac{1}{3}CK + \frac{1}{16}CR$, or $\frac{1}{43}CK$.

Wedneſday, *March* 2. 8^h. P. M. the diſtance of the Comet in S from the Star C, was nearly $\frac{4}{9}FC$; the diſtance of the Star F from the right line CS produc'd was $\frac{1}{24}FC$; and the diſtance of the Star B from the ſame right line was five times greater than the diſtance of the Star F. And the right line NS produc'd paſs'd between the Stars H and I, five or ſix times nearer to the Star H than to the Star I.

Saturday, *March* 5. 11$\frac{1}{2}^h$ P. M. when the Comet was in T, the right-line MT was equal to $\frac{1}{2}ML$, and the right-line LT produc'd paſs'd between B and F, four or five times nearer to F than to B, cutting off from BF a fifth or ſixth part thereof towards F: and MT produc'd paſs'd on the out-ſide of the ſpace BF, towards the Star B, four times nearer to the Star B than to the Star F. M was a very ſmall Star ſcarcely to be ſeen by the teleſcope, but the Star L was greater, and of about the eighth magnitude.

Monday, *March* 7. 9$\frac{1}{2}$ P. M. The Comet being in V, the right line $V\alpha$ produced did paſs between B and F, cutting off, from BF towards F, $\frac{1}{10}$ of BF, and was to the right line $V\beta$ at 5 to 4. And the diſtance of the Comet from the right line $\alpha\beta$ was $\frac{1}{2}V\beta$.

Wedneſday, *March* 9. 8$\frac{1}{2}^h$ P. M. the Comet being in X, the right line γX was equal to $\frac{1}{4}\gamma\delta$, and the perpendicular let fall from the Star δ upon the right γX was $\frac{1}{5}$ of $\gamma\delta$.

The ſame night at 12^h, the Comet being in Υ, the right line $\gamma\Upsilon$ was equal to $\frac{1}{3}$ of $\gamma\delta$, or a little leſs, as perhaps $\frac{1}{16}$ of $\gamma\delta$, and a perpendicular let fall from the Star δ on the right line $\gamma\Upsilon$ was equal to about $\frac{1}{6}$ or $\frac{1}{7}\gamma\delta$. But the Comet being then extremely near the horizon was ſcarcely diſcernable, and therefore

its place could not be determined with that certainty as in the foregoing obſervations.

From theſe obſervations, by conſtructions of figures and calculations, I deduced the longitudes and latitudes of the Comet: and Mr. *Pound* by correcting the places of the fixed Stars hath determined more correctly the places of the Comet, which correct places are ſet down above. Though my micrometer was none of the beſt, yet the errors in longitude and latitude (as derived from my obſervations) ſcarcely exceed one minute. The Comet (according to my obſervations) about the end of its motion, began to decline ſenſibly towards the north, from the parallel which it deſcrib'd about the end of *February*.

Now in order to determine the orbit of the Comet out of the obſervations above deſcrib'd; I ſelected thoſe three which *Flamſtead* made, *Dec.* 21. *Jan.* 5. and *Jan.* 25. From which I found *St* of 9842,1 parts, and *Vt* of 455, ſuch as the ſemidiameter of the *orbis magnus* contains 10000. Then for the firſt obſervation, aſſuming *tB* of 5657 of thoſe parts, I found *SB* 9747, *BE* for the firſt time 412, *Sμ* 9503, *iλ* 413, *BE* for the ſecond time 421, *OD* 10186, *X* 8528,4; *PM* 8450, *MN* 8475, *NP* 25. From whence, by the ſecond operation, I collected the diſtance *tb* 5640. And by this operation, I at laſt deduced the diſtances *TX* 4775 and *τZ* 11322. From which limiting the orbit, I found its deſcending node in ☋ and aſcending node in ♈ 1° 53′; the inclination of its plane to the plane of the ecliptick 61°. 20′⅓; the vertex thereof (or the perihelion of the Comet) diſtant from the Node 8°. 38′, and in ♐ 27°. 43′, with latitude 7°. 34′ ſouth; its *latus rectum* 236,8; and the diurnal area deſcrib'd by a radius drawn to the Sun 93585, ſuppoſing the ſquare of the ſemidiameter of the *orbis magnus*, 100000000; that the Comet in this orbit mov'd directly according to the order of

the ſigns, and on *Dec.* 8^{d}. 00^{h}. 04′ P. M. was in the vertex or perihelion of its orbit. All which I determin'd by ſcale and compaſs, and the chords of angles, taken from the table of natural ſines, in a pretty large figure, in which, to wit, the radius of the *orbis magnus* (conſiſting of 10000 parts,) was equal to 16$\frac{1}{3}$ inches of an *Engliſh* Foot.

Laſtly, in order to diſcover whether the Comet did truly move in the orbit ſo determin'd, I inveſtigated its places in this orbit partly by arithmetical operations, and partly by ſcale and compaſs, to the times of ſome of the obſervations, as may be ſeen in the following table.

	The Comet's						
	Diſt. from Sun.	Longitude computed.	Latitud. computed.	Longitud. obſerv'd.	Latitude obſerv'd.	Dif. Lo.	Dif. Lat.
Dec. 12	2792	♒ 6°. 32′	8°.18½	♒ 6°.31$\frac{1}{2}$	8°.26	+1	— 7½
29	8403	♓ 13 . 13$\frac{2}{3}$	28. 00	♓13. 11$\frac{1}{2}$	28 .10$\frac{1}{12}$	+2	—10$\frac{1}{12}$
Febr. 5	16669	♉ 17 . 00	15. 29$\frac{2}{3}$	♉16. 59$\frac{1}{2}$	15 .27$\frac{2}{3}$	+0	+ 2$\frac{1}{4}$
Mar. 5	21737	29 . 19$\frac{3}{4}$	12. 4	29. 20$\frac{1}{2}$	12 . 3$\frac{1}{3}$	—1	$\frac{1}{3}$

But afterwards Dr. *Halley* did determine the orbit to a greater accuracy by an arithmetical calculus, than could be done by linear deſcriptions; and retaining the place of the nodes in ♋ and ♈ 1° 53′, and the inclination of the plane of the orbit to the ecliptic 61° 20$\frac{1}{3}$′, as well as the time of the Comets being in perihelio, *Dec.* 8^{d}. 00. 04′: he found the diſtance of the perihelion from the aſcending node meaſur'd in the Comet's orbit 9°. 20′, and the *latus rectum* of the parabola 2430 parts, ſuppoſing the mean diſtance of the Sun from the Earth to be 100000 parts. And from theſe *data*, by an accurate arithmetical calculus, he computed the places of the Comet to the times of the obſervations as follows.

True

The Comet's True Time	Dist. from the Sun.	Longitude Computed.	Latitude Computed.	Errors in Long.	Errors in Lat.
d. h. ′ ″		° ′ ″	° ′ ″	′ ″	′ ″
Dec. 12. 4.46. ½	28028	♍ 6.29.25	8.26.0 Bor.	−3.5	−2.0
21. 6.37.	61076	♒ 5.6.30	21.43.20	−1.42	+1.7
24. 6.18.	70008	18.48.20	25.22.40	−1.3	−0.25
26. 5.20.	75576	28.22.45	27.1.36	−1.28	+0.44
29. 8.3.	84021	♓ 13.12.40	28.10.10	+1.59	+0.12
30. 8.10.	86661	17.40.5	28.11.20	+1.45	−0.33
Jan. 5. 6.1. ½	101440	♈ 8.49.49	26.15.15	+0.56	+0.8
9. 7.0.	110959	18.44.36	24.12.54	+0.32	+0.58
10. 6.6.	113162	20.41.0	23.44.10	+0.10	+0.18
13. 7.9.	120000	26.0.21	22.17.30	+0.33	+0.2
25. 7.59.	145370	♉ 9.33.40	17.57.55	−1.20	+1.25
30. 8.22.	155303	13.17.41	16.42.7	−2.10	−0.25
Febr. 2. 6.35. ½	160951	15.11.11	16.4.15	−2.42	+0.11
5. 7.4. ½	166686	16.58.55	15.29.13	−0.41	+2.14
25. 8.41.	202570	26.15.46	12.48.0	−2.49	+1.10
Mar. 5.11.39.	216205	29.18.35	12.5.40	+0.35	+2.14

This Comet alſo appeared in the *November* before, and at *Coburg* in *Saxony* was obſerved by Mr. *Gottfried Kirch* on the 4*th* of that Month, on the 6*th* and 11*th O. S*; from its poſitions to the neareſt fixed Stars obſerved with ſufficient accuracy, ſometimes with a two foot, and ſometimes with a ten foot teleſcope; from

from the difference of longitudes of *Coburg* and *London*, 11°, and from the places of the fixed Stars obſerved by Mr. *Pound*, Dr. *Halley* has determined the places of the Comet as follows.

Nov. 3*d.* 17ʰ. 2′, apparent time at *London*, the Comet was in ♌ 29 deg. 51′, with 1 deg. 17′. 45″. latitude north.

November 5. 15ʰ. 58′ the Comet was in ♍ 3°. 23′, with 1°. 6′. north lat.

November 10. 16ʰ. 31′, the Comet was equally diſtant from two Stars in ♌ which are σ and τ in *Bayer*; but it had not quite touched the right line that joins them, but was very little diſtant from it. In *Flamſtead's* catalogue this Star σ was then in ♍ 14°. 15′, with 1 deg. 41′. lat. north nearly, and τ in ♍ 17°. 3′½ with 0. deg. 34′. lat. ſouth. And the middle point between thoſe Stars was ♍ 15°. 39′¼, with 0°. 33′½ lat. north. Let the diſtance of the Comet from that right line be about 10′ or 12′; and the difference of the longitude of the Comet and that middle point will be 7′; and the difference of the latitude nearly, 7′½. And thence it follows, that the Comet was in ♍ 15°. 32′, with about 26′ lat. north.

The firſt obſervation from the poſition of the Comet with reſpect to certain ſmall fixed Stars had all the exactneſs that could be deſired. The ſecond alſo was accurate enough. In the third obſervation, which was the leaſt accurate, there might be an error of 6 or 7 minutes, but hardly greater. The longitude of the Comet, as found in the firſt and moſt accurate obſertion, being computed in the aforeſaid parabolic orbit, comes out ♌ 29°. 30′. 22″, its latitude north 1°. 25′. 7″, and its diſtance from the Sun 115546.

Moreover, Dr. *Halley* obſerving that a remarkable Comet had appeared four times at equal intervals of 575 years, that is, in the Month of *September* after *Julius Cæſar* was killed, *An. Chr.* 531 in the conſulate of *Lam-*

Lampadius and *Oreſtes*, *An. Chr.* 1106 in the Month of *February*, and at the end of the year 1680; and that with a long and remarkable tail (except when it was ſeen after *Cæſar*'s death, at which time, by reaſon of the inconvenient ſituation of the Earth, the tail was not ſo conſpicuous:) ſet himſelf to find out an elliptic orbit whoſe greater axis ſhould be 1382957 parts, the mean diſtance of the Earth from the Sun containing 10000 ſuch; in which orbit a Comet might revolve in 575 years. And placing the aſcending node in ☋ 2°, 2′; the inclination of the plane of the orbit to the plane of the ecliptic in an angle of 61°. 6′. 48″; the perihelion of the Comet in this plane in ♐ 22°. 44′. 25″; the equal time of the perihelion *December* 7^{d}. 23^{h}. 9′; the diſtance of the perihelion from the aſcending node in the plane of the ecliptic 9°. 17′. 35″; and its conjugate axis 18481, 2; he computed the motions of the Comet in this ecliptic orbit. The places of the Comet, as deduced from the obſervations and as ariſing from computation made in this orbit, may be ſeen in the following table.

True

To face Page 352

True time			Long. obs.			Lat. Nor. obs.			Long. comp.			Lat. comp.			Errors in Long.		Errors in Lat.	
d	h	′	☊	°	′	″	°	′	″	☊	°	′	″	°	′	″	′	″	′	″
Nov. 3	16	47	☊	29	51	0	1	17	45	☊	29	51	22	1	17	32 N	+ 0	22	— 0	13
5	15	37	♍	3	23	0	1	6	0	♍	3	24	32	1	6	9	+ 1	32	+ 0	9
10	16	18		15	32	0	0	27	0		15	33	2	0	25	7	+ 1	2	— 1	53
16	17	00	-	-	-	-	-	-	-	♎	8	16	45	0	53	7 S	-	-	-	-
18	21	34	-	-	-	-	-	-	-		18	52	15	1	26	54	-	-	-	-
20	17	0	-	-	-	-	-	-	-		28	10	36	1	53	35	-	-	-	-
23	17	5	-	-	-	-	-	-	-	♏	13	22	42	2	29	0	-	-	-	-
Dec. 12	4	46	♑	6	32	30	8	28	0	♑	6	31	20	8	29	6 N	— 1	10	+ 1	6
21	6	37	♒	5	8	12	21	42	13	♒	5	6	14	21	44	42	— 1	58	+ 2	29
24	6	18		18	49	23	25	23	5		18	47	30	25	23	35	— 1	53	+ 0	30
26	5	21		28	24	13	27	0	52		28	21	42	27	2	1	— 2	31	+ 1	9
29	8	3	♓	13	10	41	28	9	58	♓	13	11	14	28	10	38	+ 0	33	+ 0	40
30	8	10		17	38	0	28	11	53		17	38	27	28	11	37	+ 0	7	— 0	16
Jan. 5	6	1½	♈	8	48	53	26	15	7	♈	8	48	51	26	14	57	— 0	2	— 0	10
9	7	1		18	44	4	24	11	56		18	43	51	24	12	17	— 0	13	+ 0	21
10	6	6		20	40	50	23	43	32		20	40	23	23	43	25	— 0	27	— 0	7
13	7	9		25	59	48	22	17	28		26	0	8	22	16	32	+ 0	20	— 0	56
25	7	59	♉	9	35	0	17	56	30	♉	9	34	11	17	56	6	— 0	49	— 0	24
30	8	22		13	19	51	16	42	18		13	18	28	16	40	5	— 1	23	— 2	13
Feb. 2	6	35		15	13	53	16	4	1		15	11	59	16	2	7	— 1	54	— 1	54
5	7	4½		16	59	6	15	27	3		16	59	17	15	27	0	+ 0	11	— 0	3
25	8	41		26	18	35	12	46	46		26	16	59	12	45	22	— 1	36	— 1	24
Mar. 1	11	10		27	52	42	12	23	40		27	51	47	12	22	28	— 0	55	— 1	12
5	11	39		29	18	0	12	3	16		29	20	11	12	2	50	+ 2	11	— 0	26
9	8	38	♊	0	43	4	11	45	52	♊	0	42	43	11	45	35	— 0	21	— 0	17

The obſervations of this Comet from the beginning to the end agree as perfectly with the motion of the Comet in the orbit juſt now deſcribed, as the motions of the Planets do with the theories from whence they are calculated, and by this agreement plainly evince that it was one and the ſame Comet that appeared all that time; and alſo that the orbit of that Comet is here rightly defined.

In the foregoing table we have omitted the obſervations of *Nov.* 16, 18, 20 and 23 as not ſufficiently accurate. For at thoſe times ſeveral perſons had obſerved the Comet. *Nov.* 17. *O. S. Pontheus* and his Companions at 6^h in the morning at *Rome* (that is 5^h. 10′ at *London)* by threads directed to the fixt Stars, obſerv'd the Comet in ♎ 8°. 30′. with latitude, 0°. 40′. ſouth. Their obſervations may be ſeen in a treatiſe, which *Pontheus* publiſh'd concerning this Comet. *Cellius* who was preſent, and communicated his obſervations in a Letter to *Caſſini*, ſaw the Comet at the ſame hour in ♎ 8°. 30′. with latitude 0°. 30′ ſouth. It was likewiſe ſeen by *Galletius* at the ſame hour at *Avignon* (that is at 5^h. 42′. morning at *London)* in ♎ 8°. without latitude. But by the theory the Comet was at that time in ♎ 8°. 16′. 45″. and its latitude was 0°. 53′. 7″. ſouth.

Nov. 18. at 6^h. 30′ in the morning at *Rome* (that is, at 5^h. 40′. at *London) Pontheus* obſerv'd the Comet in ♎ 13°. 30′. with latitude 1°. 20′. ſouth; and *Cellius* in ♎ 13°. 30′. with latitude 1°. 00′. ſouth. But at 5^h 30′. in the morning at *Avignon Galletius* ſaw it in ♎ 13°. 00′. with latitude 1°. 00′ ſouth. In the univerſity of *La Fleche* in *France*, at 5^h in the morning (that is at 5^h. 9′. at *London*) it was ſeen by *P. Ango*, in the middle between two ſmall Stars, one of which is the middle of the three which lye in a right-line in the ſouthern hand of Virgo, *Bayers* ψ, and the other the outmoſt of the wing, *Bayers* θ. Whence the

Comet was then in ♎ 12°. 46′. with latitude 50′ ſouth. And I was informed by Dr. *Halley* that on the ſame day, at *Boſton* in *New-England*, in the latitude of 42 ½ deg. at 5ʰ in the morning, (that is, at 9ʰ. 44′ in the morning at *London*,) the Comet was ſeen near ♎ 14°, with latitude 1°. 30′ ſouth.

Nov. 19. at 4ʰ ½ at *Cambridge*, the Comet (by the obſervation of a young man) was diſtant from *Spica* ♍ about 2° towards the north-weſt. Now the ſpike was at that time in ♎ 19°. 23′. 47″. with latitude 2°. 1′. 59″. ſouth. The ſame day at 5ʰ in the morning at *Boſton* in *New-England*, the Comet was diſtant from *Spica* ♍ 1° with the difference of 40′ in latitude. The ſame day in the iſland of *Jamaica*, it was about 1° diſtant from *Spica* ♍. The ſame day Mr. *Arthur Storer* at the river *Patuxent* near *Hunting Creek* in *Maryland* in the confines of *Virginia* in lat. 38½° at 5 in the morning (that is at 10ʰ. at *London*) ſaw the Comet above *Spica* ♍, and very nearly join'd with it, the diſtance between them being about ¼ of one deg. And from theſe obſervations compar'd I conclude, that at 9ʰ 44′ at *London*, the Comet was in ♎ 18°. 50′ with about 1°. 25′ latitude ſouth. Now by the theory the Comet was at that time in ♎ 18°. 52′. 15″. with 1°. 26′. 54″. lat. ſouth.

Nov. 20. *Montenari* profeſſor of aſtronomy at *Padua*, at 6ʰ in the morning at *Venice* (that is 5ʰ. 10′ at *London*) ſaw the Comet in ♎ 23°. with latitude 1°. 30′ ſouth. The ſame day at *Boſton*, it was diſtant from *Spica* ♍ by about 4° of longitude eaſt, and therefore was in ♎ 23°. 24′ nearly.

Nov. 21. *Ponthæus* and his companions at 7¼ʰ in the morning, obſerv'd the Comet in ♎ 27°. 50′ with latitude 1°. 16′. ſouth. *Cellius* in ♎ 28°. *P. Ango* at 5ʰ in the morning, in ♎ 27°. 45′. *Montenari* in ♎ 27°. 51′. The ſame day in the iſland of *Jamaica*, it was ſeen near the beginning of ♏ and of about the ſame latitude

titude with *Spica* ♍, that is, 2°. 2′. The ſame day at 5ʰ morning at *Ballaſore* in the *Eaſt-Indies* (that is at 11ʰ, 20′ of the night preceding at *London)* the diſtance of the Comet from *Spica* ♍ was taken 7°. 35′. to the eaſt. It was in a right line between the ſpike and the ballance, and therefore was then in ♎ 26°. 58′. with about 1°. 11′. lat. ſouth; and after 5ʰ. 40′. (that is at 5ʰ morning at *London)* it was in ♎ 28°. 12′. with 1°. 16′. lat. ſouth. Now by the theory the Comet was then in ♎ 28°. 10′. 36″ with 1°. 53′. 35″ lat. ſouth.

Nov. 22. The Comet was ſeen by *Montenari* in ♏ 2°. 33′. But at *Boſton* in *New-England*, it was found in about ♏ 3°, and with almoſt the ſame latitude as before, that is, 1°. 30′. The ſame day at 5ʰ morning at *Ballaſore* the Comet was obſerv'd in ♏ 1°. 50′; and therefore at 5ʰ morning at *London* the Comet was in ♏ 3°. 5′ nearly. The ſame day at $6\frac{1}{2}$ʰ in the morning at *London*, Dr. *Hook* obſerv'd it in about ♏ 3°. 30′; and that in the right line which paſſeth through *Spica* ♍ and *Cor Leonis*; not indeed exactly, but deviating a little from that line towards the north. *Montenari* likewiſe obſerv'd, that this day and ſome days after, a right line drawn from the Comet through *Spica*, paſs'd by the ſouth ſide of *Cor Leonis*, at a very ſmall diſtance therefrom. The right line through *Cor Leonis* and *Spica* ♍ did cut the ecliptic in ♍ 3°. 46′ at an angle of 2°. 51′. And if the Comet had been in this line and in ♏ 3°. its latitude would have been 2°. 26′. But ſince *Hook* and *Montenari* agree, that the Comet was at ſome ſmall diſtance from this line towards the north, its latitude muſt have been ſomething leſs. On the 20th, by the obſervation of *Montenari*, its latitude was almoſt the ſame with that of *Spica*, that is about 1°. 30′. But by the agreement of *Hook*, *Montenari* and *Ango*, the latitude was continually increaſing and therefore muſt now on the 22d, be ſenſibly greater than

1°. 30′. And taking a mean between the extreme limits but now ſtated 2°. 26′ and 1°. 30′, the latitude will be about 1°. 58′. *Hook* and *Montenari* agree that the tail of the Comet was directed towards *Spica* ♍, declining a little from that Star towards the ſouth according to *Hook*, but towards the north, according to *Montenari*. And therefore that declination was ſcarcely ſenſible; and the tail lying nearly parallel to the equator, deviated a little from the oppoſition of the Sun, towards the north.

Nov. 23. *O. S.* At 5ʰ morning at *Nuremberg* (that is at $4^{h}\frac{1}{2}$ at *London*) Mr. *Zimmerman* ſaw the Comet in ♏ 8°. 8′ with 2°. 31′ ſouth lat. its place being collected by taking its diſtances from fixed Stars.

Nov. 24. Before Sun-riſing the Comet was ſeen by *Montenari* in ♏ 12°. 52′ on the north ſide of the right line through *Cor Leonis* and *Spica* ♍, and therefore its latitude was ſomething leſs than 2°. 38′. And ſince the latitude, as we ſaid, by the concurring obſervations of *Montenari*, *Ango*, and *Hook*, was continually increaſing; therefore it was now on the 24th ſomething greater than 1°. 58′; and, taking the mean quantity, may be reckon'd 2°. 18′, without any conſiderable error. *Ponthæus* and *Galletius* will have it that the latitude was now decreaſing; and *Cellius* and the obſerver in *New-England*, that it continued the ſame, *viz.* of about 1°, or $1\frac{1}{4}$°. The obſervations of *Ponthæus* and *Cellius* are more rude, eſpecially thoſe which were made by taking the azimuths and altitudes; as are alſo the obſervations of *Galletius*. Thoſe are better which were made by taking the poſition of the Comet to the fixt Stars by *Montenari*, *Hook*, *Ango*, and the obſerver in *New-England*, and ſometimes by *Ponthæus* and *Cellius*. The ſame day, at 5ʰ morning at *Ballaſore* the Comet was obſerved in ♏ 11°. 45′; and therefore at 5ʰ morning at *London* was in ♏ 13° nearly. And by

by the theory, the Comet was at that time in ♏ 13°. 22'. 42".

Nov. 25. Before Sun-riſe *Montenari* obſerv'd the Comet in ♏ 17° ¼ nearly; and *Cellius* obſerv'd at the ſame time that the Comet was in a right line between the bright Star in the right thigh of Virgo and the ſouthern ſcale of Libra; and this right line cuts the Comet's way in ♏ 18°. 36'. And by the theory the Comet was in ♏ 18° ⅓ nearly.

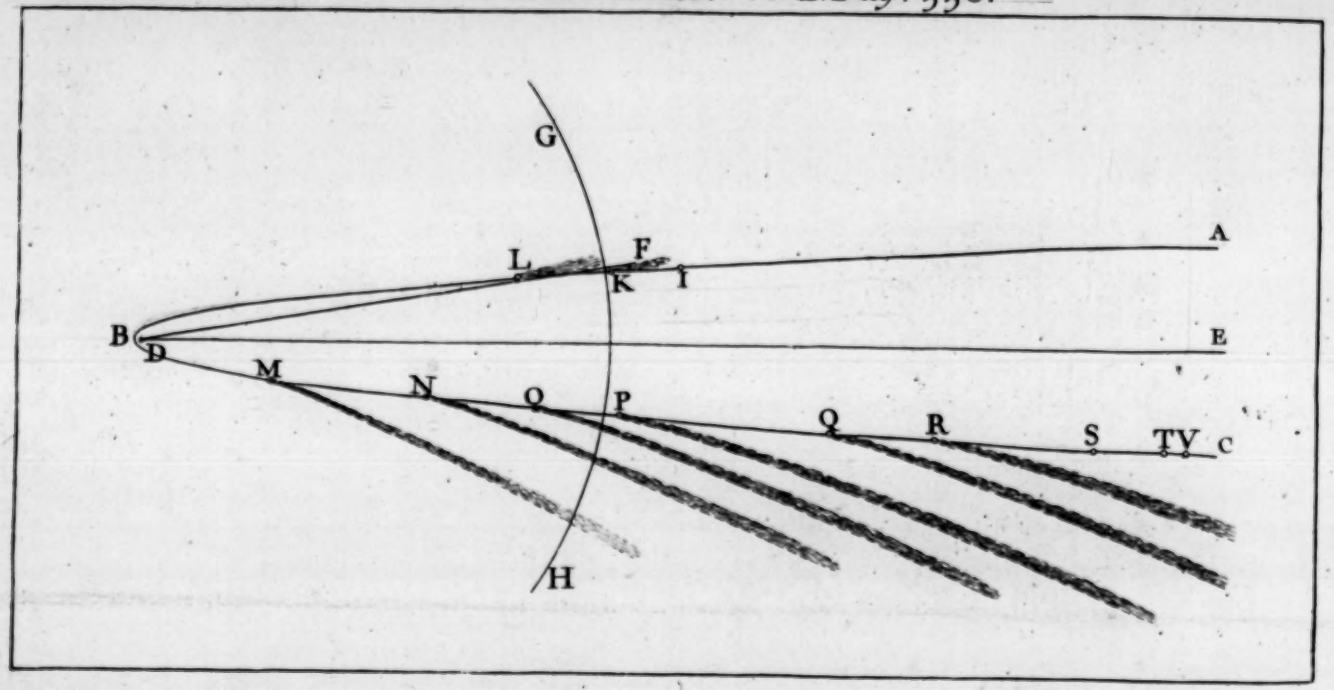
From all this it is plain that theſe obſervations agree with the theory, ſo far as they agree with one another, and by this agreement it is made clear that it was one and the ſame Comet that appeared all the time from *Nov.* 4. to *Mar.* 9. The path of this Comet did twice cut the plane of the ecliptic, and therefore was not a right line. It did cut the ecliptic, not in oppoſite parts of the heavens, but in the end of Virgo and beginning of Capricorn, including an arc of about 98°. And therefore the way of the Comet did very much deviate from the path of a great circle. For in the month of *Nov.* it declined at leaſt 3° from the ecliptic towards the ſouth; and in the month of *Dec.* following it declined 29° from the ecliptic towards the north; the two parts of the orbit in which the Comet deſcended towards the Sun, and aſcended again from the Sun, declining one from the other by an apparent angle of above 30°, as obſerv'd by *Montenari.* This Comet travel'd over 9 ſigns, to wit, from the laſt deg. of ♌ to the beginning of ♊, beſide the ſign of ♋, thro' which it paſs'd before it began to be ſeen. And there is no other theory by which a Comet can go over ſo great a part of the heavens with a regular motion. The motion of this Comet was very unequable. For about the 20th of *Nov.* it deſcrib'd about 5° a day. Then its motion being retarded, between *Nov.* 26. and *Dec.* 12. to wit, in the ſpace of 15 ½ days, it deſcrib'd only 40°. But the motion thereof being afterwards accelerated, it

deſcrib'd near 5° a day, till its motion began to be again retarded. And the theory which juſtly correſponds with a motion ſo unequable, and through ſo great a part of the heavens, which obſerves the ſame laws with the theory of the Planets, and which accurately agrees with accurate aſtronomical obſervations, cannot be otherwiſe than true.

And thinking it would not be improper, I have giv'n (*Pl.* 18.) a true repreſentation of the orbit which this Comet deſcrib'd, and of the tail which it emitted in ſeveral places, in the annexed figure; protracted in the plane of the trajectory. In this ſcheme *A B C* repreſents the trajectory of the Comet, *D* the Sun, *D E* the axis of the trajectory, *D F* the line of the nodes, *G H* the interſection of the ſphere of the *orbis magnus* with the plane of the trajectory, *I* the place of the Comet *Nov.* 4. *Ann.* 1680, *K* the place of the ſame *Nov.* 11. *L* the place of the ſame *Nov.* 19. *M* its place *Dec.* 12. *N* its place *Dec.* 21. *O* its place *Dec.* 29. *P* its place *Jan.* 5. following, *Q* its place *Jan.* 25. *R* its place *Feb.* 5. *S* its place *Feb.* 25. *T* its place *March* 5. and *V* its place *March* 9. In determining the length of the tail I made the following obſervations.

Nov. 4. and 6. the tail did not appear; *Nov.* 11. the tail juſt begun to ſhew itſelf, but did not appear above ½ deg. long through a 10 foot teleſcope; *Nov.* 17. the tail was ſeen by *Ponthæus* more than 15° long; *Nov.* 18. in *New-England* the tail appear'd 30° long, and directly oppoſite to the Sun, extending itſelf to the planet Mars, which was then in ♍ 9°. 54′; *Nov.* 19. in *Mary-Land*, the tail was found 15° or 20° long. *Dec.* 10. (by the obſervation of Mr. *Flamſtead*) the tail paſs'd through the middle of the diſtance intercepted between the tail of the Serpent of *Ophiuchus* and the Star δ in the ſouth wing of *Aquila*, and did terminate near the Stars *A*, ω, *b*, in *Bayer*'s tables. Therefore the end of the tail was in ♎ 19½°, with latitude about 34½°

Plate XVIII. *Vol.* II. *Pag.* 358.



$34\frac{1}{4}°$ north; *Dec.* 11. it aſcended to the head of *Sagitta* (*Bayer*'s α, β) terminating in ♑ 26°. 43′, with latitude 38°. 34′ north; *Dec.* 12. it paſs'd through the middle of *Sagitta*, nor did it reach much farther; terminating in ♒ 4°, with latitude $42\frac{1}{2}°$ north nearly. But theſe things are to be underſtood of the length of the brighter part of the tail. For with a more faint light, obſerv'd too perhaps in a ſerener sky, at *Rome*, *Dec.* 12. 5^h. 40′. by the obſervation of *Pontæus*, the tail aroſe to 10° above the rump of the ſwan, and the ſide thereof towards the weſt and towards the north was 45′ diſtant from this ſtar. But about that time the tail was 3° broad towards the upper end; and therefore the middle thereof was 2°. 15′ diſtant from that ſtar towards the ſouth, and the upper end was ♓ in 22° with latitude 61° north. And thence the tail was about 70° long. *Dec.* 21. it extended almoſt to *Caſſiopeia*'s chair, equally diſtant from β and from *Schedir*, ſo as its diſtance from either of the two was equal to the diſtance of the one from the other, and therefore did terminate in ♈ 24° with latitude $47\frac{1}{3}°$. *Dec.* 29. it reach'd to a contact with *Scheat* on its left, and exactly fill'd up the ſpace between the two ſtars in the northern foot of *Andromeda*, being 54° in length; and therefore terminated in ♉ 19° with 35° of latitude. *Jan.* 5. it touch'd the Star π in the breaſt of *Andromeda* on its right ſide, and the Star μ of the girdle on its left; and according to our obſervations, was 40° long; but it was curved, and the convex ſide thereof lay to the ſouth. And near the head of the Comet, it made an angle of 4° with the circle which paſs'd through the Sun and the Comet's head. But towards the other end, it was inclin'd to that circle in an angle of about 10° or 11°. And the chord of the tail contain'd with that circle an angle of 8°. *Jan.* 13. the tail terminated between *Alamech* and *Algol*, with a light that was ſenſible enough; but with a faint light it ended over againſt the Star $\varkappa$ in

Perſeus's ſide. The diſtance of the end of the tail from the circle paſſing through the Sun and the Comet, was 3°. 50′. And the inclination of the chord of the tail to that circle was $8\frac{1}{2}$°. *Jan.* 25. and 26. it ſhone with a faint light to the length of 6° or 7°. And for a night or two after when there was a very clear sky, it extended to the length of 12°, or ſomething more, with a light that was very faint and very hardly to be ſeen. But the axe thereof was exactly directed to the bright Star in the eaſtern ſhoulder of *Auriga*, and therefore deviated from the oppoſition of the Sun towards the north, by an angle of 10°. Laſtly, *Feb.* 10. with a teleſcope I obſerv'd the tail 2° long. For that fainter light which I ſpoke of, did not appear through the glaſſes. But *Pontheus* writes that on *Feb.* 7. he ſaw the tail 12° long. *Feb.* 25. the Comet was without a tail, and ſo continued till it diſappeared.

Now if one reflects upon the orbit deſcrib'd, and duly conſiders the other appearances of this Comet, he will be eaſily ſatisfy'd that the bodies of Comets are ſolid, compact, fixt and durable, like the bodies of the Planets. For if they were nothing elſe but the vapours or exhalations of the Earth, of the Sun, and other Planets, this Comet in its paſſage by the neighbourhood of the Sun, would have been immediately diſſipated. For the heat of the Sun is as the denſity of its rays, that is, reciprocally as the ſquare of the diſtance of the places from the Sun. Therefore, ſince on *Dec.* 8. when the Comet was in its perihelion, the diſtance thereof from the centre of the Sun was to the diſtance of the Earth from the ſame as about 6 to 1000, the Sun's heat on the Comet was at that time to the heat of the Summer-Sun with us, as 1000000 to 36, or as 28000 to 1. But the heat of boiling water is about 3 times greater than the heat which dry earth acquires from the Summer-Sun, as I have try'd; and the heat of red-hot iron (if my conjecture is right) is about

about three or four times greater than the heat of boiling water. And therefore the heat, which dry earth on the Comet, while in its perihelion, might have conceived from the rays of the Sun, was about 2000 times greater than the heat of red-hot iron. But by ſo fierce a heat, vapours and exhalations, and every volatile matter muſt have been immediately conſum'd and diſſipated.

This Comet therefore muſt have conceiv'd an immenſe heat from the Sun, and retain that heat for an exceeding long time. For a globe of iron of an inch in diameter, expos'd red-hot to to the open air, will ſcarcely loſe all its heat in an hour's time; but a greater globe would retain its heat longer in the proportion of its diameter, becauſe the ſurface (in proportion to which it is cool'd by the contact of the ambient air) is in that proportion leſs in reſpect of the quantity of the included hot matter. And therefore a globe of red-hot iron, equal to our Earth, that is, about 40000000 feet in diameter, would ſcarcely cool in an equal number of days, or in above 50000 years. But I ſuſpect that the duration of heat may, on account of ſome latent cauſes, increaſe in a yet leſs proportion than that of the diameter; and I ſhould be glad that the true proportion was inveſtigated by experiments.

It is further to be obſerv'd, that the Comet in the month of *December*, juſt after it had been heated by the Sun, did emit a much longer tail, and much more ſplendid, than in the month of *November* before, when it had not yet arriv'd at its perihelion. And univerſally, the greateſt and moſt fulgent tails always ariſe from Comets, immediately after their paſſing by the neighbourhood of the Sun. Therefore the heat received by the Comet conduces to the greatneſs of the tail. From whence I think I may infer, that the tail is nothing elſe but a very fine vapour, which the head or nucleus of the Comet emits by its heat.

But

But we have had three ſeveral opinions about the tails of Comets. For ſome will have it, that they are nothing elſe but the beams of the Sun's light tranſmitted through the Comet's heads, which they ſuppoſe to be tranſparent; others that they proceed from the refraction which light ſuffers in paſſing from the Comet's head to the Earth: and laſtly others, that they are a ſort of clouds or vapour conſtantly riſing from the Comet's heads, and tending towards the parts oppoſite to the Sun. The firſt is the opinion of ſuch, as are yet unacquainted with optics. For the beams of the Sun are ſeen in a darkned room only in conſequence of the light that is reflected from them by the little particles of duſt and ſmoak which are always flying about in the air. And for that reaſon in air impregnated with thick ſmoak, thoſe beams appear with great brightneſs, and move the ſenſe vigorouſly; in a yet finer air they appear more faint, and are leſs eaſily diſcerned; but in the heavens, where there is no matter to reflect the light, they can never be ſeen at all. Light is not ſeen as it is in the beam, but as it is thence reflected to our eyes. For viſion can be no otherwiſe produced than by rays falling upon the eyes. And therefore there muſt be ſome reflecting matter in thoſe parts where the tails of the Comets are ſeen: for otherwiſe, ſince all the celeſtial ſpaces are equally illuminated by the Sun's light, no part of the heavens could appear with more ſplendor than another. The ſecond opinion is liable to many difficulties. The tails of Comets are never ſeen variegated with thoſe colours which commonly are inſeparable from refraction. And the diſtinct tranſmiſſion of the light of the fixt Stars and Planets to us, is a demonſtration that the æther or celeſtial medium is not endow'd with any refractive power. For as to what is alledg'd that the fixt Stars have been ſometimes ſeen by the *Egyptians*, environ'd with a *Coma*, or *Capillitium*, becauſe that has but rarely happen'd, it

is

is rather to be aſcrib'd to a caſual refraction of clouds; and ſo the radiation and ſcintillation of the fixt Stars, to the refractions both of the eyes and air. For upon laying a teleſcope to the eye thoſe radiations and ſcintillations immediately diſappear. By the tremulous agitation of the air and aſcending vapours, it happens that the rays of light are alternately turn'd aſide from the narrow ſpace of the pupil of the eye; but no ſuch thing can have place in the much wider aperture of the object-glaſs of a teleſcope. And hence it is, that a ſcintillation is occaſion'd in the former caſe, which ceaſes in the latter. And this ceſſation in the latter caſe is a demonſtration of the regular tranſmiſſion of light through the heavens, without any ſenſible refraction. But to obviate an objection that may be made from the appearing of no tail, in ſuch Comets as ſhine but with a faint light; as if the ſecondary rays were then too weak to affect the eyes, and for that reaſon it is that the tails of the fixt Stars do not appear; we are to conſider, that by the means of teleſcopes the light of the fixt Stars may be augmented above an hundred fold, and yet no tails are ſeen; that the light of the Planets is yet more copious without any tail; but that Comets are ſeen ſometimes with huge tails, when the light of their heads is but faint and dull. For ſo it happen'd in the Comet of the year 1680, when in the month of *Dec.* it was ſcarcely equal in light to the Stars of the ſecond magnitude, and yet emitted a notable tail, extending to the length of 40°, 50°, 60° or 70°, and upwards; and afterwards on the 27 and 28 of *January* when the head appear'd but as a Star of the 7th magnitude, yet the tail (as was ſaid above) with a light that was ſenſible enough, though faint, was ſtretcht out to 6 or 7 degrees in length, and with a languiſhing light that was more difficultly ſeen, ev'n to 12°. and upwards. But on the 9 and 10 of *February*, when to the naked eye the head appear'd no more, through a tele-

a teleſcope I view'd the tail of 2° in length. But farther, if the tail was owing to the refraction of the celeſtial matter, and did deviate from the oppoſition of the Sun, according to the Figure of the heavens; that deviation in the ſame places of the heavens ſhould be always directed towards the ſame parts. But the Comet of the year 1680 *December* 28^d, $8\frac{1}{2}^h$. *P. M.* at *London* was ſeen in ♐ 8°. 41′. with latitude north 28°. 6′; while the Sun was in ♍ 18°. 26′. And the Comet of the year 1577 *Dec.* 29^d. was in ♐ 8°. 41^s, with latitude north 28°. 40′, and the Sun as before in about ♍ 18°. 26′. In both caſes the ſituation of the Earth was the ſame, and the Comet appear'd in the ſame place of the heavens: Yet in the former caſe the tail of the Comet (as well by my obſervations as by the obſervations of others) deviated from the oppoſition of the Sun towards the north, by an angle of $4\frac{1}{2}$ degrees, whereas in the latter, there was (according to the obſervations of *Tycho*) a deviation of 21 degrees towards the ſouth. The refraction therefore of the heavens being thus diſprov'd, it remains that the *phænomena* of the tails of Comets muſt be deriv'd from ſome reflecting matter.

And that the tails of Comets do ariſe from their heads, and tend towards the parts oppoſite to the Sun, is further confirm'd from the laws which the tails obſerve. As that lying in the planes of the Comet's orbits which paſs through the Sun, they conſtantly deviate from the oppoſition of the Sun towards the parts which the Comet's heads in their progreſs along theſe orbits have left. That to a ſpectator, plac'd in thoſe planes, they appear in the parts directly oppoſite to the Sun; but as the ſpectator recedes from thoſe planes, their deviation begins to appear, and daily becomes greater. That the deviation, *cæteris paribus*, appears leſs, when the tail is more oblique to the orbit of the Comet, as well as when the head of the Comet approaches

nearer to the Sun, eſpecially if the angle of deviation is eſtimated near the head of the Comet. That the tails which have no deviation appear ſtreight, but the tails which deviate are likewiſe bended into a certain curvature. That this curvature is greater when the deviation is greater; and is more ſenſible, when the tail, *cæteris paribus*, is longer: for in the ſhorter tails the curvature is hardly to be perceiv'd. That the angle of deviation is leſs near the Comet's head, but greater towards the other end of the tail; and that becauſe the convex ſide of the tail regards the parts, from which the deviation is made, and which lye in a right line drawn out infinitely from the Sun through the Comet's head. And that the tails that are long and broad, and ſhine with a ſtronger light, appear more reſplendent and more exactly defin'd on the convex than on the concave ſide. Upon which accounts, it is plain that the *phænomena* of the tails of Comets, depend upon the motions of their heads, and by no means upon the places of the heavens in which their heads are ſeen, and that therefore the tails of Comets do not proceed from the refraction of the heavens, but from their own heads, which furniſh the matter that forms the tail. For, as in our air, the ſmoak of a heated body aſcends, either perpendicularly if the body is at reſt, or obliquely, if the body is mov'd obliquely; ſo in the heavens, where all bodies gravitate towards the Sun, ſmoak and vapour muſt (as we have already ſaid) aſcend from the Sun, and either riſe perpendicularly, if the ſmoaking body is at reſt; or obliquely, if the body, in all the progreſs of its motion, is always leaving thoſe places from which the upper or higher parts of the vapour had riſen before. And that obliquity will be leaſt, where the vapour aſcends with moſt velocity, to wit near the ſmoaking body, when that is near the Sun. But becauſe the obliquity varies, the column of vapour will be incurvated; and becauſe the vapour in the preceding ſide is

ſome-

ſomething more recent, that is, has aſcended ſomething more late from the body, it will therefore be ſomething more denſe on that ſide, and muſt on that account reflect more light, as well as be better defin'd. I add nothing concerning the ſudden uncertain agitation of the tails of Comets, and their irregular figures, which Authors ſometimes deſcribe, becauſe they may ariſe from the mutations of our air, and the motions of our clouds, in part obſcuring thoſe tails; or perhaps from parts of the *Via Lactea*, which might have been confounded with and miſtaken for parts of the tails of the Comets as they paſſed by.

But that the atmoſpheres of Comets may furniſh a ſupply of vapour, great enough to fill ſo immenſe ſpaces, we may eaſily underſtand from the rarity of our own air. For the air near the ſurface of our Earth, poſſeſſes a ſpace 850 times greater than water of the ſame weight. And therefore a cylinder of air 850 feet high, is of equal weight with a cylinder of water, of the ſame breadth and but one foot high. But a cylinder of air, reaching to the top of the atmoſphere, is of equal weight with a cylinder of water, about 33 feet high: and therefore, if from the whole cylinder of air, the lower part of 850 feet high is taken away, the remaining upper part will be of equal weight with a cylinder of water 32 feet high. And from thence (and by the hypotheſis, confirm'd by many experiments, that the compreſſion of air is as the weight of the incumbent atmoſphere, and that the force of gravity is reciprocally as the ſquare of the diſtance from the center of the Earth) raiſing a calculus, by cor. prop. 22. book 2. I found, that at the heighth of one ſemidiameter of the Earth, reckon'd from the Earth's ſurface, the air is more rare than with us, in a far greater proportion than of the whole ſpace within the orb of Saturn to a ſpherical ſpace of one inch in diameter. And therefore if a ſphere of our air, of but one inch in thickneſs

neſs, was equally rarify'd with the air at the heighth of one ſemi-diameter of the Earth from the Earth's ſurface, it would fill all the regions of the Planets to the orb of Saturn and far beyond it. Wherefore ſince the air at greater diſtances is immenſely rarify'd, and the *coma* or atmoſphere of Comets is ordinarily about ten times higher, reckoning from their centers, than the ſurface of the nucleus, and the tails riſe yet higher, they muſt therefore be exceedingly rare. And tho' on account of the much thicker atmoſpheres of Comets and the great gravitation of their bodies towards the Sun, as well as of the particles of their air and vapours mutually one towards another, it may happen that the air in the celeſtial ſpaces and in the tails of Comets, is not ſo vaſtly rarify'd; yet from this computation it is plain, that a very ſmall quantity of air and vapour is abundantly ſufficient to produce all the appearances of the tails of Comets. For that they are indeed of a very notable rarity appears from the ſhining of the Stars through them. The atmoſphere of the Earth, illuminated by the Suns light, tho' but of a few miles in thickneſs, quite obſcures and extinguiſhes the light not only of all the Stars, but ev'n of the Moon itſelf: whereas the ſmalleſt Stars are ſeen to ſhine through the immenſe thickneſs of the tails of Comets, likewiſe illuminated by the Sun, without the leaſt diminution of their ſplendor. Nor is the brightneſs of the tails of moſt Comets ordinarily greater than that of our air an inch or two in thickneſs, reflecting in a darken'd room the light of the Sun beams let in by an hole of the window-ſhut.

And we may pretty nearly determine the time ſpent during the aſcent of the vapour from the Comet's head to the extremity of the tail, by drawing a right line from the extremity of the tail to the Sun, and marking the place where that right line interſects the Comet's orbit. For the vapour that is now in the extremity of the tail,

tail, if it has aſcended in a right line from the Sun, muſt have begun to riſe from the head, at the time when the head was in the point of interſection. It is true, the vapour does not riſe in a right line from the Sun, but retaining the motion which it had from the Comet before its aſcent, and compounding that motion with its motion of aſcent, ariſes obliquely. And therefore, the ſolution of the problem will be more exact, if we draw the line which interſects the orbit parallel to the length of the tail; or rather (becauſe of the curvilinear motion of the Comet,) diverging a little from the line or length of the tail. And by means of this principle I found, that the vapour which *Jan.* 25. was in the extremity of the tail, had begun to riſe from the head before *Dec.* 11. and therefore had ſpent in its whole aſcent 45 days; but that the whole tail which appear'd on *Dec.* 10. had finiſh'd its aſcent in the ſpace of the two days then elaps'd from the time of the Comet's being in its perihelion. The vapour therefore, about the beginning and in the neighbourhood of the Sun, roſe with the greateſt velocity, and afterwards continu'd to aſcend with a motion conſtantly retarded by its own gravity; and the higher it aſcended, the more it added to the length of the tail. And while the tail continu'd to be ſeen, it was made up of almoſt all that vapour, which had riſen ſince the time of the Comet's being in its perihelion; nor did that part of the vapour which had riſen firſt, and which form'd the extremity of the tail, ceaſe to appear, till its too great diſtance, as well from the Sun from which it receiv'd its light, as from our eyes, render'd it inviſible. Whence alſo it is, that the tails of other Comets which are ſhort, do not riſe from their heads with a ſwift and continual motion, and ſoon after diſappear; but are permanent and laſting columns of vapours and exhalations; which aſcending from the heads with a ſlow motion of many days, and partaking of the motion of the

the heads which they had from the beginning, continue to go along together with them through the heavens. From whence again we have another argument proving the celeſtial ſpaces to be free and without reſiſtance, ſince in them not only the ſolid bodies of the Planets and Comets, but alſo the extremely rare vapours of Comets tails, maintain their rapid motions with great freedom, and for an exceeding long time.

Kepler aſcribes the aſcent of the tails of the Comets to the atmoſpheres of their heads; and their direction towards the parts oppoſite to the Sun, to the action of the rays of light carrying along with them the matter of the Comet's tails. And without any great incongruity we may ſuppoſe, that in ſo free ſpaces, ſo fine a matter as that of the æther may yield to the action of the rays of the Sun's light, though thoſe rays are not able ſenſibly to move the groſs ſubſtances in our parts, which are clogg'd with ſo palpable a reſiſtance. Another author thinks, that there may be a ſort of particles of matter endow'd with a principle of levity, as well as others are with a power of gravity; that the matter of the tails of Comets may be of the former ſort, and that its aſcent from the Sun, may be owing to its levity. But conſidering that the gravity of terreſtrial bodies is as the matter of the bodies, and therefore can be neither more nor leſs in the ſame quantity of matter, I am inclin'd to believe that this aſcent may rather proceed from the rarefaction of the matter of the Comet's tails. The aſcent of ſmoak in a chimney is owing to the impulſe of the air, with which it is entangled. The air rarefy'd by heat aſcends, becauſe its ſpecific gravity is diminiſh'd, and in its aſcent carries along with it the ſmoak, with which it is engag'd. And why may not the tail of a Comet riſe from the Sun after the ſame manner? For the Sun's rays do not act upon the mediums which they pervade otherwiſe than

by reflection and refraction. And thoſe reflecting particles heated by this action, heat the matter of the æther which is involv'd with them. That matter is rarefied by the heat which it acquires; and becauſe by this rarefaction the ſpecific gravity with which it tended towards the Sun before is diminiſh'd, it will aſcend therefrom, and carry along with it the reflecting particles, of which the tail of the Comet is compos'd. But the aſcent of the vapours is further promoted by their circumgyration about the Sun, in conſequence whereof they endeavour to recede from the Sun, while the Sun's atmoſphere and the other matter of the heavens are either altogether quieſcent, or are only mov'd with a ſlower circumgyration deriv'd from the rotation of the Sun. And theſe are the cauſes of the aſcent of the tails of the Comets in the neighbourhood of the Sun, where their orbits are bent into a greater curvature, and the Comets themſelves are plung'd into the denſer, and therefore heavier parts of the Sun's atmoſphere; upon which account they do then emit tails of an huge length. For the tails which then ariſe, retaining their own proper motion, and in the mean time gravitating towards the Sun, muſt be revolv'd in ellipſes about the Sun in like manner as the heads are, and by that motion muſt always accompany the heads, and freely adhere to them. For the gravitation of the vapours towards the Sun can no more force the tails to abandon the heads, and deſcend to the Sun, than the gravitation of the heads can oblige them to fall from the tails. They muſt by their common gravity, either fall together towards the Sun, or be retarded together in their common aſcent therefrom. And therefore, (whether from the cauſes already deſcrib'd, or from any others) the tails and heads of Comets may eaſily acquire, and freely retain any poſition one to the other, without diſturbance or impediment from that common gravitation.

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The tails therefore that riſe in the perihelion poſitions of the Comets will go along with their heads into far remote parts, and together with the heads will either return again from thence to us, after a long courſe of years; or rather, will be there rarefied, and by degrees quite vaniſh away. For afterwards in the deſcent of the heads towards the Sun, new ſhort tails will be emitted from the heads with a ſlow motion; and thoſe tails by degrees will be augmented immenſly, eſpecially in ſuch Comets as in their perihelion diſtances deſcend as low as the Sun's atmoſphere. For all vapour in thoſe free ſpaces is in a perpetual ſtate of rarefaction and dilatation. And from hence it is, that the tails of all Comets are broader at their upper extremity, than near their heads. And it is not unlikely, but that the vapour, thus perpetually rarefy'd and dilated, may be at laſt diſſipated, and ſcatter'd through the whole heavens, and by little and little be attracted towards the Planets by its gravity, and mixed with their atmoſphere. For as the ſeas are abſolutely neceſſary to the conſtitution of our Earth, that from them, the Sun, by its heat, may exhale a ſufficient quantity of vapours, which being gather'd together into clouds, may drop down in rain, for watering of the earth, and for the production and nouriſhment of vegetables; or being condens'd with cold on the tops of mountains, (as ſome philoſophers with reaſon judge) may run down in ſprings and rivers; ſo for the conſervation of the ſeas, and fluids of the Planets, Comets ſeem to be requir'd, that from their exhalations and vapours condens'd, the waſtes of the Planetary fluids, ſpent upon vegetation and putrefaction, and converted into dry earth, may be continually ſupplied and made up. For all vegetables entirely derive their growths from fluids, and afterwards in great meaſure are turn'd into dry earth by putrefaction; and a ſort of ſlime is always found to ſettle at the bottom of putrified fluids. And hence it is, that the bulk of the ſolid earth is

continually increaſed, and the fluids, if they are not ſupplied from without, muſt be in a continual decreaſe, and quite fail at laſt. I ſuſpect moreover, that 'tis chiefly from the Comets that ſpirit comes, which is indeed the ſmalleſt, but the moſt ſubtle and uſeful part of our air, and ſo much required to ſuſtain the life of all things with us.

The atmoſpheres of Comets, in their deſcent towards the Sun, by running out into the tails are ſpent and diminiſh'd, and become narrower, at leaſt on that ſide which regards the Sun; and in receding from the Sun, when they leſs run out into the tails, they are again enlarg'd, if *Hevelius* has juſtly mark'd their appearances. But they are ſeen leaſt of all juſt after they have been moſt heated by the Sun, and on that account then emit the longeſt and moſt reſplendent tails; and perhaps at the ſame time the nuclei are environ'd with a denſer and blacker ſmoak, in the lowermoſt parts of their atmoſphere. For ſmoak that is rais'd by a great and intenſe heat, is commonly the denſer and blacker. Thus the head of that Comet which we have been deſcribing, at equal diſtances both from the Sun and from the Earth, appear'd darker after it had paſs'd by its perihelion, than it did before. For in the month of *December* it was commonly compar'd with the Stars of the third magnitude, but in *November*, with thoſe of the firſt or ſecond. And ſuch as ſaw both appearances, have deſcrib'd the firſt, as of another and greater Comet than the ſecond. For *November* 19. this Comet appear'd to a young man at *Cambridge*, though with a pale and dull light, yet equal to *Spica Virginis*; and at that time it ſhone with greater brightneſs than it did afterwards. And *Montenari*, *Nov.* 20. ſt. vet. obſerved it larger than the Stars of the firſt magnitude, its tail being then 2 deg. long. And Mr. *Storer*, (by letters which have come into my hands) writes, that in the month of *Dec.* when the tail appear'd of the greateſt bulk

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and ſplendor, the head was but ſmall, and far leſs than that which was ſeen in the month of *November* before Sun-riſing; and conjecturing at the cauſe of the appearance, he judg'd it to proceed from there being a greater quantity of matter in the head at firſt, which was afterwards gradually ſpent.

And, which further makes for the ſame purpoſe, I find, that the heads of other Comets, which did put forth tails of the greateſt bulk and ſplendor, have appeared but obſcure and ſmall. For in *Braſile*, *March* 5. 1668. 7^{h} *P. M. St. N. P. Valentinus Eſtancius* ſaw a Comet near the horizon, and towards the ſouth weſt, with a head ſo ſmall as ſcarcely to be diſcern'd, but with a tail above meaſure ſplendid, ſo that the reflection thereof from the ſea was eaſily ſeen by thoſe who ſtood upon the ſhoar. And it look'd like a fiery beam extended 23° in length from weſt to ſouth, almoſt parallel to the horizon. But this exceſſive ſplendor continu'd only three days, decreaſing apace afterwards; and while the ſplendor was decreaſing, the bulk of the tail increas'd. Whence in *Portugal*, it is ſaid to have taken up one quarter of the heavens, that is, 45 degrees, extending from weſt to eaſt with a very notable ſplendor, though the whole tail was not ſeen in thoſe parts, becauſe the head was always hid under the horizon. And from the increaſe of the bulk, and decreaſe of the ſplendor of the tail, it appears that the head was then in its receſs from the Sun, and had been very near to it in its perihelion, as the Comet of 1680 was. And we read, in the *Saxon* chronicle, of a like Comet appearing in the year 1106, *the Star whereof was ſmall and obſcure*, (as that of 1680.) *but the ſplendour of its tail was very bright, and like a huge fiery beam ſtretch'd out in a direction between the eaſt and north*, as *Hevelius* has it alſo from *Simeon* the monk of *Durham*. This Comet appear'd in the beginning of *February*, about the evening, and towards the ſouth weſt part of heaven:

From whence, and from the poſition of the tail, we infer, that the head was near the Sun. *Matthew Paris* ſays, *It was diſtant from the Sun by about a cubit, from three of the clock* (rather ſix) *till nine, putting forth a long tail.* Such alſo was that moſt reſplendent Comet, deſcribed by *Ariſtotle*, lib. 1. *Meteor.* 6. *The head whereof could not be ſeen, becauſe it had ſet before the Sun, or at leaſt was hid under the Sun's rays; but next day it was ſeen as well as might be. For having left the Sun but a very little way, it ſet immediately after it. And the ſcatter'd light of the head, obſcur'd by the too great ſplendor* (of the tail) *did not yet appear. But afterwards* (as *Ariſtotle* ſays) *when the ſplendor* (of the tail) *was now diminiſh'd* (the head of) *the Comet recover'd its native brightneſs; and the ſplendour* (of its tail) *reach'd now to a third part of the heavens* (that is, to 60°.) *This appearance was in the winter ſeaſon,* (an. 4. olymp. 101.) *and riſing to* Orion's *girdle, it there vaniſh'd away.* It is true that the Comet of 1618, which came out directly from under the Sun's rays, with a very large tail, ſeem'd to equal, if not to exceed, the Stars of the firſt magnitude. But then abundance of other Comets have appear'd yet greater than this, that put forth ſhorter tails; ſome of which are ſaid to have appear'd as big as Jupiter; others as big as Venus, or even as the Moon.

We have ſaid, that Comets are a ſort of Planets, revolv'd in very eccentric orbits about the Sun. And as in the Planets which are without tails, thoſe are commonly leſs, which are revolv'd in leſſer orbits, and nearer to the Sun; ſo in Comets it is probable, that thoſe which in their perihelion approach nearer to the Sun, are generally of leſs magnitude, that they may not agitate the Sun too much by their attractions. But as to the tranſverſe diameters of their orbits, and the periodic times of their revolutions, I leave them to be determin'd by comparing Comets together which after long intervals of time return again in the ſame orbit.

bit. In the mean time, the following propoſition may give ſome light in that enquiry.

Proposition XLII. Problem XXII.

To correct a Comet's trajectory found as above.

Operation 1. Aſſume that poſition of the plane of the trajectory which was determin'd according to the preceding propoſition. And ſelect three places of the Comet, deduc'd from very accurate obſervations, and at great diſtances one from the other. Then ſuppoſe A to repreſent the time between the firſt obſervation and the ſecond; and B the time between the ſecond and the third. But it will be convenient that in one of thoſe times the Comet be in its perigeon, or at leaſt not far from it. From thoſe apparent places find by trigonometric operations the three true places of the Comet in that aſſum'd plane of the trajectory; then through the places found, and about the center of the Sun as the focus, deſcribe a conic ſection by arithmetical operations, according to prop. 21. book 1. Let the areas of this figure which are terminated by radij drawn from the Sun to the places found, be D and E, to wit, D the area between the firſt obſervation and the ſecond, and E the area between the ſecond and third. And let T repreſent the whole time, in which the whole area D + E ſhould be deſcribed with the velocity of the Comet found by prop. 16. book 1.

Oper. 2. Retaining the inclination of the plane of the trajectory to the plane of the ecliptic, let the longitude of the nodes of the plane of the trajectory be increas'd by the addition of 20 or 30 minutes, which call P. Then from the foreſaid three obſerv'd

places of the Comet, let the three true places be found (as before) in this new plane, as alſo the orbit paſſing through thoſe places, and the two areas of the ſame deſcrib'd between the two obſervations, which call d and e; and let t be the whole time in which the whole area $d + e$ ſhould be deſcrib'd.

Oper. 3. Retaining the longitude of the nodes in the firſt operation, let the inclination of the plane of the trajectory to the plane of the ecliptic be increas'd by adding thereto 20′ or 30′, which call Q. Then from the foreſaid three obſerv'd apparent places of the Comet, let the three true places be found in this new plane, as well as the orbit paſſing through them, and the two areas of the ſame deſcrib'd between the obſervation, which call δ and ε, and let τ be the whole time in which the whole area $\delta + \varepsilon$ ſhould be deſcrib'd.

Then taking C to 1, as A to B; and G to 1, as D to E; and g to 1, as d to e; and γ to 1, as δ to ε; let S be the true time between the firſt obſervation and the third; and obſerving well the ſigns + and —, let ſuch numbers m and n be found out as will make $2G - 2C = mG - mg + nG - n\gamma$; and $2T - 2S = mT - mt + nT - n\tau$. And, if in the firſt operation I repreſents the inclination of the plane of the trajectory to the plane of the ecliptic, and K the longitude of either node, then $I + nQ$ will be the true inclination of the plane of the trajectory to the plane of the ecliptic; and $K + mP$ the true longitude of the node. And laſtly, if in the firſt, ſecond, and third operations, the quantities R, r, and ϱ, repreſent the parameters of the trajectory, and the quantities $\frac{1}{L}$, $\frac{1}{l}$, $\frac{1}{\lambda}$, the tranſverſe diameters of the ſame; then $R + mr - mR + n\varrho - nR$ will be the true parameter, and $\frac{1}{L + ml - mL + n\lambda - nL}$ will be the true tranſverſe diameter of the trajectory

which

Apparent time at Dantzic	*The observed distances of the Comet from*		*The observ'd Places*	*The places computed in the orb.*
Decemb. $3^d.18^h.29'\frac{1}{2}$	*The Lyon's heart*	46°.24'.20"	*Long.* ♎ 7°.01'.00"	♎ 7°.01'.29"
	The Virgin's spike	22.52.10	*Lat. S.* 21.39.00	21.38.50
$4.18.01\frac{1}{2}$	*The Lyon's heart*	46.02.45	*Long.* ♎ 6.15.00	♎ 6.16.05
	The Virgin's spike	23.52.40	*Lat. S.* 22.24.00	22.24.00
7.17.48	*The Lyon's heart*	44.48.00	*Long.* ♎ 3.06.00	♎ 3.07.33
	The Virgin's spike	27.56.40	*Lat. S.* 25.22.00	25.21.40
17.14.43	*The Lyon's heart*	53.15.15	*Long.* ♌ 2.56.00	♌ 2.56.00
	Orion's right shoulder	45.43.30	*Lat. S.* 49.25.00	49.25.00
19.09.25	*Procyon*	35.13.50	*Long.* ♊ 28.40.30	♊ 28.43.00
	The bright star of the Whale's jaw	52.56.00	*Lat. S.* 45.48.00	45.46.00
$20.09.53\frac{1}{2}$	*Procyon*	40.49.00	*Long.* ♊ 13.03.00	♊ 13.05.00
	The bright star of the Whale's jaw	40.04.00	*Lat. S.* 39.54.00	39.53.00
$21.09.09\frac{1}{2}$	*Orion's right shoulder*	26.21.25	*Long.* ♊ 2.16.00	♊ 2.18.30
	The bright star of the Whale's jaw	29.28.00	*Lat. S.* 33.41.00	33.39.40
22.09.00	*Orion's right shoulder*	29.47.00	*Long.* ♉ 24.24.00	♉ 24.27.00
	The bright star of the Whale's jaw	20.29.30	*Lat. S.* 27.45.00	27.46.00
26.07.58	*The bright star of Aries*	23.20.00	*Long.* ♉ 9.00.00	♉ 9.02.28
	Aldebaran	26.44.00	*Lat. S.* 12.36.00	12.34.13
27.06.45	*The bright star of Aries*	20.45.00	*Long.* ♉ 7.05.40	♉ 7.08.45
	Aldebaran	28.10.00	*Lat. S.* 10.23.00	10.23.13
28.07.39	*The bright star of Aries*	18.29.00	*Long.* ♉ 5.24.45	♉ 5.27.52
	Palilicium	29.37.00	*Lat. S.* 8.22.50	8.23.37
31.06.45	*Andromeda's girdle*	30.48.10	*Long.* ♉ 2.07.40	♉ 2.08.20
	Palilicium	32.53.30	*Lat. S.* 4.13.00	4.16.25
Jan. 1665. $7.07.37\frac{1}{2}$	*Andromeda's girdle*	25.11.00	*Long.* ♈ 28.24.47	♈ 28.24.00
	Palilicium	37.12.25	*Lat. N.* 0.54.00	0.53.00
13.07.00	*Andromeda's head*	28.07.10	*Long.* ♈ 27.06.54	♈ 27.06.39
	Palilicium	38.55.20	*Lat. N.* 3.06.50	3.07.40
24.7.29	*Andromeda's girdle*	20.32.15	*Long.* ♈ 26.29.15	♈ 26.28.50
	Palilicium	40.05.00	*Lat. N.* 5.25.50	5.26.00
Feb. 7.08.37			*Long.* ♈ 27.04.46	♈ 27.24.55
			Lat. N. 7.03.29	7.03.15
22.08.46			*Long.* ♈ 28.29.46	♈ 28.29.58
			Lat. N. 8.12.36	8.10.25
March 1.08.16			*Long.* ♈ 29.18.15	♈ 29.18.20
			Lat. N. 8.36.26	8.36.12
7.08.37			*Long.* ♉ 0.02.48	♉ 0.2,42
			Lat. N. 8.56.30	8.56.56

which the Comet deſcribes. And from the tranſverſe diameter given the periodic time of the Comet is alſo given, *Q. E. I.* But the periodic times of the revolutions of Comets, and the tranſverſe diameters of their orbits, cannot be accurately enough determin'd, but by comparing Comets together which appear at different times. If after equal intervals of time, ſeveral Comets are found to have deſcrib'd the ſame orbit, we may thence conclude, that they are all but one and the ſame Comet revolv'd in the ſame orbit. And then from the times of their revolutions, the tranſverſe diameters of their orbits will be given; and from thoſe diameters the elliptic orbits themſelves will be determin'd.

To this purpoſe, the trajectories of many Comets ought to be computed, ſuppoſing thoſe trajectories to be parabolic. For ſuch trajectories will always nearly agree with the *phænomena*, as appears not only from the parabolic trajectory of the Comet of the year 1680, which I compar'd above with the obſervations, but likewiſe from that of the notable Comet, which appear'd in the years 1664, and 1665, and was obſerv'd by *Hevelius*; who, from his own obſervations, calculated the longitudes and latitudes thereof, though with little accuracy. But from the ſame obſervations Dr. *Halley* did again compute its places; and from thoſe new places determin'd its trajectory; finding its aſcending node in ♊ 21°. 13′. 55″; the inclination of the orbit to the plane of the ecliptic 21°. 18′. 40″; the diſtance of its perihelion from the node, eſtimated in the Comet's orbit 49°. 27′. 30″. its perihelion in ☊ 8°. 40′. 30″; with heliocentric latitude ſouth, 16°. 01′. 45″; the Comet to have been in its perihelion *Nov.* 24^{d}. 11^{h}. 52′; *P. M.* equal time at *London*, or 13^{h}. 8′, at *Dantzick*, *O. S.* and that the *latus rectum* of the parabola was 410286 ſuch parts as the Sun's mean diſtance from the Earth is ſuppos'd to contain 100000. And how nearly the places of the Comet computed in this orbit agree with the obſervations, will appear from the annexed table, calculated by Dr. *Halley*.

In *February*, the beginning of the year 1665. the 1ſt Star of Aries, which I ſhall hereafter call γ, was in Υ 28°. 30′. 15″, with 7°. 8′. 58″. north lat. The 2d Star of Aries was in Υ 29°. 17′. 18″, with 8°. 28′. 16″. north lat. And another Star of the ſeventh magnitude which I call A, was in Υ 28°. 24′. 45″, with 8°. 28′. 33″, north lat. The Comet *Feb.* 7^{d}. 7^{h}. 30′, at *Paris* (that is *Feb.* 7^{d}. 8^{h}. 37′, at *Dantzick*) O. S. made a triangle with thoſe Stars γ and A, which was right-angled in γ. And the diſtance of the Comet from the Star γ was equal to the diſtance of the Stars γ and A, that is 1°. 19′. 46″, of a great circle; and therefore in the parallel of the latitude of the Star γ it was 1°. 20′. 26″. Therefore if from the longitude of the Star γ there be ſubducted the longitude 1°. 20′. 26″, there will remain the longitude of the Comet Υ 27°. 9′. 49″. M. *Auzout*, from this obſervation of his, placed the Comet in Υ 27°. 0′, nearly. And by the ſcheme in which Dr. *Hooke* delineated its motion, it was then in Υ 26°. 59′. 24″. I place it in Υ 27°. 4′. 46″, taking the middle between the two extremes.

From the ſame obſervation, M. *Auzout* made the latitude of the Comet at that time, 7° and 4′ or 5′ to the north. But he had done better to have made it 7°. 3′. 29″, the difference of the latitudes of the Comet and the Star γ being equal to the difference of the longitude of the Stars γ and A.

Feb. 22^{d}. 7^{h}. 30′, at *London*, that is, *Feb.* 22^{d}. 8^{h}. 46′, at *Dantzick*, the diſtance of the Comet from the Star A, according to Dr. *Hooke*'s obſervation, as was delineated by himſelf in a ſcheme, and alſo by the obſervations of M. *Auzout*, delineated in like manner by M. *Petit*, was a 5th part of the diſtance between the Star A and the firſt Star of Aries, or 15′. 57″; and the diſtance of the Comet from a right line joining the Star A and the firſt of Aries, was a fourth part of the ſame 5th parr, that is 4′. And therefore the Comet was in Υ 28°. 29′. 46″, with 8°. 12′. 36″, north lat.

Mar.

Mar. 1. 7^h. 0′, at *London*, that is, *Mar.* 1. 8^h. 16′, at *Dantzick*, the Comet was obſerv'd near the 2d Star in Aries, the diſtance between them being to the diſtance between the firſt and ſecond Stars in Aries, that is, to 1°. 33′, as 4 to 45 according to Dr. *Hooke*, or as 2 to 23 according to M. *Gottignies*. And therefore the diſtance of the Comet from the 2d Star in Aries was 8′. 16″, according to Dr. *Hooke*, or 8′. 5″, according to M. *Gottignies*; or taking a mean between both 8′. 10″. But according to M. *Gottignies*, the Comet had gone beyond the 2d Star of Aries, about a 4th or a 5th part of the ſpace, that it commonly went over in a day, to wit, about 1′. 35″; (in which he agrees very well with M. *Auzout*) or according to Dr. *Hooke*, not quite ſo much, as perhaps only 1′. Wherefore if to the longitude of the 1ſt Star in Aries, we add 1′, and 8′. 10″, to its latitude, we ſhall have the longitude of the Comet ♈ 29°. 18′, with 8°. 36′. 26″, north lat.

Mar. 7. 7^h. 30′, at *Paris* (that is, *Mar.* 7. 8^h. 37′, at *Dantzick*) from the obſervations of M. *Auzout*, the diſtance of the Comet from the 2d Star in Aries, was equal to the diſtance of that Star from the Star A, that is, 52′. 29″; and the difference of the longitude of the Comet and the 2d Star in Aries was 45′, or 46′, or taking a mean quantity 45′. 30″. And therefore the Comet was in ♉ 0°. 2′. 48″. From the ſcheme of the obſervations of M. *Auzout*, conſtructed by M. *Petit*, *Hevelius* collected the latitude of the Comet 8°. 54′. But the engraver did not rightly trace the curvature of the Comet's way toward the end of the motion: and *Hevelius* in the ſcheme of M. *Auzout*'s obſervations which he conſtructed himſelf, corrected this irregular curvature, and ſo made the latitude of the Comet 8°. 55′. 30″. And by farther correcting this irregularity the latitude may become 8°. 56′, or 8°. 57′.

This Comet was alſo ſeen *Mar.* 9, and at that time its place muſt have been in ♉ 0°. 18′ with 9°. 3′ ½ north lat. nearly.

This Comet appeared three months together, in which ſpace of time it travell'd over almoſt ſix ſigns, and in one of the days thereof deſcrib'd almoſt 20 deg. Its courſe did very much deviate from a great circle, bending towards the north, and its motion towards the end from retrograde became direct. And notwithſtanding its courſe was ſo uncommon, yet by the table it appears that the theory, from beginning to end, agrees with the obſervations no leſs accurately than the theories of the Planets uſually do with the obſervations of them. But we are to ſubduct about 2′. when the Comet was ſwifteſt, which we may effect by taking off 12″ from the angle between the aſcending node and the perihelion, or by making that angle 49°. 27′. 18″. The annual parallax of both theſe Comets (this and the preceding) was very conſpicuous, and by its quantity demonſtrates the annual motion of the Earth in the *orbis magnus*.

This theory is likewiſe confirm'd by the motion of that Comet, which in the year 1683 appear'd retrograde, in an orbit whoſe plane contain'd almoſt a right angle with the plane of the ecliptic, and whoſe aſcending node (by the computation of Dr. *Halley*) was in ♍ 23°. 23′; the inclination of its orbit to the ecliptic 83°. 11′; its perihelion in ♊ 25°. 29′. 30″; its perihelion diſtance from the Sun 56020 of ſuch parts as the radius of the *orbis magnus* contains 100000; and the time of its perihelion *July* 2ᵈ. 3ʰ. 50′. And the places thereof computed by Dr. *Halley* in this orbit, are compar'd with the places of the ſame obſerv'd by Mr. *Flamſteed*, in the following table.

1683

1683 Eq. Time	Sun's Place.	Comet's Long. comp.	Lat. Nor. comput.	Comet's Long. obſer.	Lat. Nor. obſerv'd	Diff. Long.	Diff. Lat.
d. h. ′	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	′ ″
Jul. 13.12.55	♌ 1.02.30	♋ 13.05.42	29.28.13	♋ 13.06.42	29.28.20	+ 1.00	+ 0.07
15.11.15	2.53.12	11.37.48	29.34.00	11.39.43	29.34.50	+ 1.55	+ 0.50
17.10.20	4.45.45	10.07.06	29.33.30	10.08.40	29.34.00	+ 1.34	+ 0.30
23.13.40	10.38.21	5.10.27	28.51.42	5.11.30	28.50.28	+ 1.03	− 1.14
25.14. 5	12.35.28	3.27.53	24.24.47	3.27.00	28.23.40	− 0.53	− 1.07
31. 9.42	18.09.22	♊ 27.55.03	26.22.52	♊ 27.54.24	26.22.25	− 0.39	− 0.27
31.14.55	18.21.53	27.41.07	26.16.57	27.41.08	26.14.50	+ 0.01	− 2.07
Aug. 2.14.56	20.17.16	25.29.32	25.16.19	25.28.46	25.17.28	− 0.46	+ 1.09
4.10.49	22.02.50	23.18.20	24.10.49	23.16.55	24.12.19	− 1.25	+ 1.30
6.10. 9	23.56.45	20.42.23	22.47.05	20.40.32	22.49.05	− 1.51	+ 2.00
9.10.26	26.50.52	16.07.57	20.06.37	16.05.55	20.06.10	− 2.02	− 0.27
15.14. 1	♍ 2.47.13	3.30.48	11.37.33	3.26.18	11.32.01	− 4.30	− 5.32
16.15.10	3.48.02	0.43.07	9.34.16	0.41.55	9.34.13	− 1.12	− 0.03
18.15.44	5.45.33	♉ 24.52.53	5.11.15	♉ 24.49.05	5.09.11	− 3.48	− 2.04
			South		South		
22.14.44	9.35.49	11.07.14	5.16.58	11.07.12	5.16.58	− 0.02	− 0.03
23.15.52	10.36.48	7.02.18	8.17.09	7.01.17	8.16.41	− 1.01	− 0.28
26.16. 2	13.31.10	♈ 24.45.31	16.38.00	♈ 24.44.00	16.38.20	− 1.31	+ 0.20

This theory is yet further confirm'd by the motion of that retrograde Comet, which appear'd in the year 1682. The aſcending node of this (by Dr. *Halley*'s computation) was in ☊ 21°. 16′. 30″; the inclination of its

its orbit to the plane of the ecliptic 17°. 56. 00″; its perihelion in ♒ 2°. 52′. 50″; its perihelion distance from the Sun 58328 parts, of which the radius of the *orbis magnus* contains 100000; the equal time of the Comet's being in its perihelion *Sept.* 4ᵈ. 7ʰ. 39′. And its places, collected from Mr. *Flamsteed*'s observations, are compar'd with its places computed from our theory, in the following table.

1682 App. Time	Sun's Place.	Comet's Lon. comp.	Lat. Nor. comp.	Com. Long. observ'd.	Lat. Nor. observ'd.	Diff. Longit.	Diff. Latitude
d h ′	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	′ ″
Aug. 19.16.38	♍ 7.00.07	♌ 18.14.28	25.50.07	♌ 18.14.40	25.49.55	— 0.12	+ 0.12
20.15.38	7.55.52	24.46.23	26.14.42	24.46.22	26.12.52	+ 0.01	+ 1.50
21.08.21	8.36.14	29.37.15	26.20.03	29.38.02	26.17.37	— 0.47	+ 2.26
22.08.08	9.33.55	♍ 6.29.53	26.08.42	♍ 6.30.03	26.07.12	— 0.10	+ 1.30
29.08.20	16.22.40	♎ 12.37.54	18.37.47	♎ 12.37.49	18.34.05	+ 0.05	+ 3.42
30.07.45	17.19.41	15.36.01	17.26.43	15.35.18	17.27.17	+ 0.43	— 0.34
Sept. 1.07.33	19.16.09	20.30.53	15.13.00	20.27.04	15.09.49	+ 3.49	+ 3.11
4.07.22	22.11.28	25.42.00	12.23.48	25.40.58	12.22.00	+ 1.02	+ 1.48
5.07.32	23.10.29	27.00.46	11.33.08	26.59.24	11.33.51	+ 1.22	— 0.43
8.07.16	26.05.58	29.58.44	9.26.46	29.58.45	9.26.43	— 0.01	+ 0.03
9.07.26	27.05.09	♏ 0.44.10	8.49.10	♏ 0.44.04	8.48.25	+ 0.06	+ 0.45

This

This theory is alſo confirmed by the retrograde motion of the Comet that appeared in the year 1723. The aſcending node of this Comet (according to the computation of Mr. *Bradley*, Savilian Profeſſor of Aſtronomy at *Oxford*) was in ♈ 14°. 16′. The inclination of the orbit to the plane of the ecliptic 49°. 59′. Its perihelion was in ♉ 12°. 15′. 20″. Its perihelion diſtance from the Sun 998651 parts, of which the radius of the *orbis magnus* contains 1000000, and the equal time of its perihelion *September* 16ᵈ. 16ʰ. 10′. The places of this Comet computed in this orbit by Mr. *Bradley*, and compared with the places obſerved by himſelf, his uncle Mr. *Pound*, and Dr. *Halley*, may be ſeen in the following table.

1723. Eq. Time.	Comet's Long. obſ.	Lat. Nor. obſ.	Comet's Lon. com.	Lat. Nor. comp.	Diff. Lon.	Diff. Lat.
d h ′	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
Oct. 9.8. 5	♎ 7.22.15	5. 2. 0	♎ 7.21.26	5. 2.47	+49	−47
10.6.21	6.41.12	7.44.13	6.41.42	7.43.18	−50	+55
12.7.22	5.39.58	11.55. 0	5.40.19	11.54.55	−21	+ 5
14.8.57	4.59 49	14.43.50	5. 0.37	14.44. 1	−48	−11
15.6.35	4.47.41	15 40.51	4.47.45	15.40.55	− 4	− 4
21.6.22	4. 2.32	19.41.49	4. 2.21	19.42. 3	+11	−14
22.6.24	3.59. 2	20. 8.12	3.59.10	20. 8.17	− 8	− 5
24.8. 2	3.55.29	20.55.18	3.55.11	20.55. 9	+18	+ 9
29.8.56	3.56.17	22.20.27	3.56.42	22.20.10	−25	+17
30.6.20	3.58. 9	22.32.28	3.58.17	22.32.12	− 8	+16
Nov. 5.5.53	4.16.30	23.38.33	4.16.23	23.38. 7	+ 7	+26
8.7. 6	4.29.36	24. 4.30	4.29 54	24. 4.40	−18	−10
14.6.20	5. 2.16	24.48.46	5. 2.51	24.48.16	−35	+30
20.7.45	5.42.20	25.24.45	5.43.13	25.25.17	−53	−32
Dec. 7.6.45	8. 4.13	26.54.18	8. 3.55	26.53.42	+18	+36

From theſe examples it is abundantly evident, that the motions of Comets are no leſs accurately repreſented by our theory, than the motions of the Planets commonly are by the theories of them. And therefore, by means of this theory, we may enumerate the orbits of Comets, and ſo diſcover the periodic time of a Comet's revolution in any orbit; whence at laſt we

ſhall

ſhall have the tranſverſe diameters of their elliptic orbits and their aphelion diſtances.

That retrograde Comet which appear'd in the year 1607, deſcrib'd an orbit whoſe aſcending node (according to Dr. *Halley's* computation) was in ♉ 20°. 21′; and the inclination of the plane of the orbit to the plane of the ecliptic 17°. 2′; whoſe perihelion was in ♒ 2°. 16′; and its perihelion diſtance from the Sun 58680 of ſuch parts as the radius of the *orbis magnus* contains 100000. And the Comet was in its perihelion *October* 16ᵈ. 3ʰ. 50′. Which orbit agrees very nearly with the orbit of the Comet which was ſeen in 1682. If theſe were not two different Comets, but one and the ſame, that Comet will finiſh one revolution in the ſpace of 75 years. And the greater axe of its orbit will be to the greater axe of the *orbis magnus*, as $\sqrt[3]{75\times75}$ to 1, or as 1778 to 100, nearly. And the aphelion diſtance of this Comet from the Sun will be to the mean diſtance of the Earth from the Sun as about 35 to 1. From which data it will be no hard matter to determine the elliptic orbit of this Comet. But theſe things are to be ſuppoſed, on condition, that after the ſpace of 75 years the ſame Comet ſhall return again in the ſame orbit. The other Comets ſeem to aſcend to greater heights, and to require a longer time to perform their revolutions.

But becauſe of the great number of Comets, of the great diſtance of their aphelions from the Sun, and of the ſlowneſs of their motions in the aphelions, they will, by their mutual gravitations, diſturb each other: ſo that their eccentricities and the times of their revolutions will be ſometimes a little increaſed, and ſometimes diminiſhed. Therefore we are not to expect that the ſame Comet will return exactly in the ſame orbit, and in the ſame periodic times. It will be ſufficient if we find the changes no greater, than may ariſe from the cauſes juſt ſpoken of.

And

And hence a reaſon may be aſſign'd why Comets are not comprehended within the limits of a zodiac as the Planets are; but, being confin'd to no bounds, are with various motions diſpers'd all over the heavens; namely, to this purpoſe, that in their aphelions, where their motions are exceeding ſlow, receding to greater diſtances one from another they may ſuffer leſs diſturbance from their mutual gravitations. And hence it is, that the Comets which deſcend the loweſt, and therefore move the ſloweſt in their aphelions, ought alſo to aſcend the higheſt.

The Comet which appear'd in the year 1680. was in its perihelion leſs diſtant from the Sun than by a ſixth part of the Sun's diameter: and becauſe of its extreme velocity in that proximity to the Sun, and ſome denſity of the Sun's atmoſphere, it muſt have ſuffer'd ſome reſiſtance and retardation; and therefore, being attracted ſomething nearer to the Sun in every revolution, will at laſt fall down upon the body of the Sun. Nay in its aphelion, where it moves the ſloweſt, it may ſometimes happen to be yet farther retarded by the attractions of other Comets, and in conſequence of this retardation deſcend to the Sun. So fixed Stars that have been gradually waſted by the light and vapours emitted from them for a long time, may be recruited by Comets that fall upon them; and from this freſh ſupply of new fewel, thoſe old Stars, acquiring new ſplendor, may paſs for new Stars. Of this kind are ſuch fixed Stars as appear on a ſudden and ſhine with a wonderful brightneſs at firſt, and afterwards vaniſh by little and little. Such was that Star which appeared in *Caſſiopeias* chair; which *Cornelius Gemma* did not ſee upon the 8th of *November* 1572, though he was obſerving that part of the heavens upon that very night, and the skie was perfectly ſerene; but the next night (*Nov.* 9.) he ſaw it ſhining much brighter than any of the fixed Stars,

and ſcarcely inferiour to *Venus* in ſplendor. *Tycho Brahe* ſaw it upon the 11th of the ſame month when it ſhone with the greateſt luſtre; and from that time he obſerv'd it to decay by little and little; and in 16 months time it entirely diſappear'd. In the month of *November*, when it firſt appeared, its light was equal to that of *Venus*. In the month of *December* its light was a little diminiſhed, and was now become equal to that of *Jupiter*. In *January* 1573. it was leſs than *Jupiter* and greater than *Sirius*, and about the end of *February* and the beginning of *March* became equal to that Star. In the months of *April* and *May* it was equal to a Star of the 2d magnitude. In *June*, *July* and *Auguſt* to a Star of the 3d magnitude. In *September*, *October* and *November* to thoſe of the 4th magnitude, in *December* and *January* 1574. to thoſe of the 5th, in *February* to thoſe of the 6th magnitude, and in *March* it entirely vaniſhed. Its colour at the beginning was clear, bright and inclining to white, afterwards it turned a little yellow, and in *March* 1573. it became ruddy like *Mars* or *Aldebaran*; in *May* it turned to a kind of dusky whiteneſs like that we obſerve in *Saturn*, and that colour it retained ever after, but growing always more and more obſcure. Such alſo was the Star in the right foot of *Serpentarius*, which *Kepler*'s ſcholars firſt obſerved *September* 30. *O. S.* 1604, with a light exceeding that of *Jupiter*, tho' the night before it was not to be ſeen. And from that time it decreas'd by little and little, and in 15 or 16 months entirely diſappeared. Such a new Star, appearing with an unuſual ſplendor, is ſaid to have moved *Hipparchus* to obſerve, and make a catalogue of, the fixed Stars. As to thoſe fixed Stars that appear and diſappear by turns, and encreaſe ſlowly and by degrees, and ſcarce ever exceed the Stars of the 3d magnitude, they ſeem to be of another kind, which revolve about their axes, and having a light and a dark ſide, ſhew thoſe two different

different ſides by turns. The vapours which ariſe from the Sun, the fixed Stars, and the tails of the Comets, may meet at laſt with, and fall into, the atmoſpheres of the Planets by their gravity; and there be condenſed and turned into water and humid ſpirits, and from thence by a ſlow heat paſs gradually into the form of ſalts, and ſulphurs, and tinctures, and mud, and clay, and ſand, and ſtones, and coral, and other terreſtial ſubſtances.

General Scholium.

The hypotheſis of Vortices is preſs'd with many difficulties. That every Planet by a radius drawn to the Sun may deſcribe areas proportional to the times of deſcription, the periodic times of the ſeveral parts of the Vortices ſhould obſerve the duplicate proportion of their diſtances from the Sun. But that the periodic times of the Planets may obtain the ſeſquiplicate proportion of their diſtances from the Sun, the periodic times of the parts of the Vortex ought to be in the ſeſquiplicate proportion of their diſtances. That the ſmaller Vortices may maintain their leſſer revolutions about *Saturn*, *Jupiter*, and other Planets, and ſwim quietly and undiſturb'd in the greater Vortex of the Sun, the periodic times of the parts of the Sun's Vortex ſhould be equal. But the rotation of the Sun and Planets about their axes, which ought to correſpond with the motions of their Vortices, recede far from all theſe proportions. The motions of the Comets are exceeding regular, are govern'd by the ſame laws with the motions of the Planets, and can by no means be accounted for by the hypotheſis of Vortices. For Comets are carry'd with very eccentric motions through all parts of the heavens indifferently, with a freedom that is incompatible with the notion of a Vortex.

Bodies, projected in our air, ſuffer no reſiſtance but from the air. Withdraw the air, as is done in Mr. *Boyle*'s vacuum, and the reſiſtance ceaſes. For in this void a bit of fine down and a piece of ſolid gold deſcend with equal velocity. And the parity of reaſon muſt take place in the celeſtial ſpaces above the Earth's atmoſphere; in which ſpaces, where there is no air to reſiſt their motions, all bodies will move with the greateſt freedom; and the Planets and Comets will conſtantly purſue their revolutions in orbits given in kind and poſition, according to the laws above explain'd. But though theſe bodies may indeed perſevere in their orbits by the mere laws of gravity, yet they could by no means have at firſt deriv'd the regular poſition of the orbits themſelves from thoſe laws.

The ſix primary Planets are revolv'd about the Sun, in circles concentric with the Sun, and with motions directed towards the ſame parts and almoſt in the ſame plane. Ten Moons are revolv'd about the Earth, Jupiter and Saturn, in circles concentric with them, with the ſame direction of motion, and nearly in the planes of the orbits of thoſe Planets. But it is not to be conceived that mere mechanical cauſes could give birth to ſo many regular motions: ſince the Comets range over all parts of the heavens, in very eccentric orbits. For by that kind of motion they paſs eaſily through the orbs of the Planets, and with great rapidity; and in their aphelions, where they move the ſloweſt, and are detain'd the longeſt, they recede to the greateſt diſtances from each other, and thence ſuffer the leaſt diſturbance from their mutual attractions. This moſt beautiful Syſtem of the Sun, Planets and Comets, could only proceed from the counſel and dominion of an intelligent and powerful being. And if the fixed Stars are the centers of other like ſyſtems, theſe being form'd by the like wiſe counſel, muſt be all ſubject to the dominion

of

of One; eſpecially, ſince the light of the fixed Stars is of the ſame nature with the light of the Sun, and from every ſyſtem light paſſes into all the other ſyſtems. And leſt the ſyſtems of the fixed Stars ſhould, by their gravity, fall on each other mutually, he hath placed thoſe Syſtems at immenſe diſtances one from another.

This Being governs all things, not as the ſoul of the world, but as Lord over all: And on account of his dominion he is wont to be called *Lord God* παντοκράτωρ, or *Univerſal Ruler*. For *God* is a relative word, and has a reſpect to ſervants; and *Deity* is the dominion of God, not over his own body, as thoſe imagine who fancy God to be the ſoul of the world, but over ſervants. The ſupreme God is a Being eternal, infinite, abſolutely perfect; but a being, however perfect, without dominion, cannot be ſaid to be Lord God; for we ſay, my God, your God, the God of *Iſrael*, the God of Gods, and Lord of Lords; but we do not ſay, my Eternal, your Eternal, the Eternal of *Iſrael*, the Eternal of Gods; we do not ſay, my Infinite, or my Perfect: Theſe are titles which have no reſpect to ſervants. The word *God* uſually [a] ſignifies *Lord*; but every lord is not a God. It is the dominion of a ſpiritual being which conſtitutes a God; a true, ſupreme or imaginary dominion makes a true, ſupreme or imaginary God. And from his true dominion it follows, that the true God is a Living, Intelligent and Powerful Being; and from his other perfections, that he is Supreme or moſt Perfect. He is Eternal and Infinite, Omnipotent and Omniſcient; that is, his duration reaches from Eternity to Eternity; his

[a] Dr. *Pocock* derives the Latin word *Deus* from the *Arabic du*, (in the oblique caſe *di*,) which ſignifies *Lord*. And in this ſenſe Princes are called *Gods*, *Pſal.* lxxxii. ver. 6. and *John* x. ver. 35. And *Moſes* is called a *God* to his brother *Aaron*, and a *God* to *Pharaoh* (*Exod.* iv. ver. 16. and vii. ver. 8. And in the ſame ſenſe the ſouls of dead Princes were formerly, by the Heathens, called *gods*, but falſly, becauſe of their want of dominion.

preſence from Infinity to Infinity; he governs all things, and knows all things that are or can be done. He is not Eternity or Infinity, but Eternal and Infinite; he is not Duration or Space, but he endures and is preſent. He endures for ever, and is every where preſent; and by exiſting always and every where, he conſtitutes Duration and Space. Since every particle of Space is *always*, and every indiviſible moment of Duration is *every where*, certainly the Maker and Lord of all things cannot be *never* and *no where*. Every ſoul that has perception is, though in different times and in different organs of ſenſe and motion, ſtill the ſame indiviſible perſon. There are given ſucceſſive parts in duration, co-exiſtent parts in ſpace, but neither the one nor the other in the perſon of a man, or his thinking principle; and much leſs can they be found in the thinking ſubſtance of God. Every man, ſo far as he is a thing that has perception, is one and the ſame man during his whole life, in all and each of his organs of ſenſe. God is the ſame God, always and every where. He is omnipreſent, not *virtually* only, but alſo *ſubſtantially*; for virtue cannot ſubſiſt without ſubſtance. In him [b] are all things contained and moved; yet neither affects the other: God ſuffers nothing from the motion of bodies; bodies find no reſiſtance from the omnipreſence of God. 'Tis allowed by all that the ſupreme God exiſts neceſſarily; and by the ſame neceſſity he exiſts

[b] This was the opinion of the Ancients. So *Pythagoras* in *Cicer. de Nat. Deor.* lib. i. *Thales*, *Anaxagoras*, *Virgil*, Georg. lib. iv. ver. 220. and Æneid. lib. vi. ver. 721. *Philo Allegor.* at the beginning of lib. i. *Aratus* in his Phænom. at the beginning. So alſo the ſacred Writers, as St. *Paul*, *Acts* xvii. ver. 27, 28. St. *John*'s Goſp. chap. xiv. ver. 2. *Moſes* in *Deut.* iv. ver. 39. and x. ver. 14. *David*, *Pſal.* cxxxix. ver. 7, 8, 9. *Solomon*, 1 *Kings* viii. ver. 27. *Job* xxii. ver. 12, 13, 14. *Jeremiah* xxiii. ver. 23, 24. The Idolaters ſuppoſed the Sun, Moon and Stars, the Souls of Men, and other parts of the world, to be parts of the ſupreme God, and therefore to be worſhipped: but erroneouſly.

always and *every where*. Whence alſo he is all ſimilar, all eye, all ear, all brain, all arm, all power to perceive, to underſtand, and to act; but in a manner not at all human, in a manner not at all corporeal, in a manner utterly unknown to us. As a blind man has no idea of colours, ſo have we no idea of the manner by which the all-wiſe God perceives and underſtands all things. He is utterly void of all body and bodily figure, and can therefore neither be ſeen, nor heard, nor touched; nor ought he to be worſhipped under the repreſentation of any corporeal thing. We have ideas of his attributes, but what the real ſubſtance of any thing is, we know not. In bodies we ſee only their figures and colours, we hear only the ſounds, we touch only their outward ſurfaces, we ſmell only the ſmells, and taſte the ſavours; but their inward ſubſtances are not to be known, either by our ſenſes, or by any reflex act of our minds; much leſs then have we any idea of the ſubſtance of God. We know him only by his moſt wiſe and excellent contrivances of things, and final cauſes; we admire him for his perfections; but we reverence and adore him on account of his dominion. For we adore him as his ſervants; and a God without dominion, providence, and final cauſes, is nothing elſe but Fate and Nature. Blind metaphyſical neceſſity, which is certainly the ſame always and every where, could produce no variety of things. All that diverſity of natural things which we find, ſuited to different times and places, could ariſe from nothing but the ideas and will of a Being neceſſarily exiſting. But by way of allegory, God is ſaid to ſee, to ſpeak, to laugh, to love, to hate, to deſire, to give, to receive, to rejoice, to be angry, to fight, to frame, to work, to build. For all our notions of God are taken from the ways of mankind, by a certain ſimilitude which, though not perfect, has ſome likeneſs however. And thus much concerning God; to diſ-

courſe of whom from the appearances of things, does certainly belong to Natural Philoſophy.

Hitherto we have explain'd the phænomena of the heavens and of our ſea, by the power of Gravity, but have not yet aſſign'd the cauſe of this power. This is certain, that it muſt proceed from a cauſe that penetrates to the very centers of the Sun and Planets, without ſuffering the leaſt diminution of its force; that operates, not according to the quantity of the ſurfaces of the particles upon which it acts, (as mechanical cauſes uſe to do,) but according to the quantity of the ſolid matter which they contain, and propagates its virtue on all ſides, to immenſe diſtances, decreaſing always in the duplicate proportion of the diſtances. Gravitation towards the Sun, is made up out of the gravitations towards the ſeveral particles of which the body of the Sun is compos'd; and in receding from the Sun, decreaſes accurately in the duplicate proportion of the diſtances, as far as the orb of Saturn, as evidently appears from the quieſcence of the aphelions of the Planets; nay, and even to the remoteſt aphelions of the Comets, if thoſe aphelions are alſo quieſcent. But hitherto I have not been able to diſcover the cauſe of thoſe properties of gravity from phænomena, and I frame no hypotheſes. For whatever is not deduc'd from the phænomena, is to be called an hypotheſis; and hypotheſes, whether metaphyſical or phyſical, whether of occult qualities or mechanical, have no place in experimental philoſophy. In this philoſophy particular propoſitions are inferr'd from the phænomena, and afterwards render'd general by induction. Thus it was that the impenetrability, the mobility, and the impulſive force of bodies, and the laws of motion and of gravitation, were diſcovered. And to us it is enough, that gravity does really exiſt, and act according to the laws which we have explained, and abundantly ſerves to account for all the motions of the celeſtial bodies, and of our ſea.

And

And now we might add ſomething concerning a certain moſt ſubtle Spirit, which pervades and lies hid in all groſs bodies; by the force and action of which Spirit, the particles of bodies mutually attract one another at near diſtances, and cohere, if contiguous; and electric bodies operate to greater diſtances, as well repelling as attracting the neighbouring corpuſcles; and light is emitted, reflected, refracted, inflected, and heats bodies; and all ſenſation is excited, and the members of animal bodies move at the command of the will, namely, by the vibrations of this Spirit, mutually propagated along the ſolid filaments of the nerves, from the outward organs of ſenſe to the brain, and from the brain into the muſcles. But theſe are things that cannot be explain'd in few words, nor are we furniſh'd with that ſufficiency of experiments which is required to an accurate determination and demonſtration of the laws by which this electric and elaſtic ſpirit operates.

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APPENDIX.

Among the Explications, (given by a Friend,) of ſome Propoſitions in this Book, not demonſtrated by the Author, the Editor finding theſe following, has thought it proper to annex them. Thus,

To Cor. 2. *Prop.* 91. *Book* 1. *Pag* 303.

1. 

O find the force whereby a ſphere (*AdBg*,) on the diameter *AB*, attracts the body *P*. (*Pl.* 19. *Fig.* 1.)

Let $SA = SB = r$, $PS = d$, $PE = x$, $PB = a = d + r$, $PA = \alpha = d - r$; Theref. $a\alpha = dd - rr$; alſo $a + \alpha = 2d$, $a - \alpha = 2r$; Therefore $aa - \alpha\alpha = 4dr$: And $SE = d - x$, $AE = x - \alpha$, $BE = a - x$.

Now the force whereby the circle, whoſe radius is *Ed*, attracts the body *P*, is as $1 - \frac{PE}{Pd}$ (by Cor. 1. Prop. 90.)

And $\overline{Ed}^2 = (AE \times EB = \overline{SA}^2 - \overline{SE}^2 = rr - dd + 2dx - xx =) - aa + 2dx - xx$. Alſo $\overline{Pd}^2 = \overline{Ed}^2 + \overline{EP}^2 = 2dx - aa - xx + xx =) 2dx - aa$: Th. $\frac{PE}{Pd} = \frac{x}{\sqrt{-aa+2dx}}$. Therefore $1 - \frac{x}{\sqrt{-aa+2dx}}$ or $\dot{x} - \frac{x\dot{x}}{\sqrt{-aa+2dx}}$ is the fluxion of the attractive force of the ſphere on the body P, or the ordinate of a curve whoſe area repreſents that force.

But the fluent of $\dot{x}$ is x; and the fluent of $\frac{x\dot{x}}{\sqrt{-aa+2dx}}$ is $\frac{aa+dx}{3dd}\sqrt{-aa+2dx}$ (by *Tab.* 1. *Form* 4. *Caſ.* 2. *Quadr. of Curv.*)

Therefore $x - \frac{aa+dx}{3dd}\sqrt{-aa+2dx}$ is the general expreſſion of the area of the curve.

Now let $x = a$, then area $= (a - \frac{aa+da}{3dd}\sqrt{-aa+2da} =) \frac{d^3+r^3}{3dd} = A$.

Alſo let $x = a$, then area $= (a - \frac{aa+da}{3dd}\sqrt{-aa+2da} =) \frac{d^3-r^3}{3dd} = B$.

And the force whereby the ſphere attracts the body P is as $(A - B$ or as $\frac{2r^3}{3d^2} =) \frac{2\overline{SA}^3}{3\overline{PS}^2}$.

2. The force whereby the ſpheroid $ADBG$, attracts the body P, may, in the ſame manner, be found thus. Let $SC = c$,

The force of a circle whoſe radius is ED, to attract P, is as $1 - \frac{PE}{PD}$, (by Cor. 1. Prop. 90.) Now $\overline{ED}^2 = \frac{\overline{SC}^2}{\overline{SA}^2} \times AEB = \frac{cc}{rr} \times \overline{-aa + 2dx - xx}$ (by the Conics;) and $\overline{PD}^2 = (\overline{ER}^2 = \overline{ED}^2 + \overline{EP}^2 = \frac{-aacc + 2dccx - ccxx}{rr} + xx =)$ $\frac{-aacc + 2dccx + \overline{rr - cc} \times xx}{rr}$. Therefore $(1 - \frac{PE}{PD} = 1 - \frac{x}{\sqrt{-\frac{aacc}{rr} + \frac{2dcc}{rr}x + \frac{rr-cc}{rr}xx}},$ or) $\dot{x} - \frac{\dot{x}x}{\sqrt{-\frac{aacc}{rr} + \frac{2dcc}{rr}x + \frac{rr-cc}{rr}xx}}$ is the fluxion of the attractive force of the ſpheroid on the body P, or the ordinate of a curve whoſe area is the meaſure of that force.

Now the fluent of $\dot{x}$ is x; and (by *Caſ.* 2. *Form* 8. *Tab.* 2. *Quad. Curv.*) the fluent of $\frac{\dot{x}x}{\sqrt{-\frac{aacc}{rr} + \frac{2dcc}{rr}x + \frac{rr-cc}{rr}xx}}$ is $(\frac{-\frac{8dcc}{rr}s + \frac{4dcc}{rr}xv - \frac{4aacc}{rr}v}{-\frac{4aacc}{rr} \times \frac{rr-cc}{rr} - \frac{4ddc^4}{r^4}} =$ $\frac{-2drrs + drrxv - aarrv}{aa \times cc - rr - ddcc} = \frac{-2ds + dxv - axv}{-cc - dd + rr}$ $=)$

$= \frac{2ds - dxv + a\alpha v}{cc + dd - rr}$. Therefore $x + \frac{dxv - a\alpha v - 2ds}{cc + dd - rr}$ is the general expreſſion for the area of the curve.

But $v = PD = ER = \sqrt{-\frac{a\alpha cc}{rr} + \frac{2dcc}{rr}x - \frac{cc - rr}{rr}xx}$ is an ordinate to a conic ſection, whoſe abſciſſa is x; and s, σ, the areas NMB, NKA, adjacent to the ordinates BM, AK: Put $D = s - \sigma$.

Let $x = a$, or $PE = PB = BM$; then $v = a$, or $PD = PB = BM$, and the area $= a + \frac{daa - \alpha aa - 2ds}{cc + dd - rr} = A$.

And let $x = \alpha$, or $PE = PA = AK$; then $v = \alpha$, or $PD = PA = AK$, and the area $= \alpha + \frac{d\alpha\alpha - a\alpha\alpha - 2d\sigma}{cc + dd - rr} = B$.

And the attractive force of the ſpheroid on P, is as $\left(A - B = a - \alpha + \frac{dxaa - \alpha\alpha - a\alpha xa - \alpha - 2dxs - \sigma}{cc + dd - rr}\right.$ $\left. = 2r + \frac{2ddr + 2r^3 - 2dD}{cc + dd - rr}\right) = \frac{2rcc + 2dx\overline{2dr - D}}{cc + dd - rr}$.

But $2d = (a + \alpha =)\ BM + AK$, therefore $2dr =$ trapezium $ABMK$; and $D = (s - \sigma =)$ area $AKRMB$; therefore $D - 2dr =$ mixtilinear area $KRMLK = C$; conſequently $2dr - D = -C$; therefore $2dx\overline{2dr - D} = -2dC$; therefore the attractive force of the ſpheroid on P, is as $\frac{2rcc - 2dC}{cc + dd - rr} = \frac{2AS \times \overline{SC}^2 - 2PS \times KRMK}{\overline{SC}^2 + \overline{PS}^2 - \overline{AS}^2}$. Conſequently, the attractive force of the ſpheroid upon the body P will be to the attractive force of a ſphere, whoſe diameter is AB, upon the ſame body P, as $\frac{rcc - dC}{cc + dd - rr}$ to $\frac{r^3}{3dd}$, or as $\frac{AS \times \overline{SC}^2 - PS \times KRMK}{\overline{SC}^2 + \overline{PS}^2 - \overline{AS}^2}$ to $\frac{\overline{AS}^3}{3\overline{PS}^2}$.

To

To Schol. Prop. 34. *Book* 2. *p.* 119. *l.* 20.

For let it be propoſed to find the vertex of the cone, a fruſtum of which has the deſcrib'd property.

Let *CFGB* be the fruſtum, and *S* the vertex required. (*Pl.* 19. *Fig.* 2.)

Now conceive the medium to conſiſt of particles which ſtrike the ſurface of a body (moving in it) in a direction oppoſite to that of the motion; then the reſiſtance will be the force which is made up of the efficacy of the forces of all the ſtrokes.

In any line *Pp*, parallel to the axis of the cone, and meeting its ſurface in *p*, take *pm* of a given length, for the ſpace deſcrib'd by each point of the cone in a given time: Draw *mq* perpendicular to the ſide (*CF*) of the cone, and *qn* perpendicular to *pm*.

Therefore the line *pm* will repreſent the velocity, or force, with which a particle of the medium ſtrikes the ſurface of the cone obliquely in *p*.

But the force *mp* is equivalent to two forces, the one (*mq*) perpendicular, the other (*pq*) parallel to the ſide of the cone; which laſt is therefore of no effect.

And the perpendicular force *mq* is equivalent to two forces, the one (*mn*) parallel to the axis of the cone, the other (*qn*) perpendicular to it; which alſo is deſtroy'd by the contrary action of another particle on the oppoſite ſide of the cone.

There remains only the force *mn*, which has any effect in reſiſting or moving the cone in the direction of its axis.

Therefore the whole force of a ſingle particle, or the effect of the perpendicular ſtroke of a particle, upon the baſe of a circumſcribing cylinder, is to the effect of the oblique ſtroke upon the ſurface of the cone (in *p*) as *mp* to *mn*, or as $\overline{mp}^2$ to ($mp \times mn =$) $\overline{mq}^2$, or as $\overline{CF}^2$ to $\overline{CH}^2$.

Now the number of particles ſtriking in a parallel direction on any ſurface, is as the area of a plane figure perpendicular to that direction, and that would juſt receive thoſe ſtrokes.

Therefore, the number of particles ſtriking againſt the fruſtum, that is, againſt the ſurfaces deſcrib'd by the rotation of FD, and CF, each particle with the forces mp, and mn reſpectively, is as the circle deſcrib'd by (FD or) OH, and the annulus deſcribed by CH, that is, as $\overline{OH}^2$ to $\overline{CO}^2 - \overline{OH}^2$.

But the whole force of the medium in reſiſting, is the ſum of the forces of the ſeveral particles.

Therefore, the reſiſtance of the medium, or the whole efficacy of the force of all the ſtrokes againſt the end FG of the fruſtum, is to the reſiſtance againſt the convex ſurface thereof, as $(mp \times \overline{OH}^2$ to $mn \times \overline{\overline{CO}^2 - \overline{OH}^2}$ or as $\overline{CF}^2 \times \overline{OH}^2$ to $\overline{CH}^2 \times \overline{\overline{CO}^2 - \overline{OH}^2}$ or as) $\overline{OH}^2$ to $\frac{\overline{CH}^2 \times \overline{\overline{CO}^2 - \overline{OH}^2}}{\overline{CF}^2}$.

Theref. the whole reſiſtance of the medium againſt the fruſtum may be repreſented by $(\overline{OH}^2 + \frac{\overline{CH}^2 \times \overline{\overline{CO}^2 - \overline{OH}^2}}{\overline{CF}^2}$

$= \frac{\overline{CF}^2 \times \overline{OH}^2 - \overline{CH}^2 \times \overline{OH}^2 + \overline{CH}^2 \times \overline{OC}^2}{\overline{CF}^2}$

$=) \frac{\overline{HF}^2 \times \overline{OH}^2 + \overline{CH}^2 \times \overline{OC}^2}{\overline{CF}^2}$, which call z; that is, (putting $OC = r$, $OD = 2a$, $OS = y$, then $CH = (\frac{OC \times FH}{OS} =) \frac{2ar}{y}$, and $OH = \frac{ry - 2ar}{y}$,) $z = \frac{r^4 + r^2y^2 - 4ar^2y + 4a^2r^2}{r^2 + y^2}$; therefore $r^4 + r^2y^2 - 4ar^2y + 4a^2r^2 = r^2z + y^2z$: Conſequently

$2r^2$

$2rr\dot{y}\dot{y} - 4ar^2\dot{y} = 2yz\dot{y} + y^2\dot{z} + r^2\dot{z}$; But z is a minimum; therefore $rr\dot{y} - 2ar\dot{r} = z\dot{y}$; consequently $(z =) \frac{r^2y - 2ar^2}{y} = \frac{r^4 + r^2y^2 - 4ar^2y + 4a^2r^2}{rr + yy}$.

Hence $yy - 2ay = rr$; and making $OQ = QD = a$; then $(y - a =) QS = (\sqrt{rr + aa} =) QC$.

To the ſame Schol. p. 120. *l.* 10.

On the right-line BC, (*Pl.*19.*Fig* 3.) ſuppoſe the parallelograms $BG\gamma b$, $MN\nu m$, of the leaſt breadth, to be erected, whoſe hights BG, MN, their diſtance Mb, and half the ſum of their baſes $\frac{1}{2}Mm + \frac{1}{2}Bb = a$, are given: Let half the difference of the baſes $\frac{1}{2}Mm - \frac{1}{2}Bb$ be called x: Let G and N be points in the curve GND; and producing $b\gamma$, and $m\nu$ to g and n, (ſo that $\gamma g = \nu n = b$,) the points g and n may alſo be in the ſame curve.

Now if the figure $CDNGB$, revolving about the axis BC, generates a ſolid, and that ſolid moves forwards in a rare and elaſtic medium from C towards B, (the poſition of the right-line BC remaining the ſame;) then will the ſum of the reſiſtances againſt the ſurfaces generated by the lineolæ Gg, Nn, be the leaſt poſſible, when $\overline{Gg}^4$ is to $\overline{Nn}^4$ as $BG \times Bb$ to $MN \times Mm$.

For the force of a particle on Gg and Nn, to move them in the direction BC, is as $\frac{1}{\overline{Gg}^2}$ and $\frac{1}{\overline{Nn}^2}$; and the number of particles that ſtrike in the ſame time on the ſurfaces generated by Gg and Nn, are as (the annuli deſcrib'd by $g\gamma$ and $n\nu$, that is, as $BG \times g\gamma$ and $MN \times n\nu$, or as) BG and MN; therefore the reſiſtances againſt thoſe ſurfaces are as $\frac{BG}{\overline{Gg}^2}$ to $\frac{MN}{\overline{Nn}^2}$, that is (putting y for $\overline{Gg}^2$, and z for $\overline{Nn}^2$,) as $\frac{BG}{y}$ to $\frac{MN}{Z}$.

But the ſum of theſe reſiſtances $(\frac{BG}{y} + \frac{MN}{Z})$ is a minimum. Therefore $-BG \times \frac{\dot{y}}{yy} - MN \times \frac{\dot{z}}{zz} = 0$, or $MN \times \frac{\dot{z}}{zz} = -BG \times \frac{\dot{y}}{yy}$: But $y = (\overline{Gg}^2 = \overline{Bb}^2 + \overline{\gamma g}^2 =) aa - 2ax + xx + bb$; and $z = (\overline{Nn}^2 = \overline{Mm}^2 + \overline{\nu n}^2 =) aa + 2ax + xx + bb$; therefore $\dot{y} = 2\dot{x}\dot{x} - 2a\dot{x}$, and $\dot{z} = 2a\dot{x} + 2\dot{x}\dot{x}$: conſequently $\frac{MN}{zz} \times 2\dot{x} \times \overline{a+x} = \frac{BG}{yy} \times 2\dot{x} \times \overline{a-x}$; or $(\frac{MN}{zz} \times \overline{a+x} =) \frac{MN}{zz} \times Mm = (\frac{BG}{yy} \times \overline{a-x} =) \frac{BG}{yy} \times Bb$. Therefore $(yy)\ \overline{Gg}^4 : (zz)\ \overline{Nn}^4 :: BG \times Bb : MN \times Mm$.

Conſequently, that the ſum of the reſiſtances againſt the ſurfaces generated by the lineolæ Gg and Nn, may be the leaſt poſſible, $\overline{Gg}^4$ muſt be to $\overline{Nn}^4$ as GBb to NMm.

Wherefore, if γg be made equal to γG, ſo that the angle γGg may be 45°, and the angle BGg 135°; alſo $\overline{Gg}^2 = 2\overline{\gamma g}^2$, and $\overline{Gg}^4 = 4\overline{\gamma g}^4$; then $4\overline{\gamma g}^4 : \overline{Nn}^4 :: GBb : NMm$; and ſince GR is parallel to Nn, and BG, BR parallel to $n\nu$, $N\nu$; alſo $n\nu = g\gamma = \gamma G$; it follows that $(n\nu = \gamma G =)\ Bb : (N\nu =)\ Mm :: BG : BR$; therefore $Bb = \frac{BG \times Mm}{BR}$; alſo $(n\nu =)\ \gamma G : Nn :: BG : GR$. Conſequently $\frac{4\overline{\gamma g}^4}{\overline{Nn}^4} =) \frac{4\overline{BG}^4}{\overline{GR}^4} = (\frac{GBb}{NMm} =) \frac{\overline{BG}^2}{MN \times BR}$. Therefore $4\overline{BG}^2 \times BR$ is to $\overline{GR}^3$ as GR to MN.

FINIS.

Fig. 1.

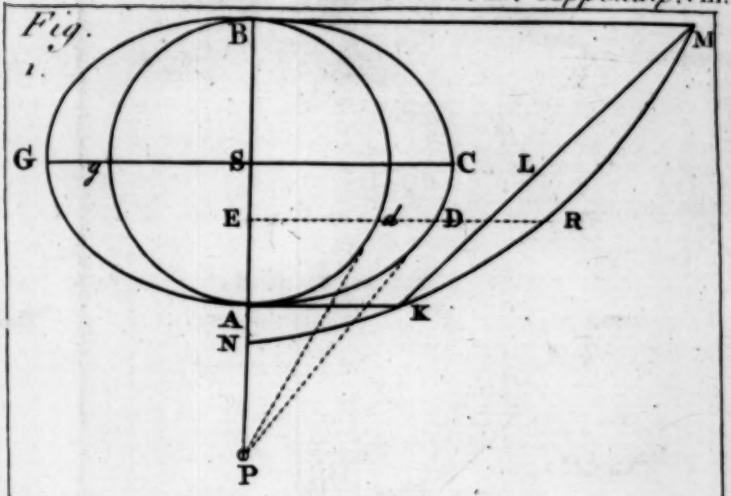


Fig. 2.

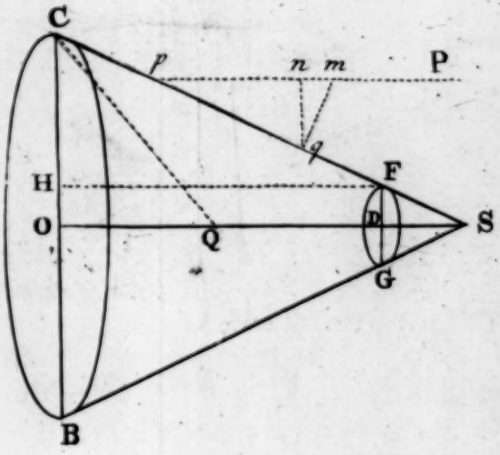
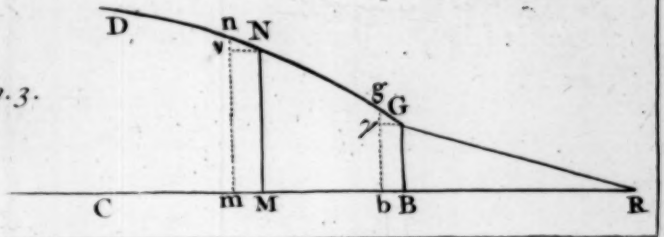


Fig. 3.



535. C. 18.
2

THE LAWS OF THE MOON's MOTION According to GRAVITY.

The LAWS of the

MOON'S

MOTION.

The LAWS of the MOON's MOTION.

IN juſtice to the editor of this tranſlation of Sir *Iſaac Newton*'s *Principia*, it is proper to acquaint the reader, that it was with my conſent, he publiſhed an advertiſement, at the end of a volume of miſcellanies, concerning a ſmall tract which I intended to add to his book by way of appendix; my deſign in which was to deliver ſome general elementary propoſitions, ſerving, as I thought, to explain and demonſtrate the truth of the rules in Sir *Iſaac Newton*'s Theory of the Moon.

THE occaſion of the undertaking was merely accidental; for he ſhewing me a paper which I communicated to the author, in the year 1717, relating to the motion of the nodes of the Moon's orbit; I recollected, that the method made uſe of in ſettling the Equation for that motion, was equally applicable to any other motion of revolution. And therefore I thought that it would not be unacceptable to a reader of the *Principia*, to ſee the uſes of the ſaid method explained in the other Equations of the Moon's motion: Eſpecially ſince the greateſt part of the Theory of the Moon is laid down without any proof; and ſince thoſe propoſitions relating to the Moon's motion, which are demonſtrated in the *Principia*, do generally depend upon calculations very intricate and abſtruſe, the truth of which is not eaſily examined, even by thoſe that are moſt skilful; and which however might be eaſily deduced from other principles.

But in my progreſs in this deſign, happening to find ſeveral general propoſitions relating to the Moon's motions, which ſerve to determine many things, which have hitherto been taken from the obſervations of Aſtronomers: And having

having reaſon to think, that the Theory of the Moon might by theſe means, be made more perfect and compleat than it is at preſent; I retarded the publication of the book, 'till I could procure due ſatisfaction by examining obſervations on places of the Moon. But finding this to be a work requiring a conſiderable time, not only in procuring ſuch places as are proper, but alſo in performing calculations, upon a new method, not yet accommodated to practiſe by convenient rules, or aſſiſted by tables; I thought it therefore more convenient for the Bookſeller, not to ſtop the publication of his impreſſion any longer upon this account. But that I may in ſome meaſure, ſatisfy thoſe who are well converſant in Sir *Iſaac Newton*'s *Principia*, (and I could wiſh that none but ſuch would look over theſe papers,) that the ſaid advertiſement was not without ſome foundation; and that I may remove any ſuſpicion that the deſign is entirely laid aſide, I have put together, altho' in no order, as being done upon a ſudden reſolution, ſome of the Propoſitions, among many others, that I have by me, which ſeem chiefly to be wanting in a Theory of the Moon, as it is a ſpeculation

founded on a phyſical cauſe; and thoſe are what relate to the ſtating of the mean motions. For altho' it be of little or no uſe in Aſtronomy to know the rules for aſcertaining the mean motions of the Node or Apogee, ſince the fact is all that is wanting, and that is otherwiſe known by comparing the obſervations of former ages with thoſe of the preſent; yet in matter of ſpeculation, this is the chief and moſt neceſſary thing required: ſince there is no other way to know that the cauſe is rightly aſſigned, but by ſhewing that the motions are ſo much and no more than what they ought to be.

But that it may not be altogether without its uſe, I have added all the rules for the equation of the Moon's motion, except two; one of which is a monthly equation of the variation depending on the Moon's anomaly; and the other an equation ariſing from the Earth's being not in the focus of the Moon's orbit, as it has been ſuppoſed to be, in all the modern theories ſince *Horrox*.

For not having had time to examine over the obſervations which are neceſſary, but being oblig'd inſtead thereof, to take Sir *Iſaac Newton*'s theory for my chief guide and direction, I cannot venture

venture to depart from it too far, in eſtabliſhing equations entirely new; ſince I am well aſſured, upon the beſt authority, that it is never found to err more than ſeven or eight minutes.

And therefore, hoping that the reader, who conſiders the ſudden occaſion and neceſſity of my publiſhing theſe Propoſitions at this time, will make due allowance for the want of order and method, and look upon them only as ſo many diſtinct Rules and Propoſitions not connected: I ſhall begin, without any other preface, with ſhewing the origine of that inequality, which is called the Variation or Reflection of the Moon.

The Variation of the Moon.

THE variation or reflection is that monthly inequality in the Moon's motion, wherein it more manifeſtly differs from the laws of the motion of a planet in an elliptic orbit. *Tycho Brahe* makes this inequality to ariſe from a kind of libratory motion backwards and forwards, whereby the Moon is accelerated and retarded by turns, moving ſwifter in the firſt and third quarter, and ſlower in the ſecond and fourth, which inequality is principally obſerved in the octants.

Sir *Iſaac Newton* accounts for the

variation from the different force of gravity of the Moon and Earth to the Sun, ariſing from the different diſtances of the Moon in its ſeveral aſpects.

The mean gravity of the Moon to the Sun, he ſuppoſes, is ſatisfied by the annual motion of the Moon round the Sun; the gravity of the Moon to the Earth, he ſuppoſes, is ſatisfied by a revolution of the Moon about the Earth. But the difference of the Moon's gravity to the Sun more or leſs than the Earth's gravity, he ſuppoſes, produces two effects; for as this difference of force may be reſolved into two forces, one acting in the way, or contrary to the way, of the Moon about the Earth, and the other acting in the line to or from the Earth: the firſt cauſes the Moon to deſcribe a larger or ſmaller area in the ſame time about the Earth, according as it tends to accelerate or retard it; the other changes the form of the lunar orbit from what it ought to be merely from the Moon's gravity to the Earth, and both together make up that inequality which is called the variation.

But ſince the real motion of the Moon, tho' a ſimple motion, cauſed by a continual deflection from a ſtreight line, by the

the joint force of its gravity to the Sun and Earth, thereby deſcribing an orbit, which incloſes not the Earth but the Sun, is yet conſidered as a compound motion, made from two motions, one about the Sun, and the other about the Earth; becauſe two ſuch motions are requiſite to anſwer the two forces of its gravity, if ſeparately conſidered: For the very ſame reaſon, the Moon's motion ought to be reſolved into a third motion of revolution, ſince there remains a third force to be ſatisfied, and that is the force ariſing from the alteration of the Moon's gravity to the Sun. And this when conſidered, will require a motion in a ſmall ellipſis, in the manner here deſcribed.

THE circle *ADFH* repreſents the orbit of the Moon about the Earth in the center *T*, as it would be at a mean diſtance, ſuppoſing the Moon had no gravity to any other body but the Earth. The diameter *ATF* divides that part of the orbit which is towards the Sun, ſuppoſe *ADF*, from the part oppoſite to the Sun, ſuppoſe *AHF*. The diameter at right angles *HTD*, is the line of the Moon's conjunction with or oppoſition to the Sun. The figure *PQLK* is an Ellipſis, whoſe cen- Fig. 1.

ter

ter is carried round the Earth in the orbit *ABDEFH*, having its longer axis *PL* in length double of the ſhorter axis *Q K*, and lying always parallel to *T D*, the line joining the centers of the Earth and Sun. Whilſt the ſaid figure is carried from *A* to *B*, the Moon revolves the contrary way from *Q* to *N*, ſo as to deſcribe equal areas in equal times about the centre of it; and to perform its revolution in the ſame time as the center of the ſaid Elliptic epicycle (if it may be ſo called,) performs its revolution; the Moon being always in the remoter extremity of its ſhorter axis in *Q* and *K* when it is in the quarters, and in the neareſt extremity of its longer axis at the time of the new and full Moon.

THE ſhorter ſemiaxis of this Ellipſis *AQ* is to the diſtance of its center from the Earth *AT*, in the duplicate proportion of the Moon's periodical time about the Earth to the Sun's periodical time: Which proportion, if there be 2139 revolutions of the Moon to the Stars in 160 ſydereal years, is that of 47 to 8400.

THE figure which is deſcribed by this compound motion of the Moon in the Elliptic epicycle, whilſt the center of it is carried round the Earth, very nearly repreſents the form of the Lunar orbit; ſuppoſing it without eccentricity, and that

th

the plane was coincident with the plane of the ecliptic, and that the Sun continu'd in the ſame place during the whole revolution of the Moon about the Earth.

FROM the above conſtruction it appears, that the proportion between the mean diſtance of the Moon and its greateſt or leaſt diſtances, is eaſily aſſigned; being ſomething larger than that which is aſſigned by Sir *Iſaac Newton* in the 8th propoſition of his third book. But as the computation there given, depends upon the ſolution of a biquadratic equation, affected with numeral coefficients; which renders it impoſſible to compare the proportions with each other, ſo as to ſee their agreement or diſagreement, except in a particular application to numbers; I ſhall therefore ſet down a rule, in general terms, derived from his method, which will be exact enough, unleſs the periods of the Sun and Moon ſhould be much nearer equal than they are. Let L be the periodical time of the Moon, S the period of the Sun, M the ſynodical period of the Moon to the Sun, and D be the difference of the periods of the Sun and Moon; then, according to Sir *Iſaac Newton*'s method, the difference of the two axes of the Moon's elliptic orbit, as it is contracted by the action

action of the Sun, is to the ſum of the ſaid axes as $3L \times \frac{M+L}{2}$ to $4DD - SS$. But according to the conſtruction before laid down, the ſaid proportion is as $3LL$ to $2SS - LL$.

By Sir *Iſaac Newton*'s rule, the difference will be to the ſum, nearly as 5 to 694; and conſequently the diameters will be nearly as 689 to 699, or 69 to 70: But by the latter rule, the difference will be to the ſum, nearly as 1 to 119; and the diameters or diſtances of the Moon, in its conjunction and quadrature with the Sun, will be as 59 to 60. Dr. *Halley*, (who in his remarks upon the Lunar theory, at the end of his catalogue of the Southern ſtars, firſt took notice of this contraction of the Lunar orbit in the Syzygies from the phenomena of the Moon's motion) makes the difference of the diameters to the ſum, as 1 to 90; and conſequently the greater axis to the leſſer, as $45\frac{1}{2}$ to $44\frac{1}{2}$.

But the difference, in theſe proportions of the extream diſtances, tho' it may appear conſiderable, is not, however, to be diſtinguiſh'd by the obſervations on the diameters of the Moon, whilſt the variations of the diameters, from

from this cauſe, are intermixt with the other much greater variations, ariſing from the eccentricity of the orbit.

THE angle of the Moon's elongation from the center, deſigned by *BTN*, is properly the variation or reflection of the Moon. The properties of which are evident from the deſcription. Fig. 1.

FIRST, It is as the ſine of the double diſtance of the Moon from the quadrature or conjunction with the Sun: For it is the difference of the two angles *BTA* and *NTA*, whoſe tangents, by the conſtruction, are in a given proportion.

SECONDLY, The variation is, *cæteris paribus*, in the duplicate proportion of the ſynodical time of the Moon's revolution to the Sun. For the variation is in proportion to the mean diameter of the epicycle, and that is in the duplicate proportion of the ſynodical time of revolution.

THE greateſt variation is an angle, whoſe ſine is to the radius, as the difference of the greateſt and leaſt diſtances *TQ* and *TL*, that is 3*AQ*, to their ſum. According to the proportion of the lines before deſcribed, this rule makes the elongation near 29 minutes; which would be

be the variation, ſuppoſing the Moon perform'd its revolution to the Sun in the time of its revolution round the Earth. But if that elongation of 29 Minutes be increaſed in the duplicate proportion of the ſynodical time to the periodical time of revolution, it will produce near 34 minutes for the variation.

IT is to be noted, that what is ſaid of the epicycle, is upon ſuppoſition, that the Earths orbit round the Sun is a circle; if the eccentricity of the annual orbit be conſidered, the mean diameter of the epicycle muſt increaſe or diminiſh reciprocally in the triplicate proportion of the Sun's diſtance.

The method of finding the inequalities in any Revolution.

THE conſtruction which I communicated to Sir *Iſaac Newton*, for the annual motion of the nodes of the Moon's orbit, (which is printed in the ſcholium to the 33d propoſition of his 3d book) is a caſe of a general method, for ſhewing the inequality of any motion round a center, when the hourly motion or velocity of the object varies, according to any rule, depending on its aſpect to ſome other object. For in any revolution, the mean motion and inequality are to be aſſigned by means of a curvilinear figure

figure, wherein equal areas are deſcribed about the center in equal times; the property of which figure is, that the rays from the center, are always reciprocally in the ſubduplicate proportion of the hourly motion or velocity about the center.

Thus in the figure deſcribed in my conſtruction, where $T N$ is the line of the nodes, $T A$ the line drawn to the Sun, is ſuppoſed to revolve round the center T, with the velocity of the Sun's motion from the node; and the ray $T B$, which is taken always in the ſubduplicate proportion of that velocity, will deſcribe equal areas in equal times; ſo that the ſector NTB will be the mean motion of the Sun; the ſector NTA the motion of the Sun from the node; and conſequently the area NAB the motion of the node; which will be a retrograde motion if the area be within the circle, and direct if it falls without. From whence it follows, Fig. 2.

1. That the periodical time of the Sun's revolution to the node, will be to the periodical time of the Sun's revolution, as the area of the curvilinear figure, to the area of the circle.

2. That if a circle be deſcribed, whoſe area is equal to the area of the curvilinear

near figure, it will cut that figure in the place where the Sun has the mean motion from the node.

3. If an angle *NTF* be made, which ſhall comprehend an area in the ſaid circle, equal to the ſector *NTB* in the figure, that angle will be the mean motion of the Sun from the node. And conſequently,

4. The angle *FTB*, which is the difference between the Sun's true motion from the node, deſigned by *ATN*, and the Sun's mean motion from the node, deſigned by *FTN*, will be the equation for the Sun's motion from the node, when the Sun's poſition to the node is deſigned by the angle *ATN*.

FROM all which it appears, that what is ſaid of the Sun's motion from the node, will hold as to any other motion round a center; as of the Sun from the Moon, or the Moon from the node or apogee. In any ſuch revolution, a curvilinear figure may be deſcribed about the center, by the areas of which, the relation between the mean and true motion may be ſhewn; and conſequently the inequality or equation of the motion.

Thus

And as in every revolution there is a certain figure which is proper to ſhew this relation, ſuch a figure may be call'd an Equant for that motion or revolution.

And in every revolution where the Equant is a figure of the ſame property, the inequalities or equations will alter according to the ſame rule.

Thus, if the Equant be an ellipſis about the center, as in that for the motion of the Sun from the node,

Firſt, The mean motion in the whole revolution, will be a geometrical mean proportional, between the greateſt motion in the extremity of the leſſer axis, and the leaſt motion in the extremity of the longer axis: For the radius of the circle, which is equal to an ellipſis, is a mean proportional between the two ſemiaxes.

Secondly, The tangents of the angles of the mean and true motion, are in the given proportion of the two axes of the ellipſis. Thus the tangents of the angles of the true and mean motion of the Sun from the node, *viz.* the tangents of the angles *ATN* and *FTN*, are in proportion as the ordinates *BG* and *FG*, that is, as the ſemiaxes *TH* and *TN*. Fig. 2.

Thirdly, The ſine of the angle of the greateſt inequality in the octants is

to the radius, as half the ſum of the axes to half their difference.

It is to be noted, that the equant is an ellipſis about the center, in every motion, where the exceſs of the velocity about the center above the leaſt velocity, is always in the duplicate proportion of the ſine of the angle of the true motion, from the place where the velocity about the center is leaſt. From which remark, upon examination it will appear, that the following motions are to be reduced to an Elliptic equant deſcribed about the center.

The monthly motion of the Moon from the node.

The annual motion of the Sun from the node.

The motion of the Moon from the Sun, as it is accelerated or retarded, by the alteration of the area deſcrib'd about the Earth, according to Sir *Iſaac Newton*'s 26th prop. 3d book.

And the annual Motion of the Sun from the apogee. How theſe ſeveral equants are determin'd will appear by what follows.

The motion of the Nodes.

The node is in its ſwifteſt retrograde motion, when the Sun and Moon are in conjunction or oppo-

oppoſition, and in a quadrature with the line of the nodes. According to Sir *Iſaac Newton*'s method, (explain'd at the end of the thirtieth propoſition of the third book) the force of the Sun to produce a motion in the node, at this time, is equal to three times the mean Solar force; that is, by the conſtruction of the elliptic epicycle, equal to a force, which is to the force of gravity, as $3AQ$ to AT, or three times the leſſer ſemiaxis of the ellipſis to the diſtance of its center from the center of the Earth. But if the Moon revolve in the elliptic epicycle as before deſcribed, the force to make a motion in the node at the time mention'd, will be to the force of gravity, as $3DL$ to DT, or three times the longer ſemiaxis to the diſtance of the center; which is the double of the former force. But then, according to Sir *Iſaac*'s method, the motion of the node at this time, is to the Moon's motion, as the ſolar force to create a motion in the node is to the force of gravity. But if the Moon be conceived as revolving in a circle, with the velocity of its motion from the node at this time, when the node moves ſwifteſt, and the plane of the ſaid circle be ſuppoſed to have a rotation Fig. 1.

upon an axis perpendicular to the plane of the ecliptic, and the contrary way to the motion of the Moon, ſo as to produce the motion of the node, and leave the Moon to move with its own motion about the Earth; the force to make a motion in the node ſeems to be the difference of the forces to retain it with the velocity of its motion in the moveable and immoveable planes: But the velocities of bodies revolving in circles are in the ſubduplicate proportion of the central forces. From whence it follows, that

The motion of the Moon from the node at this time, when the node moves ſwifteſt, is to the motion of the Moon, in the ſubduplicate proportion of the ſum of the forces to the force of gravity, or as the ſum of TD *and* 3DL *to* TD.

And this would be the greateſt motion of the node, upon ſuppoſition that the plane of the Moon's orbit was almoſt co-incident with the plane of the ecliptic; but if the inclination be conſidered, the motive force for the node muſt be diminiſhed, in the proportion of the ſine-complement of the inclination to the radius. How much this

this motion is, will appear by the following ſhort calculation.

The diſtance TD being as before equal to 8400, and $3DL$ being 282, the inclination of the plane in this poſition is $4^{\circ}. 59'. 35''$; the ſine-complement of which is to the radius, as 525 to 527 nearly; therefore the force of gravity is to the motive force for the node thus diminiſhed, in the compound proportion of 8400 to 282, and of 527 to 525, that is, in the proportion of 4216 to 141. So that the greateſt motion of the Moon from the node is to the motion of the Moon, in the ſubduplicate proportion of 4357 to 4216, that is, in the proportion nearly of 613 to 603. According to which calculation, the greateſt hourly motion of the node ought to be $32''. 47'''$. By Sir *Iſaac Newton*'s method, it amounts to $33''. 10'''\frac{1}{2}$.

This is the ſwifteſt retrograde motion of the node, when the line of the nodes is in a quadrature with the Sun, and the Moon is in its greateſt latitude in conjunction or oppoſition to the Sun. But the equant for the motion of the Moon from the node in this month, when the line of the nodes is in quadrature with the Sun, is an ellipſis about the center; and therefore the

mean motion in this month will be known by the following rule:

The mean motion of the Moon from the node, in that month when the line of the nodes is in a quadrature with the Sun, is a geometrical mean proportional, between the greateſt motion of the Moon from the node and the motion of the Moon.

And therefore this mean motion, will be to the motion of the Moon, in the ſubduplicate proportion of 613 to 603, that is, nearly in the proportion of 1221 to 1211. So that the mean motion of the node in this month, will be to the motion of the Moon, as 10 to 1211, which makes the mean hourly motion 16″. 19‴ $\frac{1}{40}$. According to Sir *Iſaac Newton* it amounts to 16″. 35‴; but, by the corrections which he afterwards uſes, it is reduced to 16″. 16‴ $\frac{2}{3}$.

But the equant for the annual motion of the Sun from the node being alſo an ellipſis, it follows, that

The mean motion of the Sun from the node, is a geometrical mean proportional, between the motion of the Sun and the mean motion of the Sun from the node, in the month when the line of the nodes is in quadrature with the Sun.

How

How near this rule agrees with the obſervations, will appear by this calculation.

Since the mean motion of the node in that month, when the line of nodes is in quadrature to the Sun, was before ſhewn to be to the Moon's mean motion, as 10 to 1211; and the motion of the Sun is to the motion of the Moon, as 160 to 2139: it follows, that the motion of the node and the motion of the Sun will be in the proportion of 154 and 1395; and therefore, by the rule, the Sun's mean motion from the node, is to the Sun's mean motion, in the ſubduplicate proportion of 1549 to 1395, that is, nearly as 98 to 93. Which correſponds with the obſervations; there being 98 revolutions of the Sun to the node in 93 revolutions of the Sun. The ſubduplicate proportion taken more nearly, is as 941 to 893, which will produce 19°. 21′. 3″, for the motion of the node from the fix'd Stars, in a ſydereal year. The motion (as obſerv'd) is 19°. 21′. 22″.

Had the calculation from the rule, been more exactly made in large numbers, the annual motion produced would be 19°. 21′. 07″½, which is 14′

leſs than the motion, as obſerved by the Aſtronomers.

Which difference may very probably ariſe from the Sun's parallax; and if ſo, it may perhaps furniſh the beſt and moſt certain method of adjuſting and fixing the true diſtance of the Sun. For the Sun's force being ſomething more on that half of the orb which is towards the Sun, than what it is on the other half, the elliptic epicycle is accordingly larger in the firſt caſe, than in the latter. And by calculation, I find that the mean motion of the node, ariſing after conſideration is had of this difference, is more than the mean motion from the mean magnitude of the epicycle, by near 2″ in the year, for every minute in the parallactic angle of the orbit of the Moon, or for every ſecond of the Sun's parallax. And by the beſt computation I have yet made, this difference of 14″, in the annual motion of the node, will ariſe from about 8″ of parallax; which will make the Sun's diſtance above 25000 ſemi-diameters of the Earth.

In like manner as the equant for the motion of the node, in that month when the line of the nodes is in quadrature with the Sun, is an ellipſis; ſo in any other

other month it is alſo an ellipſis: the motion of the node being direct and retrograde by turns, in the Moon's paſſing from the quadrature to the Sun to the place of its node, and from the place of its node to the quadrature.

The Inclination of the Plane of the Moon's orbit to the Plane of the Ecliptic.

But theſe elliptic equants do not only ſerve to ſhew the inequality of the motion of the node, but alſo the inclination of the plane of the Moon's orbit to the plane of the ecliptic. Thus the rays in the elliptic equants, for the motion of the Moon from the node in each month, deſign the inclinations of the plane of its orbit to the plane of the ecliptic, in the ſeveral reſpective poſitions of the Moon to the line of the nodes. And the rays of the elliptic equant for the annual motion of the Sun from the node, in my Conſtruction, (in the ſchol. to prop. 33. book 3. of Sir *Iſaac Newton*'s *Principia*) deſign the different mean inclinations of the ſaid plane, to the plane of the ecliptic in each month, when the Sun is in each reſpective aſpect to the line of the nodes.

Thus if NT (the ſemi-tranſverſe axis of the elliptic equant for the motion of the Sun from the node,) deſign Fig. 2.

the

the mean inclination of the plane, or, which is the ſame thing, if it repreſent the mean diſtance between the pole of the ecliptic and the pole of the Moon's orbit, in that month when the Sun is in the line of the nodes; TH, the ſemiconjugate axis of the ſaid ellipſis, will deſign the mean inclination or mean diſtance of the poles in that month when the line of nodes is in quadrature to the Sun; and TB, any other ſemidiameter of the ſaid ellipſis, will repreſent the mean diſtance between the ſaid poles, when the Sun is in that aſpect to the line of the nodes, which is deſigned by the angle NTA. For example, if the leaſt inclination, deſigned by the ſhorter ſemiaxis TH be $5^{\circ}. 00'. 00''$; ſince TH is to TK as the motion of the Sun to the mean motion of the Sun from the node, by the property of this equant; and ſince there are 98 revolutions of the Sun to the node in 93 revolutions of the Sun; it follows, that HK, the difference between the greateſt and leaſt of the mean inclinations in the ſeveral months of the year, is to TH the leaſt, as 5 to 93; by which proportion, the ſaid difference will amount to $16'. 10''$. According to Sir *Iſaac Newton*'s computation in the 35th prop. of the third book, it is

16.

16′. 23″½. But if the ſaid number be leſſen'd in the proportion of 69 to 70, according to the author's note at the end of the 34th prop. the ſaid difference will become 16′. 9″.

And in like manner, the inclinations of the plane of the Moon's orbit, in that month when the motion of the node is ſwifteſt, (being ſituated in the line of quadratures with the Sun,) are determined by the equant for the motion of the Moon from the node, in that month.

Thus, let TH be to TN in the ſub- Fig. 2.
duplicate proportion of the Moon's motion, to its greateſt motion from the node, when the Moon is in the conjunction in TH; that is, (as was before determined) let TH be to TN in the proportion of 1211 to 1221; and the ellipſis deſcribed on the ſemiaxes TH and TN, will be the equant for the motion of the Moon from the node in that month. And the rays of the ſaid equant will deſign the inclinations of the plane in the ſeveral aſpects of the Moon to the line of the nodes. That is, if TN be the inclination of the plane, or the diſtance of the pole of the ecliptic from the pole of the Moon's orbit, when the Moon

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is in *TN* the line of the nodes, the ray *TB* will repreſent the diſtance of the ſaid poles, or the inclination of the plane, in that aſpect which is deſigned by the angle *NTB*.

WHICH being laid down, it follows that the whole variation of the inclination, in the time the Moon moves from the line of the nodes to its quadrature in *THK*, is to the leaſt inclination, as *KH* to *TH*, that is, as 10 to 1211. Wherefore if the leaſt inclination be 4°. 59′. 35″, the whole variation will be 2′. 29″. This is upon ſuppoſition that the Sun continued in the ſame poſition to the line of the nodes, during the time that the Moon moves from the node to its quadrature. But the Sun's motion protracting the time of the Moon's period to the Sun, in the proportion of 13 to 12; the variation muſt be increaſed in the ſame proportion, and will therefore be 2′. 41″. According to Sir *Iſaac Newton*'s computation, as delivered in the corollaries to the 34th prop. of the 3d book, for ſtating this greateſt variation, (the intermediate variations in this or any other month not being computed or ſhewn by any method) it amounts to 2′. 43″. But if the ſaid quantity be diminiſh'd in

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the proportion of 70 to 69, according to his note at the end of the ſaid propoſition, it will become the ſame preciſely as it is here deriv'd from the equant.

THE motion of the Moon from the Sun, as it is accelerated or retarded by the increment of the area deſcribed about the Earth, (according to the 26th prop. of the 3d book) is alſo to be reduced to an elliptic equant; by taking the ſhorter axis to the longer axis, in the ſubquadruplicate proportion of the force of the Moon's gravity to the Earth, to the ſaid force added to three times the mean Solar force, that is, as TA to the firſt of three mean proportionals between TA and $TA+3AQ$. And in the ſame proportion is the area deſcribed by the Moon about the Earth, when in quadrature with the Sun, to the mean area, or as the mean area to the area deſcribed in the ſyzygies: So that the greateſt area in the ſyzygies is to the leaſt in the quadratures, in the ſubduplicate proportion of $TA+3AQ$ to TA, or as $\sqrt{8541}$ to $\sqrt{8400}$. This is upon ſuppoſition, that the Moon revolves to the Sun in the ſame time as it revolves about the Earth; which will be found to agree

The Variation of the Area deſcribed by the Moon about the Earth.

Fig. 1.

gree very nearly with Sir *Iſaac Newton*'s computation, in the before-cited propoſition.

The Motion of the Apogee.

And after the ſame manner an elliptic equant might be conſtructed, which would very nearly ſhew the mean motion of the apogee, according to the rules deliver'd by Sir *Iſaac Newton* (in the corollaries of the 45th prop. of the firſt book) for ſtating the motion of the apogee, namely, by taking the greateſt retrograde motion of the apogee, from the force of the Sun upon the Moon in the quarters; and the greateſt direct motion, from the force of the Sun upon the Moon when in the conjunction or oppoſition; each according to his rule, deliver'd in the ſecond corollary to the ſaid propoſition. And if an ellipſis be made whoſe axes are in the ſubduplicate proportion of the Moon's motion from the apogee, when in the ſaid ſwifteſt direct and retrograde motions, the ſaid ellipſis will be nearly the equant for the motion of the Moon from the apogee, and will be found to be nearly of the form of that above for the increment of the area.

But the motion of the apogee, according to this method, will be found

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to be no more than 1°. 37′. 22″, in the revolution of the Moon from apogee to apogee, which (according to the obſervations) ought to be 3°. 4′. 7″½.

So that it ſeems there is more force neceſſary to account for the motion of the Moon's apogee, than what ariſes from the variation of the Moon's gravity to the Sun, in its revolution about the Earth.

But if the cauſe of this motion be ſuppoſed to ariſe from the variation of the Moon's gravity to the Earth, as it revolves round in the elliptic epicycle, this difference of force, which is near double the former, will be found to be ſufficient to account for the motion; but not with that exactneſs as ought to be expected. Neither is there any method that I have ever yet met with upon the commonly received principles, which is perfectly ſufficient to explain the motion of the Moon's apogee.

The rules which follow concerning the motion of the apogee, and the alteration of the eccentricity, are founded upon other principles, which I may have occaſion hereafter to explain, it being, as I apprehend, impoſſible to derive theſe, and many other ſuch propoſitions

poſitions from the laws of centripetal forces.

Fig. 1. LET TC (in the above conſtruction of the Lunar orbit) be the mean diſtance of the Moon, or half the ſum of its greateſt and leaſt diſtances, *viz.* TQ and TL; and let CL be the mean ſemidiameter of the elliptic epicycle, or half the ſum of the ſemiaxes; and take a diſtance LM, on the other ſide towards the centre, equal to CL; then,

The mean motion of the Moon from its apogee, is to the mean motion of the Moon, in the ſubduplicate proportion of TM *to* TC.

FOR example, Half the ſhorter axis or DC is $23\frac{1}{2}$; therefore TC the mean diſtance is $8376\frac{1}{2}$; CM or $2CL$, the ſum of the ſemiaxes, is 141; ſo that TM is $8235\frac{1}{2}$. Wherefore the motion of the Moon from the apogee is to the motion of the Moon, in the ſubduplicate proportion of $8235\frac{1}{2}$ to $8376\frac{1}{2}$, or of 16471 to 16753, that is, nearly as 117 to 118, or more nearly, as 352 to 355; or yet more nearly, as 1877 to 1893; ſo that there ought to be about 16 revolutions of the apogee in 1893 revolutions of the Moon; which agrees to great preciſeneſs with the moſt modern numbers of Aſtronomy; according to which proportion,

portion, the mean motion of the apogee, in a ſydereal year, ought to be 40°. 40′. 40½″. But by the numbers in Sir *Iſaac Newton*'s theory of the Moon, the ſaid motion is 40°. 40′. 43″. According to the numbers of *Tycho Brahe*, it ought to be 40°. 40′. 47″.

THE mean motion of the apogee being ſtated, I find the following rule for the alteration of the eccentricity.

The Variation of the Eccentricity.

The leaſt eccentricity is to the mean eccentricity, in the duplicate proportion of the Sun's mean motion from the apogee of the Moon's orbit, to the Sun's mean motion. Or in the duplicate proportion of the periodical time of the Sun's revolution, to the mean periodical time of its revolution to the Moon's apogee.

BY the foregoing rule for the mean motion of the apogee, there are 16 revolutions of the apogee in 1893 revolutions of the Moon; but there being 254 revolutions of the Moon in 19 revolutions of the Sun; there muſt be about 7 revolutions of the apogee in about 62 revolutions of the Sun, or rather about 20 in 177. So that the periods of the Sun to the Stars, and of the Sun to the Moon's apogee, are in pro-

portion nearly as the numbers 157 and 177. The duplicate of which proportion is that of 107 to 136; which, according to the rule, ought to be the proportion of the leaſt eccentricity to the mean eccentricity.

So that by this rule, the mean eccentricity, (or half the ſum of the greateſt and leaſt,) ought to be to the difference of the mean from the leaſt, (or half the difference of the greateſt and the leaſt,) as 136 to 29.

How near this agrees with the Obſervations, will appear from the numbers of Mr. *Horrox* or Mr. *Flamſted*, and of Sir *Iſaac Newton*.

The mean eccentricity according to Mr. *Flamſted* or Mr. *Horrox* is 0.055236, half the difference between the greateſt and leaſt is 0.011617; which numbers are in the proportion of $135\frac{1}{2}$ to $28\frac{1}{2}$ nearly.

According to Sir *Iſaac Newton*, the mean eccentricity is 0.05505, half the difference of the greateſt and leaſt is 0.01173; which numbers are in proportion nearly as $135\frac{47}{48}$ to $28\frac{47}{48}$, each of which proportions is very near that above aſſigned.

But it is to be noted, that the rule, which is here laid down, is true only upon ſuppoſition that the eccentricity is

exceeding

exceeding ſmall. There is another rule derived from a different method, which preſuppoſes the knowledge of the quantity of the mean eccentricity; and which will not only determine the variation of the eccentricity according to the laws of gravity, with greater exactneſs, but ſerve alſo to correct an hypotheſis in the modern theories of the Moon, in which their greateſt error ſeems to conſiſt; and that is, in placing the earth in the focus of that ellipſis, which is deſcribed on the extreme diameters of the lunar orbit; whereas it ought to be in a certain point nearer the perigee, as I may have occaſion to explain more fully hereafter.

THE greateſt and leaſt eccentricity being determined; the equant for the motion of the Sun from the apogee is an ellipſis, whoſe greater and leſſer axes are the greateſt and leaſt eccentricities: and therefore, by the property of ſuch an equant as before laid down,

The Equation of the Apogee.

The ſine of the greateſt equation of the apogee will be to the radius, as the difference of the axes of the equant is to their ſum; that is, as the difference of the greateſt and leaſt eccentricities to their ſum.

FOR example, ſince the difference is to the ſum as 29 to 136, by what was determined in the foregoing article, the greateſt equation of the apogee will be about 12°. 18′. 40″. Sir *Iſaac Newton* has determined it from the obſervations to be 12°. 18′.

THE greateſt and leaſt eccentricities being determined; the eccentricity and equation of the apogee, in any given aſpect of the Sun, are determined by the equant, in the following manner.

LET *TN* be the greateſt eccentricity, *TH* the leaſt, the ellipſis on the ſemi-axes *TN* and *TH*, the equant for the motion of the apogee. Fig 2.

THEN if the angle *NTF*, be made equal to the mean diſtance or mean motion of the Sun from the apogee, the angle *NTB* will be the true diſtance or motion of the Sun from the apogee; the difference *BTF*, the equation of the apogee; and the ray *TB*, the eccentricity of the orbit, in that aſpect of the Sun to the apogee deſigned by the angle *NTB*. Hence ariſes this rule.

The tangent of the mean diſtance, viz. NTF, *is to the tangent of the true diſtance* NTB, *in the given proportion of the*

the greateſt eccentricity TN *to the leaſt* TH, *that is, as* 165 *to* 107.

FROM what has been laid down concerning the general property of an equant, that it is a curve line deſcribed about the center, whoſe rays are reciprocally in the ſubduplicate proportion of the velocity at the center, or the velocity of revolution, it will not be difficult to deſcribe the proper curve for any motion that is propoſed; and where the inequality of the motion throughout the revolution is but ſmall, there is no need of any nice or ſcrupulous exactneſs in the quadrature of the curve for ſhewing what the equation is. Thus all the ſmall annual equations of the Moon's motion ariſing from the different diſtances of the Sun, at different times of the year, may be reduced to one rule exact enough for the purpoſe.

FOR ſince the Sun's force to create theſe annual alterations, is reciprocally in the triplicate proportion of the diſtance; the rays of the equant for ſuch a motion, will be in the ſeſquiplicate proportion of the diſtance. From whence it will not be difficult to prove, that if the revolution of the motion to be equated, were performed in the time of the Sun's revolution, the equation would be to the

equation of the Sun's center, nearly as 3 to 2 : and ſo if the force decreaſed as any other power of the Sun's diſtance, ſuppoſe that whoſe index is *m*, the equation would be to that of the Sun's center as *m* to 2. But if the motion be performed in any other period, the equation will be more or leſs, in the proportion of the period of the revolution to the Sun, to the period of the revolution of the motion to be equated. Thus if it were the node or apogee of the Moon's orbit, the equation is to the former as the period of the Sun to the node or apogee, to the period of the node or apogee. Which rule makes the greateſt equation for the node about 8′. 56″, being a ſmall matter leſs than that in Sir *Iſaac Newton*'s theory ; and the greateſt equation for the apogee about 21′. 57″, being ſomething larger than that in the ſame theory.

The like rule will ſerve for the annual equation of the Moon's mean motion. If inſtead of the equation for the Sun's center, another ſmall equation be taken in proportion to it as the force, by Sir *Iſaac Newton* called the mean ſolar force, to the force of the Moon's gravity, or as 47 to 8400 ; the ſaid equation increaſed in the proportion of the Sun's

Sun's period to the mean ſynodical period of the Moon to the Sun, or of 99 to 8, will be the annual equation of the Moon's mean motion. According to this, the equation, when greateſt, will be 12′. 5″.

WHAT is ſaid may be ſufficient for the preſent purpoſe, which is only to lay down the principal laws and rules of the ſeveral motions of the Moon, according to gravity. Some other propoſitions, which ſeem no leſs neceſſary than the former, for compleating the theory of the Moon's motion, as to its aſtronomical uſe, I reſerve to another time.

BUT to make ſome amends for the ſhortneſs and confuſedneſs of the preceeding propoſitions, I ſhall add one example to ſhew the uſe of the equant more at large, in what is commonly called the ſolution of the *Keplerian* problem; that being one of the things which I propoſed to explain, when the elements for the theory of the Moon were advertiſed.

An example of the uſe of the equant in finding the equation of the center.

Fig. 3. LET the figure *ADP* be the orbit in which a body revolves, deſcribing equal areas in equal times by lines drawn from a given point *S*; and let it be propos'd to find the equant for the apparent motion of the ſaid body, about any other place within the orbit, ſuppoſe *F*.

LET there be a line *FR* indefinitely produc'd, which revolves with the body as it moves through the arch *AR*; and in the ſaid line take a diſtance *Fp*, which ſhall be to *FR*, the diſtance of the body from the given point *F*, in the ſubduplicate proportion of the perpendicular let fall upon the tangent of the orbit at *R* from the point *S*, to the perpendicular on the ſaid tangent let fall from the given point *F*; and the curvilinear figure, deſcrib'd by the point *p*, ſo taken every where, will be the equant for the motion of the body about the point *F*.

FOR ſince the areas deſcribed at the diſtances *Fp* and *FR* are in the duplicate

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cate proportion of thoſe lines, that is, by the conſtruction, in the proportion of the perpendiculars on the tangents let fall from *S* and *F*; the areas which the body deſcribes, in moving through the arch *AR* about the points *S* and *F*, are in the proportion of the ſame perpendiculars. And therefore the area deſcribed by the revolution of the line *Fp* in the figure, will be equal to that which is deſcribed by the revolution of the line *SR* in the orbit. So that the areas deſcribed in the figure will be equal in equal times, as they are in the orbit. And conſequently the rays *Fp* of the figure will conſtantly be in the ſubduplicate proportion of the velocity of the motion, as it appears at the center *F*, which is the property of the equant.

FROM which conſtruction, it will be eaſy to ſhew, that in the caſe where a body deſcribes equal areas in equal times about a fixed point, there may be a place found out within the orbit, about which the body will appear to revolve with a motion more uniform than about any other place.

THUS ſuppoſe the orbit *ADP* was a figure, wherein the remoteſt and neareſt apſis *A* and *P* were diametrically oppoſite, in a line paſſing through the point *S*,

S, *viz.* the point about which the equal areas are deſcribed; then if the point *F* be taken at the ſame diſtance from the remoteſt apſis *A*, as the point *S* is from the neareſt apſis *P*, the ſaid center *F* will be the place, about which the body will appear to have the moſt uniform motion. For in this caſe the point *F* will be in the middle of the figure *LpDl*, which is the equant for the motion about that point. So that the body will appear to move about the center *F*, as ſwift when it is in its ſloweſt motion in the remoter apſis *A*, as it does when it is in its ſwifteſt motion in the neareſt apſis *P*.

For by the conſtruction, when the body is at *A*, the ray of the equant *FL* is a mean proportional between *AF* and *AS*; and when the body is at *P*, the ray of the equant *Fl* is a mean proportional between the two diſtances *PS* and *PF*, which are reſpectively equal to the former.

And in like manner in an orbit of any other given form, a place may be found about which the motion is moſt regular.

If what has been ſaid be applied to the caſe of a body revolving in an elliptic orbit, and deſcribing equal areas

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in equal times about one of the foci, as is the caſe of a planet about the Sun, and a ſecondary planet about the primary one; it will ſerve to ſhew the foundation of the ſeveral hypotheſes and rules which have been invented by the modern Aſtronomers, for the equating of ſuch motions; and likewiſe ſhew how far each of them are deficient or imperfect.

For if the ellipſis ADP be the orbit of a planet deſcribing equal areas about the Sun in the focus S, the other focus, ſuppoſe F, will be the place about which the motion is moſt regular, from what has been already ſaid; that focus being at the ſame diſtance from the aphelion A, as the Sun at S is from the perihelion P. And by the conſtruction, each ray (Fp) of the equant will always be a mean proportional between FR and RS, the two diſtances of the planet from the two foci, in that place where the ray Fp is taken. For the rays SR and RF, making equal angles with the tangent at R, by the property of the ellipſis, are in the proportion of the perpendiculars from S and F, let fall on thoſe tangents. And therefore Fp being to FR in the ſubduplicate proportion

portion of SR to FR, it will be a mean proportional between thoſe diſtances.

1. HENCE when the planet is in the aphelion A, or perihelion P, the rays of the equant FL and Fl are the ſhorteſt, each being equal to CD, the leſſer ſemi-axis of the orbit: For by the property of the ellipſis, the rectangle of the extream diſtances from the focus is equal to the ſquare of the leſſer ſemi-axis.

2. WHEN the planet is at its mean diſtance from the Sun in D or d, the extremities of the leſſer axis, the equant cuts the orbit in the ſame place; the rays of the equant being then the longeſt, being each equal to the greater ſemi-axis CA. For in thoſe points of the orbit, the diſtances from the foci and the mean proportional are the ſame.

FROM which form of the equant it appears,

1. THAT the velocity of the revolution about the focus F diminiſhes, in the motion of the planet from the aphelion or perihelion to the mean diſtance; and increaſes in paſſing from the mean diſtance to the perihelion or aphelion. For the rays of the equant increaſe in the firſt caſe, and diminiſh in the latter; and the velocity of revolution increaſes in

in the duplicate proportion, as the rays diminiſh.

2. In any place of the orbit, ſuppoſe *R*, the velocity of the revolution about the focus *F*, is in proportion to the mean velocity, as the rectangle of the ſemi-axes of the orbit *CD* and *CA*, to the rectangle of the focal diſtances *RF* and *RS*. For the equant and the orbit, being figures of the ſame area, are each equal to a circle, whoſe radius is a mean proportional between the two ſemi-axes *CD* and *CA*. But the mean motion about the focus *F*, is in thoſe places, where the ſaid circle cuts the equant; and in other places, the velocity of the revolution is reciprocally as the ſquare of the diſtance, that is, reciprocally as the rectangle of the focal diſtances *RF* and *RS*.

3. So that the planet is in its mean velocity of revolution about the focus *F*, in four places of the orbit, that is, where the rectangle of the focal diſtances is equal to the rectangle of the ſemi-axes; which places in orbits nearly circular, ſuch as thoſe of the planets, are about 45 degrees from the aphelion or perihelion; but may be aſſigned in general, if need be, by taking a point in the orbit, ſuppoſe *R*, whoſe neareſt diſtance from the leſſer axis of the orbit *CD* is to the longer

longer ſemi-axis CA, in the ſubduplicate proportion of the longer axis to the ſum of the two axes; as may be eaſily proved.

WHAT has been ſaid, may be enough to ſhew the form of the equant, and the manner of the motion about the upper focus in general. But the preciſe determination of the inequality of the motion, requires the knowledge of the quadrature of the ſeveral ſectors of the equant, or at leaſt, if any other method be taken, of that which is equivalent to ſuch a quadrature.

There are divers methods for ſhewing the relation between the mean and true motion of a planet round the Sun, or round the other focus, ſome more exact than others. But the following ſeems the moſt proper for exhibiting in one view, all the ſeveral hypotheſes, and rules, which are in common uſe in the modern Aſtronomy, whereby it may eaſily appear, how far they agree or differ from each other, and how much each of them errs from the preciſe determination of the motion, according to the true law of an equal deſcription of areas about the Sun.

UPON the center F deſcribe the ellipſis LNl, equal and ſimilar to the elliptic orbit ADP; but having its axes

FN

FN and *FL* contrarily poſited, that is, the ſhorter axis *LF* lying in the longer axis of the orbit *AP*, and the longer axis *FN* parallel to the ſhorter *CD*. Let the focus of the ſaid ellipſis be in *f*. And ſuppoſe two other ellipſis *LBl* and *Lfl*, to be drawn upon the common axis *Ll*, one paſſing through the point *B*, where the perpendicular *FN* interſects the orbit, and the other through the focus *f*. Let the line *FR*, revolving with the planet in the orbit, be indefinitely produced, till it interſect the firſt ellipſis *LNl* (which was ſimilar to the orbit) in *Q*, the equant in *p*, and the ellipſis *LBl* (drawn through the interſection *B*,) in *K*. From the point *K* let fall *KH* perpendicular to the line of apſides *AP*, and let it be produced till it interſect the firſt ellipſis *LNl* in *O*, and the ellipſis *Lfl* (paſſing through the focus *f*) in *E*. And laſtly, in the ellipſis *LNl*, let *GM* be an ordinate equal and parallel to *EH*. In which conſtruction it is to be noted, that the ellipſis *Lfl* and *LBl* are ſuppoſed as drawn only to divide the line *OKH* in given proportions, that *KH* may be to *OH*, as the latus rectum of the orbit to the tranſverſe axis; and that *EH* or *GM*, the baſe of the elliptic ſegment *GLM*, may

may be to OH, as the diſtance of the foci to the tranſverſe axis.

WHICH being premiſed, it will be eaſy to prove, that the ſector pFL in the equant, or, which is the ſame thing, the ſector RSA in the orbit, is equal to the curvilinear area $OKFMG$, that is, equal to the elliptic ſector QFL, deducting the ſegment LMG, and adding or ſubducting the trilinear ſpace QKO, according as the angle RFA is leſs or greater than a right angle. Wherein it is to be noted, that theſe ſigns of addition and ſubduction are to be uſed in general, if the angle AFR is taken from the aphelion in the firſt ſemi-circle, but towards the aphelion in the latter ſemi-circle. But if the angle AFR be taken the ſame way throughout the whole revolution, as is the method in Aſtronomical calculations, then the ſegment and the trilinear ſpace in the latter ſemi-circle muſt be taken with the contrary ſigns to what are laid down.

HENCE it appears, that the inequality in the motion of a planet about the upper focus F, conſiſts of three parts.

I. THE firſt and principal of which is the inequality in the alteration of the angle QFL, in making equal areas in the

ellipſis

ellipſis *LNl*. For if a circle equal to the ellipſis be deſcribed upon the center *F*, ſince the radius (being a mean proportional between the two ſemi-axes) will fall without the ellipſis about the line of apſides, and within it about the middle diſtances, the angle *QFL*, which is proportional to the area deſcribed in the circle, will therefore increaſe faſter about the line of apſides, and ſlower about the middle diſtances, in deſcribing equal areas in the ellipſis, than it ought to do in the hypotheſis of Biſhop *Ward*, who makes the planet revolve uniformly about the focus. The equation to rectify this inequality is determined by the following rule. Fig. 3.

THE tangent of the angle *QFL*, is to the tangent of the angle in the circle including the ſame area, as the longer axis of the ellipſis to the ſhorter axis; and the difference of the angles, whoſe tangents are in this proportion, is the equation; as is manifeſt from what was before ſaid on the properties of an elliptic equant. From the ſame it alſo follows, that

1. THE greateſt equation is an angle, whoſe ſine is to the radius as the difference of the axes to their ſum, or, which is the ſame thing, as the ſquare of the diſtance

of the foci, to the ſquare of half the ſum of the axes. So that in ellipſis nearly circular, of different eccentricities, this greateſt equation will vary nearly in the duplicate proportion of the eccentricity.

2. In ellipſis nearly circular, the equation at any given angle QFL, is to the greateſt equation, nearly as the ſine of the double of the given angle to the radius; which follows from hence, that the equation is the difference of two angles, whoſe tangents are in a given proportion, and nearly equal.

3. This equation adds to the mean motion in the firſt and third quadrant of mean anomaly, and ſubducts in the ſecond and fourth; as will eaſily appear from that the line QF, in deſcribing equal areas in the ellipſis, makes the angle to the line of the apſides, leſs acute than it would be in an uniform revolution.

This is the equation which is accounted for in the hypotheſis of *Bullialdus*. For he ſuppoſes the motion of the planet in its orbit to be ſo regulated about the upper focus, that the tangents of the angles, from the lines of apſides, ſhall always be to the tangents of the angles anſwering to the mean anomaly, in the proportion of the ordinates

in

in the ellipſis to the ordinates in the circle circumſcribed; which in effect is the ſame, as if he had made the true equant for its motion about the focus *F*, to be the ellipſis as above deſcribed.

THE ſame equation is alſo uſed by Sir *Iſaac Newton*, in his ſolution of the *Keplerian* problem, in the ſcholium to the 31ſt prop. of the 1ſt book, and is there deſigned by the letter *V*.

BUT ſince the true equant *LDl* coincides with the elliptic equant in the extremities of the ſhorter axis at *L* and *l*, and falls within the ſame at its interſection with the longer axis *FN*, it follows, that the motion of the planet in the ſemi-circle about the aphelion, is ſwifter than according to the hypotheſis of an equal deſcription of areas in the ellipſis *LNl*, and for the ſame reaſon ſlower in the other ſemi-circle about the perihelion; the velocity about the center *F* being always reciprocally in the duplicate proportion of the diſtance.

WHICH leads to the ſecond part of the inequality of the motion about the focus.

II. THE equation to rectify this inequality, is an angle anſwering to the ſegment *GLM*; which angle is to be added to the mean anomaly, to make the area of the elliptic ſector *QFL*.

THIS angle or equation is determined by the following rule. Let R be an angle ſubtended by an arch equal in length to the radius of the circle, *viz.* 57,29578 degrees; and let A be an angle, whoſe ſine is to the radius as GM, the baſe of the ſegment, to FN the ſemi-tranſverſe axis; alſo let B be an arch in proportion to R, as the ſine of the double of the angle A to the radius: Then the equation for the ſegment will be equal to $A - \frac{1}{2}B$.

THIS equation is at its maximum, when the angle LFQ is a right angle; the baſe of the ſegment becoming equal to Ff, half the diſtance of the foci, and the angle A, being in this caſe half the angle FDS formed at the extremity of the leſſer axis, and ſubtended by FS, the diſtance of the foci; which is commonly called the greateſt equation of the center. And conſequently the arch B, in this caſe, is to R, as the ſine of the ſaid greateſt equation of the center, is to the radius. So that according to this rule, for the meaſure of the ſegment, it will follow, That

1. THIS greateſt equation is in proportion to the greateſt equation of *Bullialdus*, as found in the preceding article for the elliptic equant, nearly

nearly as three times the tranſverſe axis, to eight times the diſtance of the foci. Or, otherwiſe, the greateſt equation is to the angle deſigned by *R*, as twice the cube of the diſtance between the foci, to three times the cube of the tranſverſe axis. Either of which rules may be derived from the true angle, as before determined; or by taking $\frac{1}{3}$ of the rectangle of *GM* and *LM*, the baſe and height of the ſegment, for the meaſure of that ſegment.

So that in elliptic orbits nearly circular, this greateſt equation for the ſegment is in the triplicate proportion of the eccentricity.

2. This equation at any given angle *QFL*, is to the greateſt equation, in the triplicate proportion of the ordinate *OH* to the ſemi-tranſverſe; that is, nearly as the cube of the ſine of the mean anomaly joined to the double of *Bullialdus*'s equation to the cube of the radius. For the ſegment *GML*, which is proportional to the equation, is in the triplicate proportion of its baſe nearly; and the baſe is proportional to the ordinate *OH*, by the conſtruction.

But the ordinate *QH* (in a circle deſcribed upon the radius *FN*,) becomes the ſine of an angle, whoſe tan-

gent is to the tangent of the angle QFL, in the proportion of the tranſverſe axis to the conjugate; but the tangent of the ſame angle QFL, is to the tangent of the mean motion, anſwering to the area of the elliptic equant QFL in the ſame proportion. So that the ordinate OH is to the ſine of that angle of mean motion, in the duplicate of the ſaid proportion; and conſequently the ordinate OH, in the circle on the radius FN, is the ſine of an angle, nearly equal to the mean anomaly joined to the double of *Bullialdus*'s equation.

3. This equation adds to the mean motion in paſſing from the aphelion to the perihelion, and ſubducts in paſſing from the perihelion to the aphelion; as is evident from the tranſit of the point of interſection E round the periphery of the ellipſis Lfl.

In Sir *Iſaac Newton*'s rule (in the before-cited ſcholium to the 31ſt prop. 1ſt book,) the angle X anſwers to this equation for the ſegment; excepting that it is there taken in the triplicate proportion of the ſine of the mean anomaly, inſtead of the triplicate proportion of the ordinate OH. The error of this rule makes

III. The third part of the inequality, anſwering to the trilinear ſpace QKQ, being the difference of the elliptic ſector OFQ and the triangle OFK.

The ſector OQF is proportional to an angle, which is the difference of two angles, whoſe tangents are in the given proportion of the ſemi-latus rectum FB and the ſemi-tranſverſe FN, or in the duplicate proportion of the leſſer axis to the axis of the orbit. So that this ſector, when at a maximum, is as an angle, whoſe [illegible] is to the radius, as the difference of the latus rectum and tranſverſe to their ſum; or as the difference of the ſquares of the ſemi-axes to their ſum.

The triangle OFK is proportional to the rectangle of the co-ordinates OH and HF; that is, as the rectangle of the ſine OH and its coſine, in the circle on the radius FN; or as the ſine of the double of that angle, whoſe ſine is OH; that is, the double of the angle, whoſe tangent is to the tangent of the angle QFL, in the given ratio of the greater to the leſſer axis; or whoſe tangent is the tangent of the angle of mean motion anſwering to the elliptic ſector QFL, in the duplicate of the ſaid

ratio. But this triangle OFK, when at a maximum, makes an angle of mean motion, which is to the angle called R, as BN, half the difference between the latus rectum and tranſverſe axis, is to the double of the tranſverſe axis.

So that the ſector or triangle in orbits nearly circular, is always nearly equal to the double of *Bullialdus*'s equation.

The triangle and ſector being thus determined, the equation for the trilinear ſpace is accordingly determined. From what has been ſaid, it appears, that

1. This equation for the trilinear ſpace OKQ, is to that for the triangle OKF, in a ratio compounded of BN, the difference between the ſemi-tranſverſe and ſemi-latus rectum to the ſemi-latus rectum, and of the duplicate proportion of the ſine OH to the radius; or OKQ is to OKF, in a proportion compounded of the duplicate proportion of the diſtance of the foci to the ſquare of the leſſer axis, and the duplicate proportion of the ſine OH to the radius. For the trilinear figure OKQ and the triangle OKF, are nearly as OK and KH, which are in that proportion; and conſequently it holds in this proportion to the double of *Bullialdus*'s equation.

2. This

2. This equation, in different angles, is as the content under the ſine complement and the cube of the ſine. For the triangle *OKF*, is as the rectangle of the ſine and the ſine complement.

3. It is at a maximum, at an angle whoſe ſine complement is to the radius, as the ſquare of the greater axis is to the ſum of the ſquares of the two axes; which in orbits nearly circular, is about 60 degrees of mean anomaly.

4. In orbits of different eccentricities, it increaſes in the quadruplicate proportion of the eccentricity.

5. It obſerves the contrary ſigns to that for the elliptic equant, called *Bullialdus*'s equation; ſubducting from the mean motion in the firſt and third quadrants, and adding in the ſecond and fourth, if the motion is reckoned from the aphelion.

The uſe of theſe equations, in finding the place of a planet from the upper focus, will appear from the following rules, which are eaſily proved from what has been ſaid.

Let t be equal to CA the ſemi-tranſverſe, c equal to FC the diſtance of the center from the focus, b equal to CD the ſemi-conjugate, and R an angle ſubtended by an arch equal to the

the radius, *viz.* 57°. 17'. 44". 48"', or 57, 2957795 degrees. Take an angle $T = \frac{oo}{2tt} R$; $E = \frac{b}{2t} T$; $S = \frac{4c}{3b} T$.

The angle T be will the greateſt equation for the triangle OFK; the angle S will be the greateſt equation for the ſegment LMG; and the angle E will be the greateſt equation for the area $OKFL$. Which greateſt equations being found, the equations at any angle of mean anomaly, will be determined by the following rules.

Let M be the mean anomaly; and let τ be to T as the ſine of the angle $2M$ to the radius: In which proportion, as alſo in the following, there is no need of any great exactneſs, it being ſufficient to take the proportions in round numbers.

Take e to E as the ſine of $2M \pm 2\tau$ to the radius; and s to S as the cube of the ſine of $M \pm \tau$ to the cube of the radius.

Then the angle QFL is equal to $M+e+s$, in the firſt quadrant LN, or $M-e+s$, in the ſecond quadrant Nl, or $M+e-s$ in the third quadrant, or $M-e-s$ in the fourth quadrant.

Note, That the ſmall equation τ is always of the ſame ſign with the equation e; and

and in the caſe of the planets, always near the double of that equation.

THE angle *RFA* at the upper focus *F* being known, the angle *RSA* at the Sun in the other focus, is found by the common rule of Biſhop *Ward*; *viz.* the tangent of half the angle *RSA*, is to be to the tangent of half the angle *RFA*, always in the given proportion of the perihelion diſtance *SP* to the aphelion diſtance *SA*. How theſe equations are in the ſeveral eccentricities of the Moon's orbit, will appear by the following Table.

Eccentr.	E.	S.
	′ ″	″
0.040	1.23	09
0.045	1.45	13
0.050	2.09	17
0.055	2.36	23
0.060	3.06	30
0.065	3.38	38
0.070	4.14	47

To add one example; ſuppoſe the eccentricity 0.060, the mean anomaly 30°. The ſine of the double of the mean anomaly, that is, the ſine of 60 is to the radius, nearly as 87 to 100; whence, if the equation $E=3'.06''$, be divided in that propor-

proportion, it will produce 2′.40″ nearly, for the equation *e*: the ſine of *M* is, in this caſe, equal to $\frac{1}{2}$ the radius, the cube is $\frac{1}{8}$ of the cube of the radius; whence if the equation $S=30''$ be divided in the ſame proportion, it will produce near 4″ for the equation *s*. Therefore the angle *RFA*, which is $M+e+s$, will be 30°.2′.44″; and the half is 15°.1′.22″; wherefore if the tangent of this angle be diminiſhed, in the proportion of 1.06, the aphelion diſtance, to 94 the perihelion diſtance, it will produce the tangent of 13°.23′.13″; the double of which 26°.46′.26″, is the true anomaly or angle at the Sun *RSA*. And conſequently, the equation of the center is 3°.13′.34″ to be ſubducted, at 30 degrees mean anomaly.

When the place of a planet is found by this, or any other method; the place may be corrected to any degree of exactneſs by the common property of the equant, *viz.* that the rays are reciprocally in the duplicate proportion of the velocity about the center. For in this caſe, if there be a difference between the mean motion belonging to the angle aſſumed at the upper focus, and the given mean motion, the error of the angle aſſumed is to the difference, as the rectangle of the ſemi-axes to the rect-

angle

angle of the diſtances from the foci. But in orbits like thoſe of the planets, the rules as they are delivered above are ſufficient of themſelves without further correction.

POSTSCRIPT.

UPON reviewing theſe few ſheets after they were printed off, which happened a little ſooner than I expected, I fear the apology I have offered for delivering the propoſitions relating to the Moon's motion, in this rude manner, without giving any proof of them, or ſo much as mentioning the fundamental principles of their demonſtration, will ſcarcely paſs as a ſatisfactory one; eſpecially ſince there are among theſe propoſitions, ſome which, I am apt to think, cannot eaſily be proved to be either true or falſe, by any methods which are now in common uſe.

WHEREFORE to render ſome ſatisfaction in this article, I ſhall add a few words concerning the principles from whence theſe propoſitions, and others of the like nature

nature are derived: and alſo take the opportunity to ſubjoin a few remarks, which ought to have been made in their proper places.

Firſt, THERE is a law of motion, which holds in the caſe where a body is deflected by two forces, tending conſtantly to two fixed points.

WHICH is, *That the body, in ſuch a caſe, will deſcribe, by lines drawn from the two fixt points, equal ſolids in equal times, about the line joining the ſaid fixt points.*

THE law of *Kepler*, that bodies deſcribe equal areas in equal times, about the center of their revolution, is the only general principle, in the modern doctrine of centripetal forces.

BUT ſince this law, as Sir *Iſaac Newton* has proved, cannot hold, whenever a body has a gravity or force to any other than one and the ſame point; there ſeems to be wanting ſome ſuch law as I have here laid down, that may ſerve to explain the motions of the Moon and Satellites, which have a gravity towards two different centers.

IT follows as a corollary to the law here laid down, that if a body, gravitating towards two fixt centers, be ſuppoſed, for given ſmall intervals of time,

as moving in a plane paſſing through one of the fixt centers, the inclination of the ſaid plane, to the line joining the centers, will vary according to the area deſcribed; that is, if the area be greater, the inclination will be leſs; and if the area be leſs, the inclination will be greater, in order to make the ſolids equal.

This corollary, when rightly applied, will ſerve to explain the variation of the inclination of the plane of the Moon's orbit to the plane of the ecliptic.

And how extremely difficult it is to compute the variation of the inclination in any particular caſe, without the knowledge of ſome ſuch principle as this is, will beſt appear, if any one conſider the intricacy of the calculations, uſed in the corollaries to the 34 prop. of the third book of the *Principia*, in order to ſtate the greateſt quantity of variation, in that month, when the line of the nodes is in quadrature with the Sun, and that only in particular Numbers, whereby it is determined to be 2'.43".

Whereas, there is a plain and general rule in this caſe, which follows from what is laid down, though not immediately; namely, that the greateſt variation in the ſaid poſition of the Moon's orbit, is

to the mean inclination of the plane as the difference of the greateſt and leaſt areas deſcribed in the ſame time by the Moon about the earth, when in the conjunction and in the quarters to the mean area.

WHEREFORE, if S be to L, as the Sun's period to the Moon's period: The greateſt area is to the leaſt, as $\sqrt{SS+3LL}$ to S, or as $S+\frac{3LL}{2S}$ to S nearly, by what is ſaid on this article in the 29th page. So that the difference of areas is to the mean area, as $\frac{1}{2}LL$ to $SS+\frac{1}{4}LL$; and in the ſame proportion is the greateſt variation of the inclination of the plane in this month to the mean inclination, which agrees nearly with Sir *Iſaac*'s computation.

Secondly, THERE is a general method for aſſigning the laws of the motion of a body to and from the center, abſtractly conſider'd, from its motion about the center.

THE motion to and from the center is called by *Kepler* a Libratory motion; the knowledge of which ſeems abſolutely requiſite, to define the laws of the revolution of a body, in reſpect of the apſides of its orbit.

FOR the revolution of a body, from apſis to apſis, is performed in the time of

of the whole libratory motion; the apſides of the orbit being the extreme points, wherein the libratory motion ceaſes.

So that, according to this method, the motion of a body round the center, is not conſider'd as a continued deflection from a ſtreight line; but as a motion compounded of a circulatory motion round the center, and a rectilinear motion to or from the center.

Each of which motions require a proper *Equant*. Of the equant for the motion round the center, I have already given ſeveral examples. And in the caſe of all motions, which are governed by a gravity or force tending to a fixt point, the real orbit in which the body moves, is the equant for this motion. In all other caſes it is a different figure.

The *Equant* for the libratory motion, is a curve line figure, the areas of which ſerve to ſhew the time wherein the ſeveral ſpaces of the libration are performed.

Which figure is to be determined, by knowing the law of the gravity to the center: For the libratory force, to accelerate or retard the motion to or from the center, is the difference between the gravity of the body to the center, and the centrifugal force ariſing from

the circulatory motion. But the latter is always under one rule: For in all revolutions round a center, in any curve line, whether deſcribed by a centripetal force or not, the centrifugal force is directly in the duplicate proportion of the area deſcribed in a given ſmall time, and reciprocally in the triplicate proportion of the diſtance; which is an immediate conſequence of a known propoſition of Mr. *Huygens*. The like proportion alſo holds as to the centripetal force in all circular motions, from a known propoſition of Sir *Iſaac Newton*. But what is true of the centripetal force in circles, is univerſally true of the other force in orbits of any form.

So that by knowing the gravity of the body, ſince the other force is always known, the difference, which is the abſolute force to move the body to or from the center, will be known; and from thence the velocity of the motion, and the ſpace deſcribed in any given time, may be found, and the equant deſcribed. Theſe hints may be ſufficient to ſhew what the method is.

To add an example. If the gravity be reciprocally as the ſquare of the diſtance; the equant for the libratory motion, will be found to be an ellipſis

ſimilar

ſimilar to the orbit, whoſe longer axis is the double of the eccentricity; the center of the libratory motion, that is the place where it is ſwifteſt, will be in the focus; the time of the libration, through the ſeveral ſpaces, is to be meaſured by ſectors of the ſaid ellipſis, ſimilar to thoſe deſcribed by the body round the focus of the orbit; and the period of the libratory motion will be the ſame with the period of the revolution.

In any other law of gravity, the equant for the libratory motion, will either be of a form different from the orbit, or if it be of the ſame form, it muſt not be ſimilarly divided.

I may juſt mention, that the equant for the libratory motion, in the caſe of the Moon, is a curve of the third kind, or whoſe equation is of four dimenſions; but is to be deſcribed by an ellipſis, the center of the libration not being in the focus.

From this method of reſolving the motion, it will not be difficult to ſhew the general cauſes of the alteration of the eccentricity and inequality in the motion of the apogee. For when the line of apſides is moving towards the Sun, it may be eaſily ſhewn, that ſince the external force in the apſides, is then centri-

fugal, it will contribute to lengthen the ſpace and time of the libration; by lengthening the ſpace, it increaſes the eccentricity; and by lengthening the time of the libration, it protracts the time of the revolution to the apſis, and cauſes what is improperly called a motion of the apſis forward. But when the line of apſides is moving to the quadratures, the external force in the apſides, is at that time centripetal; which will contribute to ſhorten the ſpace and time of libration; and by ſhortening the ſpace will thereby leſſen the eccentricity, and by ſhortening the time of libration, will thereby contract the time of the revolution to the apſis; and cauſe what is improperly called a retrograde motion of the apſis.

I ſhall only add a few remarks, which ought to have been made in their proper places.

As to the motion of the Moon in the elliptic epicycle (page 9.) it ſhould have been mentioned, that there is no need of any accurate and perfect deſcription of the curve called an ellipſis, it being only to ſhew the elongation of the Moon, from the center of the epicycle; which doth not require any ſuch accurate deſcription.

It

It ſhould have been ſaid, that when the Moon is in any place of its orbit, ſuppoſe ſomewhere at *N*, in that half of the orbit which is next the Sun, it then being nearer the Sun than the Earth, has thereby a greater gravity to the Sun than the Earth: which exceſs of gravity, according to Sir *Iſaac Newton*'s method, conſiſts of two parts; one acting in the line *NV*, parallel to that which joins the Earth and Sun; and the other acting in the line *VB* directed to the Earth; and theſe two forces, being compounded into one, make a force directed in the line *NB*; which is in proportion to the force of gravity, as that line *NB* is to *TB* nearly. Wherefore, as there is a force conſtantly impelling the Moon ſomewhere towards the point *B*, this force is ſuppoſed to inflect the motion of the Moon into a curve line about that point; for the ſame reaſon as the gravity of it to the Earth, is ſuppoſed to inflect its motion into a curve line about the Earth: not that the Moon can actually have ſo many diſtinct motions, but the one ſimple motion of the Moon round the Sun is ſuppoſed to ariſe from a compoſition of theſe ſeveral motions. Fig. 1.

In

In the laſt article on the ſmall annual equations, (page 38.) theſe rules ought to have been added.

Let *Æ* be the equation of the Sun's center; *P* the mean periodical time of the node or apogee; *S* the mean ſynodical time of the Sun's revolution to the node or apogee: Then will $\frac{3S}{2P}$ *Æ* be the annual equation of the node or apogee, according as *S* and *P* are expounded.

The like rule will ſerve for the annual equation of the Moon's mean motion. If *S* be put for the Sun's period; *P* for the mean ſynodical period of the Moon to the Sun; and *L* for the Moon's period to the Stars: The annual equation of the Moon's mean motion will be $\frac{3LL}{2PS}$ *Æ*.

According to theſe rules when expounded, the equation for the node will be found to be always in proportion to the equation of the Sun's center, nearly as 1 to 13.

The equation of the apogee to the equation of the Sun's center, as 10 to 53.

And the equation of the Moon's mean motion to the ſame, as 8 to 77.

It may be throughout obſerved, that the propoſitions are in general terms, ſo as to ſerve, *mutatis mutandis*, for any other ſatellite, as well as the Moon.

There might have been ſeveral other obſervations and remarks made in many other places, had there been ſufficient time for it. But perhaps what I have already ſaid may be too much, conſidering the manner in which it is delivered.

ERRATUM.

Page 11. l. 11. for 8th, read 28th.

FINIS.

MVSEVM
BRITAN
NICVM

ERRATA.

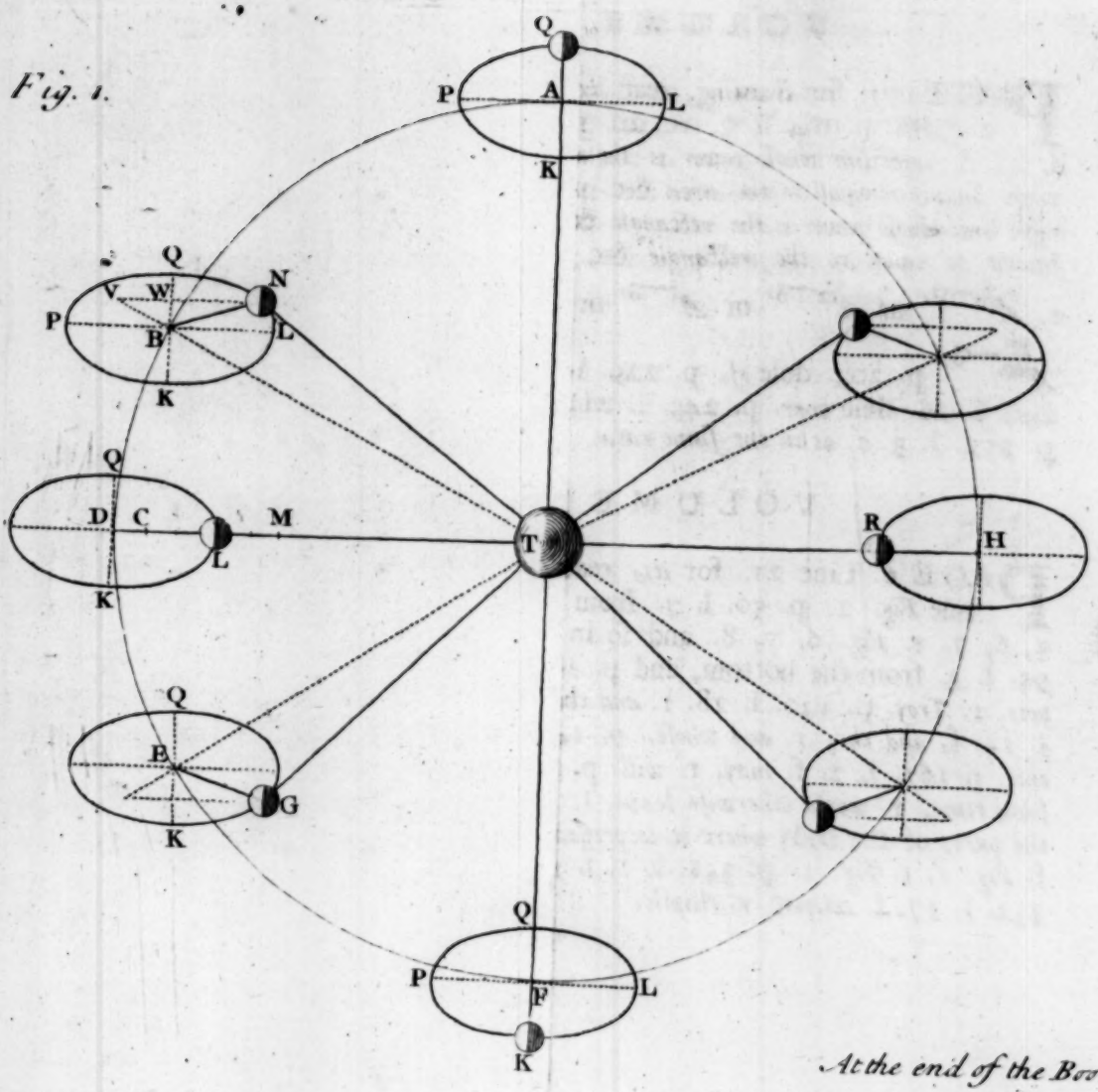
VOLUME I.

PAGE 117. for *drawing*, read *draw*. p. 156. f. $2AB$, r. $\frac{1}{2}AB$. p. 164. l. 7. 10, 20. p. 165. l. 9. p. 171. l. 27. f. *right line whoſe power is the area* &c. r. *right line whoſe ſquare is equal to the area* &c. p. 166. l. 25, 29. f. *right line whoſe power is the rectangle* &c. r. *right line whoſe ſquare is equal to the rectangle* &c. p. 192. l. 23, 24. r. $A^{\frac{1}{64}-3}$, or $A^{\frac{1}{16}-3}$, or $A^{\frac{1}{4}-3}$, or $A^{\frac{4}{9}-3}$, l. 29. r. $A^{\frac{nn}{mm}-3}$, p. 203. dele *if*. p. 229. l. penult. dele *is*. p. 240. l. 26. dele *near*. p. 243. l. 21. f. *when*. r. *becauſe*. p. 272. l. 3. r. *is in the ſame ratio*.

VOLUME II.

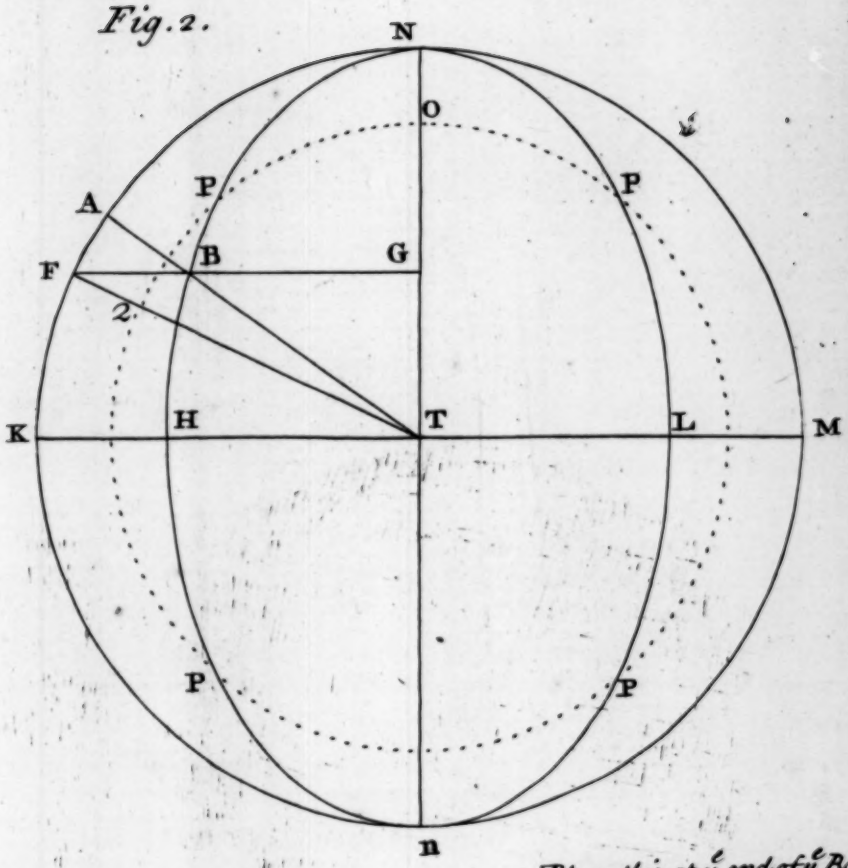
PAGE 6. Line 21. for *its*, read *the*. p. 24. l. 21. dele *Fig*. 2. p. 50. l. 7. from the bottom. f. *Fig*. 5, 6, 7. r. *Fig*. 6, 7, 8. and ſo in page following. p. 95. l. 4. from the bottom, and p. 100. l. 5. f. *Averdupois*, r. *Troy*. p. 130. l. 28. r. *and the water*, &c. p. 140. l. 14. f. *and the*, r. *and whoſe*. p. 144. l. ult. f. *but*, r. *this*. p. 161. l. 2. f. *may*, r. *will*. p. 169. l. 6. f. *leave for ſome time*, r. *would otherwiſe leave*. l. 21. r. *receding from the parts* of the body *where it is preſſed*, &c. p. 338. l. 7. f. *Fig*. 1. r. *Fig*. 2. p. 341. l. 1. f. *Fig*. 2. r. *Fig*. 3. p. 352. l. 17. f. *ecliptic*. r. *elliptic*.

Fig. 1.



At the end of the Book

Fig. 2.



Place this at y^e end of y^e Book

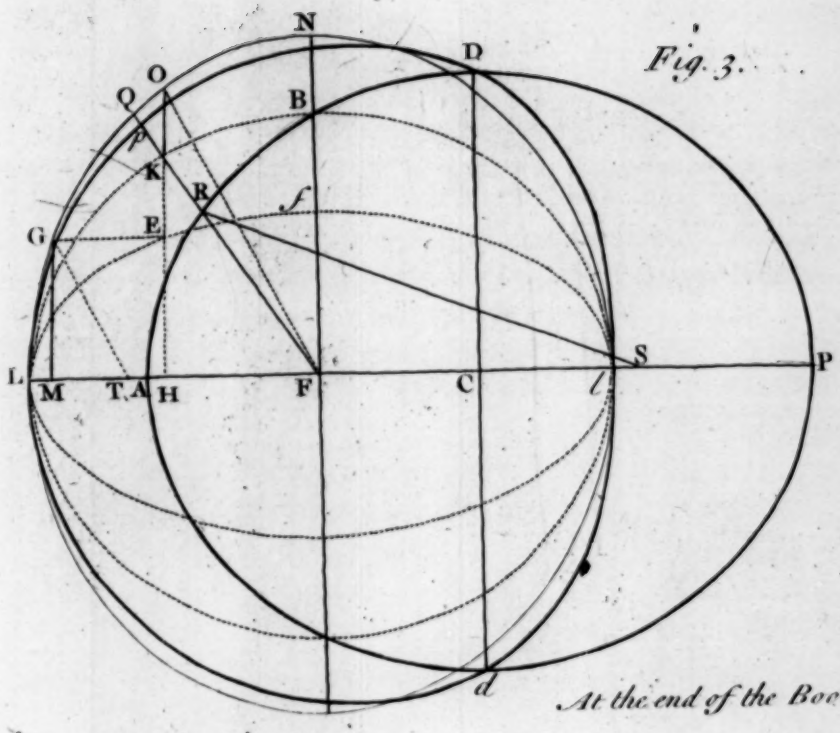


Fig. 3.

At the end of the Book